REGGE POLES IN QUANTUM FIELD THEORY

Yu. M. MALYUTA

Physics Institute, Academy of Sciences, Ukrainian S.S.R.

Submitted to JETP editor November 3, 1962

J. Exptl. Theoret. Phys. (U.S.S.R.) **44**, 1317—1319 (April, 1963)

The condition of unitarity and the dispersion relations are used to obtain the asymptotic behavior of the meson-meson scattering amplitude in the region of large momentum transfer.

1. INTRODUCTION

RECENTLY a new method has been introduced in quantum field theory which enables one to investigate bound states and resonances [1-4]. This method is based on the study of the asymptotic behavior of scattering amplitudes in the region of large momentum transfer.

From the point of view of the new method the basic objects of investigation in quantum field theory must be the Regge pole trajectories. The explicit calculation of Regge trajectories presents considerable difficulties and requires the use of approximation methods. At present there exist two approaches: 1) solution of the Bethe-Salpeter equations [5,6] and 2) summation of diagrams by the method of the renormalization group [7]. In this paper an approximation method is presented based on unitarity and dispersion relations.

2. THE CONDITION OF UNITARITY IN THE STEPWISE APPROXIMATION

We restrict ourselves to a discussion of the theory of neutral scalar mesons with contact interaction $\lambda\varphi^3$. By applying Cutkosky's rules [8] to the diagrams shown in Fig. 1 we write the condition of unitarity [1)]:

$$A_{13}(sz) = \frac{1}{16\pi^2}\sqrt{\frac{q^2}{q^2+m^2}}\int dz_1 \times \int dz_2 \frac{1}{\sqrt{k(zz_1z_2)}} A_3^{\text{II}}(sz_2)\, A_3(sz_1);$$

$$A(sz) = A(s+i\varepsilon, z), \qquad A^{\text{II}}(sz) = A(s-i\varepsilon, z),$$

$$k(zz_1z_2) = z^2 + z_1^2 + z_2^2 - 1 - 2zz_1z_2, \quad s = 4(q^2+m^2),$$

$$t = 2q^2(z-1), \quad t_1 = 2q^2(z_1-1), \; t_2 = 2q^2(z_2-1), \qquad (1)$$

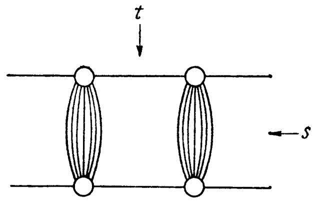

FIG. 1

where s is the square of the total energy, − t is the square of the transferred momentum, m is the meson mass. Integration in (1) is carried out over the region

$$z > z_1z_2 + \sqrt{(z_1^2-1)(z_2^2-1)}.$$

On substituting (1) into the dispersion relation

$$A_1(sz) = \frac{1}{\pi}\int dz' \frac{A_{13}(sz')}{z'-z}$$

and integrating over z′ we obtain

$$A_1(sz) = -\frac{1}{32\pi^3}\sqrt{\frac{q^2}{q^2+m^2}}\int dz_1 \int dz_2 \frac{1}{\sqrt{k(zz_1z_2)}} \times \ln\frac{z - z_1z_2 + \sqrt{k(zz_1z_2)}}{z - z_1z_2 - \sqrt{k(zz_1z_2)}} A_3^{\text{II}}(sz_2)\, A_3(sz_1). \qquad (2)$$

Noting that

$$A_1(sz) = -\tfrac{1}{2} i\,[A(sz) - A^{\text{II}}(sz)],$$

we rewrite (2) in the form

$$A(sz) = A^{\text{II}}(sz) - \frac{i}{16\pi^3}\sqrt{\frac{q^2}{q^2+m^2}}\int dz_1 \int dz_2 \frac{1}{\sqrt{k(zz_1z_2)}} \times \ln\frac{z - z_1z_2 + \sqrt{k(zz_1z_2)}}{z - z_1z_2 - \sqrt{k(zz_1z_2)}} A_3^{\text{II}}(sz_2)\, A_3(sz_1).$$

We make an approximation analogous to that which is made in obtaining equations of the Bethe-Salpeter type [9], viz., we set

$$A^{\text{II}}(sz) = -\frac{\lambda^2}{2q^2(z-1) - m^2},$$

$$A_3^{\text{II}}(sz_2) = \pi\lambda^2\delta\,[2q^2(z_2-1) - m^2].$$

[1)]In this paper we use an approximation which does not take into account the contribution to the scattering amplitude of the singularities of the invariant u.

