

INVESTIGATION OF NUCLEAR LEVELS BY NEUTRON SPECTROSCOPY. 1. NEUTRON
CROSS SECTIONS OF MOLYBDENUM ISOTOPES IN THE 7–15000 eV ENERGY RANGE

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Results of measurements of the total neutron cross sections and radiative-capture cross sections of molybdenum are presented. They were obtained with the use of a linear electron accelerator and a time-of-flight neutron spectrometer. The highest resolution in the measurements of the total cross sections was $0.006 \mu\text{sec}/\text{m}$. The parameters of a number of resonance levels are determined with allowance for interference between potential and resonance scattering. The strength function $\bar{\Gamma}_n^0/D$ for each of the isotopes is also determined. Several weak levels ascribed to the p wave have been detected. The question of the identical position of levels belonging to different isotopes is discussed.

1. INTRODUCTION

DATA on the position and parameters of nuclear levels obtained thus far by neutron spectrometry techniques largely refer to a natural mixture of isotopes. However, the investigation of a number of characteristics dealing especially with the position of the levels and the distribution of the distances between them require the study of separated isotopes by neutron spectroscopy. Knowledge of the physical characteristics of separated isotopes is also necessary in a number of cases for the practical use of materials in nuclear power.

In the present paper we consider the results of the study of the total neutron cross sections and the radiative-capture cross sections for separated isotopes of molybdenum in the 7–15 000 eV interval. The choice of molybdenum for this study was based on the following considerations: a) Molybdenum has a large number of even-even isotopes, for which the relation between the position of the levels should have a particularly simple character; b) the atomic weights of molybdenum isotopes lie in a region where, according to the optical model of the nucleus, a large number of resonances connected with the capture of p neutrons should be expected; c) the data on molybdenum available in the literature^[1,2] were obtained in 1955–1956 with poor resolution; d) molybdenum is a construction material used in nuclear power.

2. APPARATUS AND SAMPLES

The measurements were made with a neutron spectrometer^[3] with a linear electron accelerator

as a pulsed source of neutrons.¹⁾

The length of the electron pulse was $0.6 \mu\text{sec}$, the channel width of the time analyzer was $0.577 \mu\text{sec}$, and the pulse repetition rate was 100 ppm.

The total cross section measurements were made with a flight distance of 109.14 m. The samples were placed 5.5 m from the target. The collimator diameter at the position of the samples (depending on the sample dimensions) was 42 or 62 mm. The neutron beam was filtered by a layer of B^{10} of thickness $0.11 \text{ g}/\text{cm}^2$.

The neutron detector consisted of 230 proportional counters placed in a common aluminum tank filled to atmospheric pressure with BF_3 gas enriched to 80% B^{10} . The detector area was 2500 cm^2 and the detector thickness in the direction of the neutron beam was 17.6 cm; the maximum delay time for the occurrence of the counter pulse relative to the moment of neutron capture by boron was $0.5 \mu\text{sec}$. A special nonlinear circuit^[5] was used to apply the initial γ -ray pulse.

Information from the apparatus located in the neutron beam was fed to a 2048-channel analyzer with an electrostatic memory tube and a magnetic drum.

The beam spectrum, after subtraction of background but without correction for the detector efficiency, was approximated to an accuracy of better than 5% for the entire energy interval by the following function (E in eV):

¹⁾Preliminary results of the measurements were reported at a working conference on slow-neutron physics at Dubna.^[4]

$$N(E) = \text{const} (140/E^{1.3} + E/10^8).$$

With the diameter of the collimator aperture equal to 62 mm near the samples, the counting rate in the time-analyzer channel of width 0.577 μsec was $\sim 50 \text{ h}^{-1}$.

The radiative-capture cross-section measurements were made with a flight distance of 15.1 m. The γ -ray detector consisted of a NaI(Tl) crystal. The apparatus for the radiative-capture cross-section measurements has been described in detail in [6].

The separated molybdenum isotopes in the form of metallic or trioxide powders of molybdenum were poured into aluminum containers with an internal diameter of 50 or 70 mm and a plane-parallel bottom and cover (wall thickness 0.5–0.8 mm), packed down, and rolled in. We prepared 13 samples with an abundance of at least 75% for the principal isotope. The samples of natural molybdenum were prepared from metallic slabs 1 mm thick. The molybdenum purity in all the samples was at least 99.9%.

