EFFECTS OF INELASTIC PROCESSES ON ELECTRON-ELECTRON AND ELECTRON-POSITRON SCATTERING

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By means of dispersion relations it is shown that the effect of inelastic processes, including the production of strongly interacting particles, is small and cannot significantly affect the analysis of experiments made to test the validity of electrodynamics by the use of colliding beams.

1. Experiments with colliding beams of electrons and positrons at high energies, which are opening up new possibilities in the physics of unstable particles, are often regarded as a possible way of testing experimentally the validity of the laws of quantum electrodynamics. One of the processes most susceptible to analysis is the elastic scattering of electrons by positrons and of electrons by electrons (e^-e^+, e^-e^-), or, in general, ee scattering. The analysis of future experimental data on elastic ee scattering from the point of view of the validity of electrodynamics must, however, be preceded by a study of the inelastic processes which occur simultaneously with the elastic ones, and an investigation of the effect of the inelastic processes on the elastic processes.

Here we would like to examine what effects on the real part of the amplitude for elastic ee scattering are produced by certain processes which are not usually considered in the framework of the electrodynamics of electrons and photons. Because there are no experimental data, in estimating the effects of the inelastic processes we shall take for the cross sections of these processes the expressions found for them in the one-photon approximation.

The calculation of the actual effects of the inelastic processes is made below by means of the method of dispersion relations (d. r.). The complete d. r. for ee scattering are not considered. As a first step we include the contribution to the real part of the scattering amplitude from dispersion terms of the form

$$\delta D(\omega_0) = \frac{k_0^2}{4\pi^2}\int\limits_{\omega_t}^{\infty}\frac{d\omega}{k}\left[\frac{\sigma_+(\omega)}{\omega-\omega_0}+\frac{\sigma_-(\omega)}{\omega+\omega_0}\right], \quad (1)$$

where, for example, for e^+e^- (e^-e^-) scattering σ_+ is the total cross section for the e^+e^- (e^-e^-) interaction, and σ_- is that for the e^-e^- (e^+e^-) interaction. The nondispersion part of the amplitude is to be added to the expression (1), which is the d. r. for forward scattering in a system in which one of the particles is at rest, and which near threshold gives the effect of inelastic interactions for arbitrary angles.

2. We consider here effects near threshold which are due to the production of muon pairs in positron-electron collisions:

$$e^+ + e^- \to \mu^+ + \mu^-. \quad (2)$$

Near its threshold the existence of process (2) leads to an effect [1,2] in e^+e^- scattering, and we shall estimate the magnitude and energy width of this effect. In electron-electron scattering in the range of energies close to the threshold of the reaction (2) the derivative of the cross section with respect to the energy of course does not become infinite at the threshold. In the framework of the d. r. method, however, the process (2) leads through its contribution to the cross-symmetric process to some effects in e^-e^- scattering, and we shall also obtain estimates of these effects.

An expression for the cross section of process (2) has been obtained by Berestetskiĭ (cf. e. g., discussion in [3]):

$$\sigma = \frac{2\pi}{3}r_0^2\frac{m}{\omega}\left(1+\frac{\omega_t}{2\omega}\right)\left(1-\frac{\omega_t}{\omega}\right)^{1/2}, \quad (3)$$

where $\omega_t \approx 2m_\mu^2/m = 4\times10^{10}$ eV is the threshold of reaction (2) in the system where one of the electrons was at rest before the collision; m_μ and m are the masses of the muon and electron; and r_0 is the classical electron radius.

Substitution of Eq. (3) in Eq. (1) gives as the expression for the dispersion part of the amplitude of e^+e^- scattering caused by process (2)

$$\delta D_+(\omega_0) = \frac{1}{6\pi}\left(\frac{e^2}{\hbar c}\right)^2 \frac{k_0 c}{\omega_0}\frac{k_0 c}{\omega_t}\frac{\hbar c}{mc^2}\frac{\Lambda_+(y_0)}{y_0}, \qquad (4)$$

where $y_0 = \omega_t/\omega_0$, and

$$\Lambda_+ = y_0\Big\{\frac{28}{15} + (y_0 - 1)\left(\frac{8}{3} + y_0\right) - y_0(2 + y_0)(y_0 - 1)^{1/2}\operatorname{arc\,tg}(y_0 - 1)^{1/2}\Big\} \text{ for } y_0 \geqslant 1,$$

$$\Lambda_+ = y_0\Big\{\frac{28}{15} - (1 - y_0)\left(\frac{8}{3} + y_0\right) + y_0(2 + y_0)(1 - y_0)^{1/2}\ln\left|\frac{1 + \sqrt{1 - y_0}}{1 - \sqrt{1 - y_0}}\right|\Big\} \text{ for } y_0 \leqslant 1. \qquad (4')^*$$