We then obtain

$$A(sz) = -\frac{\lambda^2}{2q^2(z-1)-m^2} - \frac{i\lambda^{2\bullet}}{32\pi^2\sqrt{q^2(q^2+m^2)}}\int dz_1 \frac{1}{\sqrt{k(zz_1z_2)}} \times \ln\frac{z-z_1z_2+\sqrt{k(zz_1z_2)}}{z-z_1z_2-\sqrt{k(zz_1z_2)}}A_3(sz_1);$$

$$z_2 = 1 + m^2/2q^2. \qquad (3)$$

Equation (3) is the unitarity condition in the stepwise approximation.

3. ASYMPTOTIC BEHAVIOR FOR LARGE MOMENTUM TRANSFER

As $t \to \infty$ $(z \to \infty)$ Eq. (3) assumes the following form

$$A(st) = -\frac{\lambda^2}{t} - \frac{i\lambda^2}{8\pi^2\sqrt{s(s-4m^2)}}\int dt_1 \frac{1}{t}\ln(t_1-t)A_3(st_1). \qquad (4)$$

We multiply (4) by t and differentiate with respect to t. Then, taking into account the dispersion relation

$$A(st) = \frac{1}{\pi}\int dt_1 \frac{A_3(st_1)}{t_1-t},$$

we obtain the simple differential equation

$$t\frac{d}{dt}A(st) = \alpha(s)A(st);$$
$$\alpha(s) = -1 + i\lambda^2/8\pi\sqrt{s(s-4m^2)}. \qquad (5)$$

Equation (5) has the solution

$$A(st) = \beta(s)\,t^{\alpha(s)}.$$

The dependence of Re $\alpha(s)$ on s (the Regge trajectory) for m = 1 and $\lambda^2 = 25$ is shown [2)] in

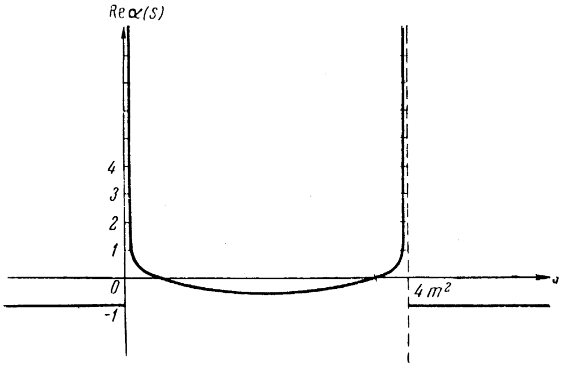

FIG. 2

Fig. 2. It can be seen from Fig. 2 that there exists a family of bound states [dRe $\alpha(s)/ds > 0$ for $2m^2 < s < 4m^2$]; there are no resonances and no "ghosts."

[2)] In the construction of the Regge trajectory $\alpha(s)$ is continued into the region $s < 4m^2$.

[1] R. Blankenbecler and M. Goldberger, Phys. Rev. **126**, 766 (1962).

[2] Chew, Frautschi and Mandelstam, Phys. Rev. **126**, 1202 (1962).

[3] Frautschi, Gell-Mann and Zachariasen, Phys. Rev. **126**, 2204 (1962).

[4] V. N. Gribov, JETP **42**, 1260 (1962), Soviet Phys. JETP **15**, 873 (1962).

[5] Bertocchi, Fubini and Tonin, Nuovo cimento **25**, 626 (1962).

[6] B. Lee and R. Sawyer, Phys. Rev. **127**, 2266 (1962).

[7] Arbusov, Logunov, Tavkhelidze, and Faustov, Phys. Letters **2**, 150 (1962).

[8] R. Cutkosky, J. Math. Phys. **1**, 429 (1960).

[9] S. Edwards, Phys. Rev. **90**, 284 (1953).

Translated by G. Volkoff
211