3. ANALYSIS OF THE RESULTS

The total neutron cross sections were determined from the attenuation of the neutron beam by the sample of the investigated material and were calculated from the usual formula:

$$\sigma_t = \frac{1}{n} \ln \frac{1}{T(E)},$$

where n is the number of atoms in 1 cm^2 of the substance and $T(E)$ is the sample transmission.

The experimental data on the total cross sections were analyzed by the area method with the aid of the well-known Hughes diagrams [7] permitting the determination of the resonance parameters with allowance for the Doppler effect and the resolution of the neutron spectrometer.

When samples of sufficiently different thickness were available, we determined the values of the radiative (Γ_γ) and neutron (Γ_n) widths. In the high-energy region, where $\Gamma_n \gg \Gamma_\gamma$, it was sufficient to use measurements with one sample; finally, in some cases for which the measured values of the areas on the transmission curves lay in the region of the Hughes diagrams where accurate values of the width could not be obtained, the analysis was made with an assumed value of the radiation width.

Additional difficulties in the analysis arose when interference between resonance and potential scattering had an appreciable effect on the shape of the resonance peak, and consequently, on the

area under the dip in the transmission curve, i.e., for neutron widths which make up an appreciable part of the total width. In these cases we used a rather simple method for preliminary analysis of the transmission curve which permitted us to take into account all factors affecting the behavior of the cross section in the resonance region, including interference, and which appreciably increased the accuracy of the determination of the resonance parameters.

The idea behind this method was to bring the resonance transmission, the deformed interference, and the overall resolving power of the apparatus into symmetric form. The sample transmission in the resonance region was, as usual, considered as a function of a dimensionless parameter

$$x = 2(E - E_0)/\Gamma$$

(E is the neutron energy, and E_0 and Γ are the position and width of the resonance level). If the energy E_0 is known, then we find the points $T(x)$ and $T(-x)$ on the transmission curve and introduce the quantity

$$T^* = \sqrt{T(x) \cdot T(-x)}.$$

The interference term is an odd function of x and vanishes here, so that by applying Hughes' method to T^* we can determine the resonance parameters.

With the use of the symmetrization procedure, the errors in the determination of the resonance parameters could, in principle, depend on the overall resolution of the apparatus and the uncertainty in the resonance energy. Numerical estimates show, however, that, in the interval of resonance parameters, sample thicknesses, and resolving power of practical importance, the error introduced by the symmetrization of the transmission curve does not exceed the errors of the area method. The method employed for data reduction has been described in detail by one of the authors. [8]

Data on the radiative-capture cross section were used in conjunction with the results of the total cross-section measurements for the identification of the levels and for the determination of the parameters of the weak levels not appearing in the total cross-section measurements. Here we used a method similar to that described by Kim et al. [9]. The total number of detector counts due to the capture of neutrons in a sample with an energy within the limits of the investigated resonance is given by the relation

$$\Sigma N_\gamma = CN(E) \sqrt{E} \varepsilon \frac{\Gamma_\gamma}{\Gamma} \int (1 - e^{-n\sigma t}) dE = C\varepsilon N(E) \sqrt{E} \frac{\Gamma_\gamma}{\Gamma} A_T,$$