The value reached by the threshold "perturbation" of the real part of the amplitude is

$$\delta D_+(\omega_t) = \frac{1}{3\pi}\frac{14}{15}\frac{e^2}{\hbar c} r_0. \qquad (5)$$

Since the magnitude of the cross section for e^+e^- scattering in the one-photon approximation falls off sharply with increasing energy, and its value when the energy of each of the colliding particles is $E_b = m_\mu$ (and for $\theta = 90°$) is about

$$\sigma_s(90°, m_\mu) = {}^9/_{16} r_0^2 (m/m_\mu)^2, \qquad (6)$$

one might think that the additional factor $e^2/\hbar c$ in Eq. (5) can be compensated by the ratio of the muon and electron masses. It is not the value (5), however, that is to be compared with the value of the cross section (6) in the c. m. s. (for colliding beams this is the laboratory system), but the result of transforming the quantities (4) and (5) to the c. m. s.

The values of the forward scattering amplitude A_L in the l. s. (system in which one particle was at rest before the collision) and that in the c. m. s. (A_C) are connected by the relation

$$A_L/A_C = k_L/k_C \approx 2E_b/m, \qquad (7)$$

where the last equality holds for high energies. We finally have to compare the threshold perturbation in the system of the colliding beams

$$\delta D_+^C(\omega_t) = \frac{m}{m_\mu}\frac{1}{6\pi} r_0 \frac{e^2}{\hbar c} \qquad (8)$$

with the approximate value

$$\sqrt{\sigma_s(90°, m_\mu)} = {}^3/_4 r_0 m/m_\mu.$$

Thus the fractional effect amounts to

$$\delta D_+^C(\omega_t)/\sqrt{\sigma_s(90°, m_\mu)} = {}^2/_9 e^2/\pi\hbar c, \qquad (9)$$

and the mass ratio has dropped out, so that the maximum effect on the cross section at $\theta = 90°$ does not exceed 0.1 percent. We note that the energy width below threshold of the effect near the threshold is given by Eq. (4) as

$$\Delta E \equiv \sqrt{\Delta(E_b^2)} = 22.4 \text{ MeV}.$$

The contribution (3) to the real part of the amplitude for e^-e^- scattering is due to the second term in Eq. (1). For this case use of Eq. (3) together with Eqs. (4) and (5) leads to expressions in which $\Lambda_+(y_0)$ is replaced by the quantity

$$\frac{\Lambda_-}{y_0} = \frac{1}{3} - 2\left(1 - \frac{y_0}{2}\right)\left[1 - \frac{\sqrt{1+y_0}}{2}\ln\frac{(1+\sqrt{1+y_0})^2}{y_0}\right]\Big\}, \qquad (10)$$

which describes a contribution positive in sign and monotonically increasing with the energy. The ratio of the threshold perturbations is an appreciable quantity:

$$\Lambda_-(\omega_t) = 0.29\,\Lambda_+(\omega_t). \qquad (11)$$

The effect of process (2) on e^-e^- scattering increases with increasing energy.

Near thresholds for production of other fermion pairs Eq. (8) can be altered by the strong interaction in the final state of the inelastic reaction. For $\bar{\Lambda}\Sigma^0$ pairs this remains valid for the relative parity $P(\Lambda\Sigma) = +1$.

3. On this dispersion effect associated with the process (2) there are superimposed effects associated with other processes. The inelastic process with lowest threshold which leads in the one-photon approximation to the production of neutral pions is

$$e^- + e^+ \to \pi^0 + \gamma \qquad (12)$$

and its total cross section (in cm^2) is [4]

$$\sigma = 2.75\cdot10^{-32}\,|G(-K^2)/G(0)|^2(1 - x^2)^{3/2} \qquad (x = m_\pi/2E_b), \qquad (13)$$

where according to an estimate by Cabibbo and Gatto the ratio of the form-factors can reach the value

$$|G(-K^2)/G(0)|^2 \approx 250.$$

Substitution of Eq. (13) in Eq. (1) gives as the formulas relating to e^+e^- scattering (δD_+) and e^-e^- scattering (δD_-) (with $y_0 = \omega_t/\omega_0$)

$$\delta D_\pm(\omega_0) = \frac{k_0^2}{4\pi^2}\left|\frac{G(-K^2)}{G(0)}\right|^2 \frac{2.75\cdot10^{-32}}{\omega_t} y_0\cdot 2\Lambda_\pm(\omega_0), \qquad (14)$$

