THEORY OF ELECTRICAL CONDUCTIVITY OF IONIC SEMICONDUCTORS IN STRONG ELECTRIC AND MAGNETIC FIELDS

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The electrical conductivity of ionic semiconductors is investigated in strong crossed electric and magnetic fields. The case is considered in which the scattering of the electron energy is due to piezoelectric or optical phonons and momentum scattering is due to ionized impurities. The mean electron energy increases with increase in the electric field strength, and the frequency of electron collisions with ionized impurities decreases. As a result, the value of the conductivity current decreases with growth of the field in a certain electric field intensity range.

As is well known, the scattering of conduction electrons in semiconductors at temperatures below the Debye temperature is almost elastic. Landau and Kompaneets [1] and Davydov [2] have shown that, as a consequence, the electrons slowly transfer to the lattice the energy gained from the external electric field **E**, and a heating of the electron gas takes place. For a sufficiently large value of the field, the mean energy of the electrons $\bar{\epsilon}$ and their mean free path l become especially dependent on the value of E, which leads to departures from Ohm's law.

In the works mentioned, the interaction with acoustic phonons was taken into account. Davydov and Shmushkevich [3] considered the case of an ionic semiconductor when the scattering of electrons was brought about by their interaction with optical phonons. The spectrum of optical vibrations of the cubic lattice has the form

$$\omega(q) = \omega_0 (1 - a^2 q^2), \qquad (1)$$

where $\omega(\mathbf{q})$ is the frequency of a phonon with wave vector $\mathbf{q}$, ω_0 is the limiting frequency, and a is a length of the order of the lattice constant.

If the mean energy of electrons $\bar{\epsilon}$ is small in comparison with the limiting energy of optical phonons $\hbar\omega_0$, then most of the electrons do not give the energy extracted from the electric field directly to the lattice. Only the electrons with energies ϵ exceeding the energy of the phonons $\hbar\omega$ can emit optical phonons. The process of energy relaxation of the electron gas in this case is that the electrons with $\epsilon \gtrsim \hbar\omega_0$, on the average, emit phonons of higher frequency than the frequency of phonons absorbed by electrons with energies $\epsilon \ll \hbar\omega_0$.

The theory of galvanomagnetic phenomena in strong electric fields in the classic case $\hbar\Omega \ll T$ was constructed by Bass; [4] here $\Omega = eH/mc$; m is the effective mass of the electron; e is its electric charge; H is the magnetic field intensity, c the velocity of light, and T the lattice temperature in energy units.

A paper by the authors [5] was devoted to the quantum theory of nonlinear galvanomagnetic phenomena in semiconductors for the case $\hbar\Omega \gg T$. We assumed that the scattering of the energy of the electrons was brought about by interaction with acoustic phonons, while the momentum scattering was brought about by phonons and impurities. It was shown that in the case $\hbar\Omega \gg T$, for $\mathbf{E} \perp \mathbf{H}$, the dependence of the current on the electric field has an S-shaped form.

The present communication is devoted to a study of the electrical conductivity of ionic semiconductors in strong crossed electric and magnetic fields. We restrict ourselves to a consideration of the low-temperature case, in which the energy relaxation of the electrons is brought about by piezoelectric phonons or by weak dispersion of optical phonons. We consider here an electric field for which the energy of the electrons is much smaller than the limiting energy of the optical phonons $\hbar\omega_0$. Furthermore, we assume that the scattering of electrons is brought about primarily by ionized impurities, but in this case we neglect collisions of electrons with one another. This is valid in the case of a compensated impurity semiconductor, in which the concentration of ionized impurities is large in comparison with the concentration of conduction electrons. Moreover, we shall assume that all impurities are ionized and that the electron concentration n does not depend on the value of the electric field.