Table I

Iso- tope	E_0 , eV	Γ_γ , MeV	Γ_n , eV	Γ_n^0 , MeV	$g\Gamma_n^1$, MeV	Method of deter- mining parameters
1	2	3	4	5	6	7
Mo ⁹²	346.6±1.0		0.028±0.004	1.5±0.2		$\Gamma_\gamma = 260 \pm 80$ MeV
	2040±30					
	3170±26		12.0±1.0	210±20		$\Gamma_n \gg \Gamma_\gamma^*$
	11610±220		64.7±13.0	600±120		$\Gamma_n \gg \Gamma_\gamma$
	13920±260		61.7±12.0	520±100		$\Gamma_n \gg \Gamma_\gamma$
Mo ⁹⁴	16660±300		42.7±9.0	330±70		$\Gamma_n \gg \Gamma_\gamma$
	1519±9		1.55±0.27	40±7		$\Gamma_\gamma = 260 \pm 80$ MeV
	1688±10		0.75±0.2	18±5		$\Gamma_\gamma = 260 \pm 80$ MeV
Mo ⁹⁵	5380±60		15.3±2.0	209±30		$\Gamma_n \gg \Gamma_\gamma$
	45.0±0.4	210±60	0.175±0.010	26.1±1.5		[12]**
	110.8±2		(0.16±0.03)·10 ⁻³	0.0155±0.0030	78	$\Gamma_\gamma \gg \Gamma_n$
	118.3±2		(0.125±0.025)·10 ⁻³	0.0113±0.0022	52	$\Gamma_\gamma \gg \Gamma_n$
	159.0±0.3	260±50	(0.0136±0.002	1.1±0.2		[12]
	220±4		(1.04±0.20)·10 ⁻³	0.070±0.014	180	$\Gamma_\gamma \gg \Gamma_n$
	249±5		(0.35±0.07)·10 ⁻³	0.022±0.004	50	$\Gamma_\gamma \gg \Gamma_n$
	267.3±0.6		(0.82±0.16)·10 ⁻³	0.050±0.010	105	$\Gamma_\gamma \gg \Gamma_n$
	335±10		(2.05±0.45)·10 ⁻³	0.112±0.025	185	$\Gamma_\gamma \gg \Gamma_n$
	358.2±1.0		0.4±0.05	21.0±2.5		$\Gamma_\gamma = 260 \pm 80$ MeV
	480±10		(5.4±1.1)·10 ⁻³	0.25±0.05	300	$\Gamma_\gamma \gg \Gamma_n$
	554.7±1.5	260±50	0.12±0.06	5±3		[12]
	635±2					
	679.3±2.5		0.80±0.12	30±4.6		$\Gamma_\gamma = 260 \pm 80$ MeV
	768±20					
	896±4		0.48±0.1	16.3		$\Gamma_\gamma = 260 \pm 80$ MeV
	-1022±5					
-1147±5						
-1413±7						
-7400±200						
Mo ⁹⁶	114.5±2		9.0±1.0	100±11		$\Gamma_n \gg \Gamma_\gamma$
	-131.1±0.2	260±50	(0.75±0.25)·10 ⁻³	0.070±0.023	340	$\Gamma_\gamma \gg \Gamma_n$
	-2371±17		0.200±0.015	17.3±1.3		[12]
	-3300±30		5±0.5	10.3±1		$\Gamma_n \gg \Gamma_\gamma$
	-3660±30		14.8±2.0	260±35		$\Gamma_n \gg \Gamma_\gamma$
Mo ⁹⁷	70.9±0.15	330±80	0.0175±0.0017	2.06±0.20		[12]
	210±6		(0.78±0.16)·10 ⁻³	0.054±0.011	145	$\Gamma_\gamma \gg \Gamma_n$
	230±6		(2.05±0.45)·10 ⁻³	0.135±0.030	330	$\Gamma_\gamma \gg \Gamma_n$
	267.3±0.6					
	285.4±0.7		0.064±0.012	3.8±0.7		$\Gamma_\gamma = 260 \pm 80$ MeV
	311.7±1.0		(8.0±2.0)·10 ⁻³	0.45±0.11	800	$\Gamma_\gamma \gg \Gamma_n$
	396.7±1.2	260±80	0.080±0.040	4±2		[12]
	505±3		0.53±0.1	23±4		$\Gamma_\gamma = 260 \pm 80$ MeV
	576±2	260±80	0.67±0.13	28±6		[12]
	695±20					
	788±3		2.15±0.20	76±7		$\Gamma_\gamma = 260 \pm 80$ MeV
1255±6		3±0.3	8.1±0.8		$\Gamma_\gamma = 260 \pm 80$ MeV	
1519±9						
2000±20						
Mo ⁹⁸	12±0.1	200±6	(0.07±0.005)·10 ⁻³	(0.02±0.0014)		analysis from shape
	429.7±1.3		0.070±0.010	3.4±0.5	900	$\Gamma_\gamma = 260 \pm 80$ MeV
	467.0±1.4	260±50	0.70±0.08	32±4		two-sample technique
	612±2		0.035±0.005	1.4±0.2		$\Gamma_\gamma = 260 \pm 80$ MeV
	816±3		0.080±0.015	2.8±0.5		$\Gamma_\gamma = 260 \pm 80$ MeV
	1103±5					
	1519±9		1.40±0.21	36.0±5.4		$\Gamma_\gamma = 260 \pm 80$ MeV
	2550±20		1.44±0.20	29±3		$\Gamma_\gamma = 260 \pm 80$ MeV
	3300±30		4.0±0.3	70±5		$\Gamma_n \gg \Gamma_\gamma$
	9000±130		50±5	30±50		$\Gamma_n \gg \Gamma_\gamma$
Mo ¹⁰⁰	99.5±0.2	260±80	(0.56±0.22)·10 ⁻³	0.056±0.022	320	$\Gamma_\gamma \gg \Gamma_n$
	363±1		1.00±0.12	52±6		[12]
	1067±5		0.42±0.08	13±2.5		$\Gamma_\gamma = 260 \pm 80$ MeV
	1255±6		0.315±0.030	9.0±0.9		$\Gamma_\gamma = 260 \pm 80$ MeV
	1668±10		1.15±0.10	28±3		$\Gamma_\gamma = 260 \pm 80$ MeV
	1936±12		3.34±0.20	76±5		$\Gamma_n \gg \Gamma_\gamma$