where for e^+e^- scattering

$$\Lambda_+ = \frac{1}{3} - (y_0 - 1) + (y_0 - 1)^{3/2}\operatorname{arc\,tg}(y_0 - 1)^{1/2}, \text{ for } y_0 \geqslant 1,$$

$$\Lambda_+ = \frac{1}{3} + (-y_0 + 1) - \frac{(1 - y_0)^{3/2}}{2}\ln\left|\frac{1 + \sqrt{1 - y_0}}{1 - \sqrt{1 - y_0}}\right|, \text{ for } y_0 \leqslant 1 \qquad (14')$$

and for e^-e^- scattering

*arctg = $\tan^{-1}$.

$$\Lambda_- = \frac{(1+y_0)^{3/2}}{2}\ln\left|\frac{1+\sqrt{1+y_0}}{\sqrt{1+y_0}-1}\right| - \frac{1}{3} - (y_0+1). \quad (15)$$

From Eqs. (14), (15), (7), (8), and (9) one finds that the contributions of process (12) for the two elastic scattering processes are comparable and very small. The conclusion that the effect of process (12) is small (< 0.1 per cent) remains true when the influence of the form-factor is taken into account.

Even if we suppose that in the production of charged-pion pairs

$$e^- + e^+ \to \pi^- + \pi^+ \quad (16)$$

the presence of the form-factor in the cross section (cm^2)

$$\sigma = 0.53\cdot 10^{-32}\, m_\pi^2\, z^{-2}\,(1 - z^{-2})^{3/2}\,|F(-4E_b^2)|^2 \quad (17)$$

(m_π in BeV, $z = E_b/m_\pi$) leads to a further factor of the order of 20,[4] the magnitude of the perturbation is

$$\delta D_\pm(\omega_0) = \frac{k_0^2}{4\pi^2}\frac{5\cdot 10^{-30}}{\omega_t}\Lambda_\pm(y_0), \quad (18)$$

where for e^+e^- scattering

$$\Lambda_+ = 2y_0\left\{y_0\left[\frac{1}{3} - (y_0-1) + (y_0-1)^{3/2}\operatorname{arc\,tg}(y_0-1)^{1/2}\right] - \frac{1}{5}\right\} \text{ for } y_0 \geqslant 1,$$

$$\Lambda_+ = 2y_0\left\{y_0\left[\frac{1}{3} + (1-y_0) - \frac{(1-y_0)^{3/2}}{2}\ln\left|\frac{1+(1-y_0)^{1/2}}{1-(1-y_0)^{1/2}}\right|\right] - \frac{1}{5}\right\} \text{ for } y_0 \leqslant 1, \quad (19)$$

and for e^-e^- scattering

$$\Lambda_- = 2y_0\left\{y_0\left[\frac{1}{3} + (1+y_0) - \frac{(1+y_0)^{3/2}}{2}\ln\left|\frac{1+(1+y_0)^{1/2}}{(1+y_0)^{1/2}-1}\right|\right] - \frac{1}{5}\right\} \quad (20)$$

and remains a small quantity. At threshold $\Lambda_-(\omega_t) = 0.21\Lambda_+(\omega_t)$, and this increases to $\Lambda_- = 6\Lambda_+$ for $\omega_0 = 2\omega_t$.

Thus we can conclude that inelastic processes which include strong interactions cannot have much effect on the results of experiments to test the validity of quantum electrodynamics by the use of colliding beams. The total effect does not exceed 1 per cent.

The d. r. method used here can also be useful for more detailed calculations when experimental data on the inelastic processes have been obtained, and also for the analysis of ee collisions at smaller energies.

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[1] E. Fermi, Ricerca sci. **7** (2), 13 (1936) (see also The Collected Papers of Enrico Fermi, Acad. Nazion. dei Lincei, Rome, Vol. **1**, page 943). E. P. Wigner, Phys. Rev. **73**, 1002 (1948). A. I. Baz', JETP **33**, 923 (1957), Soviet Phys. JETP **6**, 709 (1958).

[2] L. I. Lapidus and Chou Kuang-chao, JETP **38**, 201 (1960); **39**, 364 (1960), Soviet Phys. JETP **11**, 147 (1960); **12**, 258 (1961).

[3] A. I. Akhiezer and V. B. Berestetskiĭ, Kvantovaya élektrodinamika (Quantum Electrodynamics), 2d Ed., Fizmatgiz, 1959, Sect. 36.

[4] N. Cabibbo and R. Gatto, Phys. Rev. **124**, 1577 (1961).

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