1. In the case of crossed fields $\mathbf{E} \perp \mathbf{H}$, the conductivity current and the generation of Joule power are brought about by the scattering of the electrons. Upon scattering, the center of the electronic orbit is displaced by a quantity of the order of the Larmor radius R. As a consequence, a change in energy of the electron of order eER takes place. If this quantity exceeds the energy transferred by the electrons to the lattice in the time between two such collisions, then the electron gas is heated and its distribution function differs strongly from equilibrium. For small inelasticity of the scattering, the energy relaxation time of the electrons τ_ϵ is large in comparison with the momentum relaxation time $\tau_p : \tau_p \ll \tau_\epsilon$. Therefore, the part of the electron distribution function F_0 that is even in the momentum $\mathbf{p}$ quickly becomes symmetric about the direction of $\mathbf{p}$ and is a function only of the energy ϵ. The part of the distribution function $f(\mathbf{p}) = -f(-\mathbf{p})$ that is odd in the electron momentum, corresponding to the conduction current, is expressed in terms of the symmetric part $F_0(\epsilon)$ in the same fashion as in the theory that is linear in E.

The symmetric part of the distribution function $F_0(\epsilon)$ is found from the condition for the stationary mode, which represents the equation of continuity in energy space. It means that the power of the electric field absorbed by electrons with energy ϵ is equal to the power given to the lattice by the same electrons. It was shown in [5] that in the case of interaction of the electrons with longitudinal acoustic phonons and impurities, the equation for $F_0(\epsilon)$, for $\epsilon \gg \hbar\Omega \gg \hbar\,(\nu_i + \nu_{ph})$, has the form

$$F_0(\varepsilon) + T\left\{1 + \frac{1}{3}\left(\frac{cE}{sH}\right)^2 [1 + \nu_i(\varepsilon)/\nu_{ph}(\varepsilon)]\right\}\frac{\partial F_0}{\partial \varepsilon} = 0. \quad (2)$$

Here $\nu_i(\epsilon)$ and $\nu_{ph}(\epsilon)$ are the collision frequencies of electrons with energy ϵ with impurities and phonons, respectively; s is the speed of longitudinal sound. In the derivation of Eq. (2), it was assumed that the energy of phonons interacting with the electrons is small in comparison with the lattice temperature, while their distribution function is equilibrium. A calculation was carried out for an isotropic quadratic spectrum of conduction electrons.

As follows from considerations of symmetry,[6] in the ionic semiconductors InAs, GaAs, InP and so on, which have the structure of zinc blende, the acoustic vibrations can lead to the appearance of a macroscopic electric field. In the scattering of conduction electrons by such a piezoelectric potential, the square of the modulus of the matrix element of interaction has the form[7]

$$|C_{\mathbf{q},\alpha}|^2 = 32\pi^2 \frac{\hbar e^2\beta^2}{\varkappa^2\rho\omega_{\mathbf{q},\alpha}} [q_1q_2e_3(\mathbf{q},\alpha) + q_1q_3e_2(\mathbf{q},\alpha) + q_2q_3e_1(\mathbf{q},\alpha)]^2 q^{-4}, \quad (3)$$

where β is the piezoelectric modulus of the cubic crystal, ρ is the density, κ is the dielectric constant, $\mathbf{e}(\mathbf{q},\alpha)$ and $\omega_{\mathbf{q},\alpha}$ is the unit polarization vector and the frequency of the phonon with wave vector $\mathbf{q}$ and wave number α, while the indices 1, 2, and 3 indicate the projections of the vectors $\mathbf{q}$ and $\mathbf{e}$ on the principal crystallographic axes.

By means of the method used in [5] for the derivation of (2), and under the same assumptions, it can be shown that the equation for the symmetric part of the distribution $S_0(\epsilon)$ differs from (2) by the substitution

$$\frac{\nu_i(\varepsilon)}{\nu_{ph}(\varepsilon)} \to \frac{T}{\varepsilon}\,\frac{\tau_f}{\tau_i}, \quad (4)$$

where τ_i and τ_j are, in order of magnitude, equal to the relaxation times of electrons in their scattering by charged impurities and long-wave piezoelectric phonons in the absence of an electric field. Here s^{-2} has the meaning of the inverse square of the sound velocity, averaged over the directions of its propagation and summed over the polarizations.