* $\Gamma_\gamma \gg \Gamma_n$ are the weak levels which can be ascribed to the p wave. In these cases we have introduced the parameters $g\Gamma_n$ and $g\Gamma_n^0$ obtained from measurements of the capture cross section under the assumption that $\Gamma_\gamma = 260 \pm 80$ MeV.

**In these cases the values of the parameters were taken from [11] and the position of the resonances is based on our measurements.

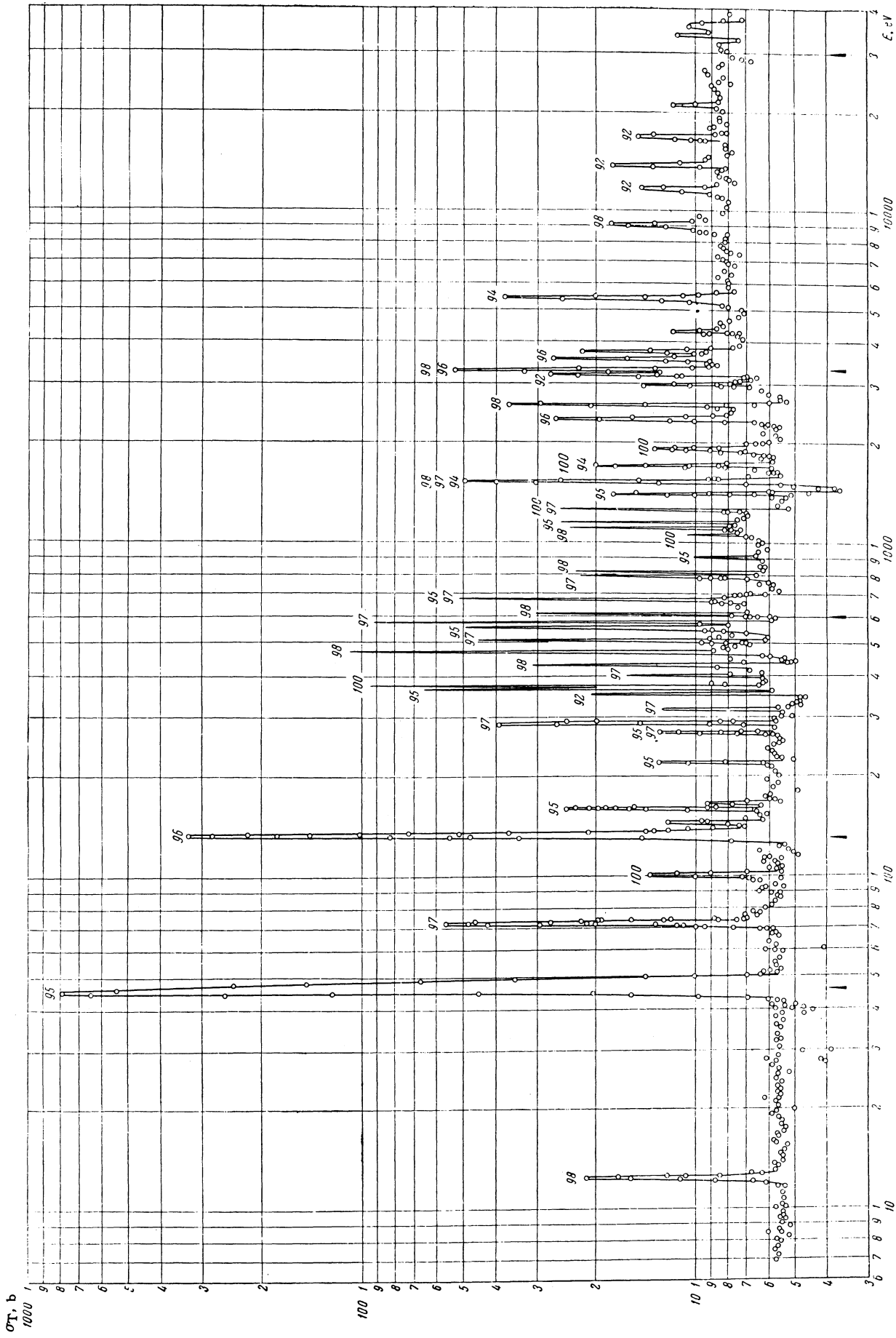
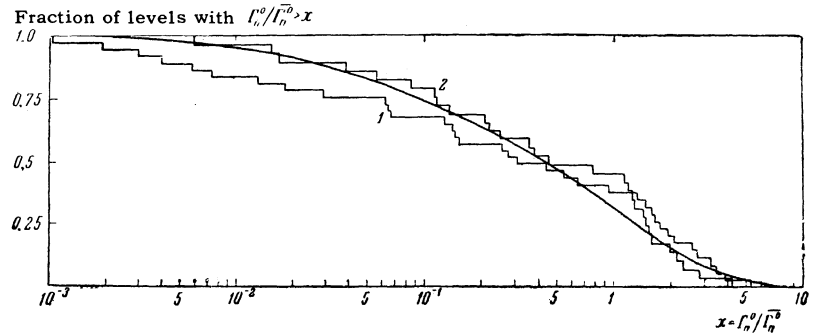


FIG. 1. Total neutron cross section for natural molybdenum.

FIG. 2. Neutron width distribution.



where ϵ is the recording efficiency of the detector for neutron capture, N_γ are the counts in the time-analyzer channel, $CN(E)\sqrt{E}$ is the total intensity of neutrons falling on the sample during the measurements ($C = \text{const}$), and A_T is the area in the dip of the transmission curve for the investigated sample.

Applying this relation to the resonance whose parameters are known from the total cross-section measurements, we can determine the quantity $C\epsilon$. For this purpose, we used the parameters of the levels of Mo^{95} at 159 eV, of Mo^{97} at 70.9 and 285 eV, and of Mo^{96} at 131 eV. Knowing the magnitude of $C\epsilon$ and taking Γ_γ to be the same for all levels, we can determine the value of A_T for the weak levels ($\Gamma_\gamma/\Gamma \approx 1$) from the radiative-capture measurements, after which the value of $g\Gamma_n$ could readily be determined from the calculated curves of A_T/Δ as functions of $n\sigma_0$ and Γ/Δ (see reference 10). The quantity $C\epsilon$ and the parameters $g\Gamma_n$ determined in this way for the weak levels depend very weakly on the value taken for Γ_γ , namely, $\Gamma_\gamma = (260 \pm 80)$ MeV.

4. RESULTS AND DISCUSSIONS

The results of the measurements and the values of the level parameters obtained from them are given in Table I.

Besides the molybdenum levels shown in Table I, we also observed levels lying at 967 ± 4 , 1200 ± 6 , 1582 ± 9 , 1761 ± 10 , and 3760 ± 34 eV, which we were unable to assign to definite isotopes.

Figure 1 shows the behavior of the total cross section for natural molybdenum.²⁾ The resonances detected only molybdenum in good agreement with our data. We take this opportunity to thank the authors for sending the preprint.

In Fig. 2 (step-like curve 1) we show the dis-

tribution of the neutron widths for 37 levels of Mo^{95} , Mo^{97} , Mo^{98} , and Mo^{100} . The Mo^{95} level at 7400 eV and the Mo^{98} level at 9000 eV have been omitted. The smooth curve corresponds to the Thomas-Porter distribution for one degree of freedom. We note that there is a large deviation of the experimental points from this curve, especially in the region of small neutron widths.

The agreement is considerably improved if we eliminate the weakest levels from the experimental data (Mo^{95} at 110.8, 118.3, 220, 249, 267.3 eV; Mo^{97} at 230 eV, Mo^{98} at 12 eV, and Mo^{100} at 99.5 eV) by considering them to correspond to the capture of p neutrons. The data for the remaining 29 levels are also shown in Fig. 2 (step-like curve 2).