Carrying out the substitution (4) in (2), and introducing the non-dimensional energy $z = \epsilon/T$, we represent the function F_0 in the case of interest to us in the form

$$F_0(z) = A\exp\left\{-\int_0^z dz'\left[1 + \eta\left(1 + \frac{1}{z'}\frac{\tau_f}{\tau_i}\right)\right]^{-1}\right\},$$

$$\eta = (cE/sH)^2/3, \quad (5)$$

where A is a normalizing constant.

We have noted above that in almost-elastic scattering of electrons the part of the distribution function f that is even in the velocity of the electron is expressed in terms of F_0 in the same way as in the theory that is linear in E. Therefore, for a fixed concentration of conduction electrons n = const, the current density $\mathbf{j}$ is determined by the expression $(\mathbf{E} \parallel x;\ \mathbf{H} \parallel z)$

$$j_x = j_x^{(0)}\int_0^\infty dz\left(\frac{1}{\tau_i} + \frac{z}{\tau_f}\right)\frac{\partial F_0}{\partial z}\Bigg/\int_0^\infty dz\left(\frac{1}{\tau_i} + \frac{z}{\tau_f}\right)\frac{\partial f_0}{\partial z}, \quad (6)$$

$$j_y = j_y^{(0)} = nec\,E/H, \quad (7)$$

where $f_0(z)$ and $\mathbf{j}^{(0)}$ are the electron distribution function and the current density vector, respectively, as $E \to 0$.

2. We first consider the case $\tau_i \gg \tau_f$, in which the impurities do not play a role in electron scattering. Then

$$F_0(z) = A_0(1 + \eta)^{-3/2}\exp\{-z/(1 + \eta)\},$$

$$A_0 = n\,(2\pi\hbar^2/mT)^{3/2}, \quad (8)$$

i.e., the electron distribution is described by an exponential with effective temperature

$$T_{eff} = (1 + \eta)\, T. \tag{9}$$

Substituting (8) in (6) for $\tau_i \gg \tau_f$, it is easy to prove that in this case

$$j_x = j_x^{(0)} \left[1 + \frac{1}{3} (cE/sH)^2 \right]^{-1/2}. \tag{10}$$

Thus, in the region of strong electric fields, $E \gg sH/c$, the density of conduction current j_x is inversely proportional to the third power of the magnetic field H, and does not depend on the value of the electric field E. The saturation of j_x as a function of E is a consequence of the fact that the probability of scattering by a piezoelectric potential is inversely proportional to the square root of the energy of the electron, and, in accord with (8), the mean energy of the electrons for $\eta \gg 1$ is proportional to E^2.

3. We now consider the case $\tau_i \ll \tau_f$. In the region of electric fields E satisfying the condition

$$\eta\tau_i/\tau_f \ll 1 \ll \eta\tau_f/\tau_i, \tag{11}$$

the scattering of electrons is essentially determined by the charged impurities. In this case, the electron distribution function is

$$F_0(z) \approx A_0 (\tau_i/2\eta\tau_f)^{3/4} \exp(-z^2\tau_i/2\eta\tau_f) \tag{12}$$

and the conduction current density is

$$j_x \approx j_x^{(0)} (\tau_i/2\eta\tau_f)^{3/4} \sim (EH)^{-1/2}. \tag{13}$$

The decrease of the conduction current with increase in the electric field (negative differential conductivity) is explained by the fact that the mean electron energy is

$$\bar{z} \sim \int_0^\infty F_0(z)\, z^{3/2} dz \sim (\eta\tau_f/\tau_i)^{1/2} \sim E, \tag{14}$$

while the frequency of collisions with charged impurities (and, consequently, the conductivity in a magnetic field) is proportional to $(\bar{z})^{-3/2}$.

Let us make precise the region of applicability of Eq. (13) for j_x. Along with the condition (11), this region is determined by the requirement that the mean collision frequency of electrons with impurities be small in comparison with the cyclotron frequency Ω, i.e.,

$$\Omega\tau_i (\eta\tau_f'/\tau_i)^{3/4} \gg 1. \tag{15}$$

In other words, (13) holds also for $\Omega\tau_i \ll 1$, if conditions (11) and (15) are satisfied.

In the experiment one usually measures the resistivity tensor

$$\rho_{ik} = E_i j_k / j^2. \tag{16}$$

In the presence of a strong magnetic field H, the angle between the vectors **E** and **j** is close to $\pi/2$ ($j_x \ll j_y$). Because of this,

$$j \approx j_y = nec\, E/H$$

and the dependence of the resistance of the crystal on the value of j has in the case (11) and (15) the form

$$\rho_{xx} \sim j^{-3/2}, \qquad \rho_{xy} = H/nec. \tag{17}$$

In Fig. 1, the dependence of the projection of the vector **E** in the direction **j** is shown as a function of the quantity j. The curve a corresponds to the case $\tau_i \gg \tau_f$, the curve b to the case $\tau_i \ll \tau_f$. Here the same value of τ_f and different values of τ_i correspond to the two curves. The merging of curve b with curve a is a consequence of the fact that, for high values of the current density j, a strong heating of the electron gas takes place and the ionized impurities cease to play a role in the scattering of the electrons.

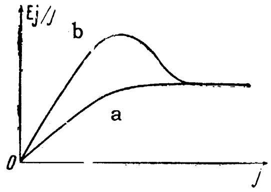

FIG. 1

4. Let us not investigate the electrical conductivity of an ionic semiconductor in the case in which the relaxation of the energy of the electrons is brought about by disperse optical phonons (1) and the relaxation of the momentum by the ionized impurities.

In accord with Davydov and Shmushkevich,[3] the function $F_0(z)$ for scattering of electrons by optical phonons in the case $z \ll \hbar\omega_0/T$ is determined by the equation

$$eEv(z) = \frac{4}{3} \lambda^2 \nu_0 \hbar\omega_0 z\, [F_0(z) + \partial F_0/\partial z], \tag{18}$$

where

$$v(z) = -\frac{2}{3} \frac{eE}{m\nu_0} z \frac{\partial F_0}{\partial z}$$

is the velocity of the electrons averaged over states with energy z;

$$\nu_0 = 2\pi (2m\hbar\omega_0)^{1/2} \left(\frac{\gamma Z e^2}{\hbar\omega_0} \right)^2 \frac{\exp(-\hbar\omega_0/T)}{M a_0^3}$$

is the frequency of collisions of electrons with optical phonons (M is the mass of the elementary cell, a_0 is the lattice constant, γ is a dimensionless coefficient of polarizability of the ions, Z is the charge); $\lambda = 2m\omega_0 a^2/\hbar$; the quantity $1/\lambda^2\nu_0$ characterizes the electron energy relaxation time.

Equation (18) represents the differential energy balance for electrons with energy z. Actually, the quantity eEv(z) is the average power of the electric field absorbed by electrons with energy z, while the right side of (18) is the power transferred by such electrons to the lattice.

In the presence of impurities, the total frequency of collisions of the electron with the scatterers is equal to $\nu_0 + \nu_i(z)$. Therefore, in crossed fields $\mathbf{E} \perp \mathbf{H}$ the velocity of the electron averaged over states with energy z has the form

$$v(z) = -\frac{2}{3}\frac{eE}{m}\frac{\nu_0 + \nu_i(z)}{\Omega^2 + [\nu_0 + \nu_i(z)]^2} z \frac{\partial F_0}{\partial z}. \qquad (19)$$