Table II lists the mean distances D between the s levels for each isotope.

For Mo^{95} and Mo^{97} we list the mean distances calculated for a one-spin state under the assumption that the density of the two systems of levels is the same.

Since the binding energies of the nucleon for molybdenum isotopes differ by approximately 2 MeV, we also calculated for the different isotopes the quantities D^* reduced to one excitation energy of 6.5 MeV (see Table II). The quantity D^* proved to be largest for Mo^{92} , which has a closed neutron shell and decreases with increasing atomic weight.

In the last row of Table II we list the values of the strength function for s-waves $S_0 = \overline{\Gamma_n^0}/D$ for the isotopes studied. It should be noted that the obtained data, which have large errors, lie appreciably above the results of other experiments^[13] for this region of atomic weights and are in better agreement with the predictions of the optical model of the nucleus.

For the case of Mo^{95} we also calculated the strength function S_1 for p-waves and obtained the value $(6.3 \pm 4) \times 10^{-4}$. The calculations were made with the formula

$$S_1 = \frac{1}{2L+1} \frac{\langle g\Gamma_n^1 \rangle}{D_1} = \frac{1}{3\Delta E} \sum g\Gamma_n^1,$$

²⁾After submission of this paper, we received a preprint from Saclay^[12] which gave data on the position of the resonance levels for natural molybdenum in good agreement with our data. We take this opportunity to thank the authors for sending the preprint.

Table II

Isotope mass number	92	94	95	96	97	98	100
D , eV	2400 ± 1000	1000 ± 400	220 ± 50	1000 ± 500	220 ± 50	270 ± 80	430 ± 150
D^* , eV	7500	1460	1170	1120	570	83	67
$S_0 \cdot 10^4$	1.0 ± 0.7	0.5 ± 0.4	0.55 ± 0.4	0.8 ± 0.7	0.6 ± 0.3	0.8 ± 0.5	0.9 ± 0.6

where $g\Gamma_n^1 = g\Gamma_n E^{-3/2} E_R$, $\Delta E = 350$ eV is the energy interval in which the p levels were observed, E_R is the energy at which the neutron wavelength is equal to the radius of the nucleus, and Γ_n^1 is the reduced neutron width for p levels.

Attention is drawn to the fact that the positions of the resonances belonging to different isotopes of molybdenum coincide. Thus we note the agreement between the levels of Mo^{95} and Mo^{98} at 470 ± 3 eV, Mo^{95} and Mo^{97} at 685 ± 6 eV, Mo^{98} and Mo^{100} at 1085 ± 15 eV, Mo^{97} and Mo^{100} at 1255 ± 7 eV, Mo^{94} and Mo^{100} at 1680 ± 10 eV and also three-fold agreement for the levels of Mo^{92} , Mo^{95} , and Mo^{100} at 355 ± 10 eV and Mo^{94} , Mo^{97} , and Mo^{98} at 1520 ± 10 eV. The presently obtainable resolution and statistical accuracy do not permit the reliable resolution of all cases of coinciding levels belonging to different isotopes.

Coinciding levels belonging to different isotopes have also been observed earlier, for example, for the case of nickel isotopes by Newson et al.^[14] It is of interest to estimate the probability of a random coincidence of levels of different isotopes. The expected number α_2 of chance coincidences of two levels in an energy interval ΔE can be estimated from the formula

$$\alpha_2 = E\Delta E \sum_{i \neq k}^m \frac{1}{D_i D_k}$$

and for a three-fold coincidence we have

$$\alpha_3 = E\Delta E^2 \sum_{i \neq k \neq l}^m \frac{1}{D_i D_k D_l},$$

where E is the considered energy interval; D_i , D_k , D_l are the mean distances between levels for the i -th k -th, and l -th isotope; and m is the number of isotopes considered.

In our case $m = 7$, and, setting $E = 2 \times 10^3$ eV and $\Delta E = 20$ eV, we obtain $\alpha_2 = 4.5$ and $\alpha_3 = 1.4$. Eleven double and two triple coincidences have been observed experimentally in the given energy interval. The available experimental data are insufficient for an unambiguous answer to the question of whether there exists an effect involving the coincidence of levels of different isotopes.

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