We substitute (19) in Eq. (18) and integrate it in the case $z \gg 1$ and

$$\nu_0 \ll \nu_i(z) \ll \Omega. \qquad (20)$$

Bearing in mind that

$$\nu_i(z) \approx 1/\tau_i z^{3/2},$$

we arrive at the following expression for the symmetric part of the distribution function:

$$F_0(z) \approx A_0 g^{-3/5} \exp\left(-\frac{2}{5} g z^{5/2}\right), \qquad (21)$$

$$g = \frac{2\Delta}{mc^2}\frac{H^2}{E^2}, \qquad (22)$$

where the quantity $\Delta = \lambda^2\nu_0\tau_i\hbar\omega_0$ characterizes the energy given off by the electron to the lattice in the time between two successive collisions with impurities.

By means of (21), it is not difficult to compute the mean energy of electrons $\bar{z}$ and the conduction current density:

$$\bar{z} = \int_0^\infty F_0(z) z^{3/2} dz \Big/ \int_0^\infty f_0(z) z^{1/2} dz \approx g^{-2/5}, \qquad (23)$$

$$j_x \approx j_x^{(0)} g^{3/5} \sim H^{-4/5} E^{-1/5}. \qquad (24)$$

Thus a negative differential conductivity occurs also in the case under consideration. The region of applicability of (23) and (24) is determined by the requirements that the mean energy of the electron $\bar{z}$ be much greater than unity and that the mean frequency of collisions with impurities be large in comparison with Ω, i.e., that

$$g \ll 1,\ \nu_0 \ll g^{3/5}\tau_i^{-1} \ll \Omega. \qquad (25)$$

If condition (25) is satisfied the dependence of the resistivity ρ_{xx} on the value of j has the form

$$\rho_{xx} = \sigma_0^{-1}(2\Delta/mc^2)^{3/5}(nec/j)^{4/5}, \qquad (26)$$

where σ_0 is the conductivity of the crystal for H = 0 and E → 0.

The dependence of the projection of the electric field **E** on the direction of **j** as a function of the magnitude of j is shown in Fig. 2.

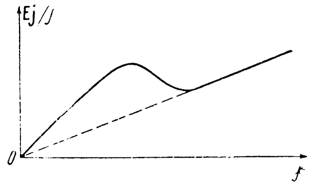

FIG. 2

For small j, Ohm's law is valid and the dependence of $\mathbf{j}\cdot\mathbf{E}/j$ on the value of j is linear. In the case $\Omega\tau_i \gg 1$, the slope of the volt-ampere characteristic is proportional to $1/\tau_i$. For large values of j, heating of the electron gas takes place. As a consequence, the probability of scattering of electrons by ionized impurities decreases and the magnetic resistance of the crystal falls off with increasing j (26). The decreasing portion of the volt-ampere characteristic corresponds to this region of values of j. With further increase in j, the electron gas is heated so much that the probability of scattering of electrons by impurities becomes less than the probability of scattering by optical phonons. Inasmuch as the frequency of collisions with optical phonons ν_0 does not depend on the energy of the electron, the magnetoresistance ceases to depend on the value of j and Ohm's law is again satisfied. However, the slope of the characteristic is now determined by collisions with optical phonons and is proportional to ν_0 and not to $1/\tau_i$.

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[1] L. Landau and A. Kompaneets, JETP **5**, 276 (1935).

[2] B. Davydov, JETP **7**, 1069 (1937).

[3] B. Davydov and N. Shmushkevich, JETP **10**, 1043 (1940).

[4] F. G. Bass, FMM **6**, 961 (1958).

[5] R. F. Kazarinov and V. G. Skobov, JETP **42**, 1047 (1962), Soviet Phys. JETP **15**, 726 (1962).

[6] L. D. Landau and E. M. Lifshitz, Mekhanika sploshnykh sred (Mechanics of Continuous Bodies), Gostekhizdat, 1957.

[7] M. Meijer and D. Polder, Physica **19**, 255 (1953).

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