RADIATION REACTION OF SOUND DUE TO THE MOTION OF SMALL BODIES IN INHOMOGENEOUS GASEOUS MEDIA

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Submitted to JETP editor October 26, 1962, resubmitted January 19, 1963

J. Exptl. Theoret. Phys. (U.S.S.R.) **44**, 1544-1551 (May, 1963)

Radiation of sound due to the motion of solid bodies in statistically inhomogeneous gaseous media is considered. The dimension of the body is assumed small in comparison with the mean free path. It is shown that, in contrast with homogeneous media, sound can be radiated in inhomogeneous media at subsonic velocities of the body.

As is well known, there exists a definite analogy between many phenomena of electrodynamics and acoustics. For example reference can be made to the Cerenkov radiation of an electric charge [1-3] and sound radiation in the supersonic motion of bodies in gaseous media. [4-6] Furthermore, in the crossing by a small body of a boundary separating two media, radiation of sound waves takes place, [6] in considerable analogy to the transition radiation of electromagnetic waves (see, for example, [7,8]). Recently, a number of works have appeared, [9-11] in which the radiation of electromagnetic waves is considered for the uniform motion of charged particles in a statistically inhomogeneous medium. It is natural to expect that a similar phenomenon occurs also in acoustics.

In the present work, the radiation of sound waves arising in the motion of small objects in gaseous media with random inhomogeneities is considered. An expression is found, by the method of the radiation reaction of sound, for the intensity of the sound waves radiated under the condition $\lambda \gg l$, where λ is the wavelength and l is the dimension of the inhomogeneities of the medium.

1. For the motion of small bodies in gaseous media, the force of frontal resistance acting on the moving body for the case of subsonic motion is determined by the relation (see [12])

$$\mathbf{F}_1^* = -\Gamma S\rho_0 c_s \mathbf{V}_0, \tag{1}$$

and for motions with velocity exceeding the velocity of sound,[1)]

$$\mathbf{F}_2^* = -\Gamma S\rho_0 V_0 \mathbf{V}_0, \tag{2}$$

where S is the frontal cross sectional area, $\mathbf{V}_0$ is the velocity of the particle, ρ_0 is the density of the medium, c_s the velocity of sound under adiabatic change of the parameters of the medium, and Γ is the coefficient of resistance, which depends on the character of the collisions of the molecules of the medium with the surface of the moving body. [12,13] Equations (1) and (2) were obtained under the condition that the dimension of the moving body L is much smaller than the mean free path λ_f, where a state of free molecular flow past the bodies exists.

In turn, the moving body will act on the gas surrounding it with a force $\mathbf{F} = -\mathbf{F}^*$. Inasmuch as we are interested in sound radiation, i.e., the generation of hydrodynamic perturbations with characteristic dimensions much larger than the length of the mean free path and, correspondingly, large in comparison with the dimensions of the moving object, we can further neglect the dimensions of the body.[2)] This allows us to represent the force acting on a unit volume of the medium in the form

$$\mathbf{f} = \mathbf{F}\delta(\mathbf{r} - \mathbf{V}_0 t), \tag{3}$$

just as was done in [7]; here $\mathbf{F} = \Gamma S\rho_0 c_s \mathbf{V}_0$, and $\delta(\mathbf{r} - \mathbf{V}_0 t)$ is the spatial Dirac delta function with dimensionality L^{-3}.

The energy of sound waves radiated per unit time can be determined by means of the reaction of the sound from the following formula:

$$\partial W/\partial t = \mathrm{Re}\,\overline{\mathbf{F}\mathbf{V}}, \tag{4}$$

1)In what follows, we shall use only Eq. (1) in the calculation of the energy of the sound waves radiated per unit time. This case is of most interest in that the presence of inhomogeneities in it leads to the phenomenon of radiation. In a homogeneous medium, there is no radiation of sound at subsonic velocities, as was shown in [7].

2)Of course, it is impossible to neglect the dimensions of the body in Eqs. (1) and (2) for the force of frontal resistance, since the force is proportional to the frontal cross sectional area S and tends to zero with decrease in S.

where V is the velocity of the particles of the medium at the point where the moving small object is located, and is computed from the hydrodynamic equations. The bar over quantities indicates statistical averaging.

2. To find V, we shall start out from the equations of hydrodynamics for an inhomogeneous nonviscous medium:

$$\rho d\mathbf{V}/dt = -\operatorname{grad} p + \mathbf{f}, \qquad (5)$$

$$\partial\rho/\partial t + \operatorname{div}(\rho\mathbf{V}) = 0, \qquad (6)$$

$$dp/dt = c_s^2\, d\rho/dt. \qquad (7)$$

Here ρ, V, and p are respectively the mass density, velocity, and pressure in the medium, c_s is the sound velocity, and f is the external force acting on the gas from the moving body, and is given by Eq. (3).

The presence of inhomogeneities can be taken into account in the following way. We shall assume that the gas is at constant pressure, while the fluctuations in the sound velocity and density are determined by temperature fluctuations. In this case, it is seen (see [14]) that

$$\Delta c_s/c_{s0} = -\mu, \qquad c_s - c_{s0} = \Delta c_s, \qquad (8)$$

$$\Delta\rho_0/\rho_{00} = 2\mu, \qquad \rho_0 - \rho_{00} = \Delta\rho_0. \qquad (9)$$

After linearization of Eqs. (5), (6), and (7), we get

$$\rho_{00}(1 + 2\mu)\,\partial\mathbf{V}/\partial t = -\operatorname{grad} p + \mathbf{f}, \qquad (5')$$

$$\partial\rho/\partial t + \rho_{00}(1 + 2\mu)\operatorname{div}\mathbf{V} + 2\rho_{00}(\mathbf{V}\operatorname{grad}\mu) = 0, \qquad (6')$$

$$\partial p/\partial t = c_s^2\,[\partial\rho/\partial t + 2\rho_{00}(\mathbf{V}\operatorname{grad}\mu)]. \qquad (7')$$

We differentiate Eq. (5′) with respect to time[3)]

$$\rho_{00}(1 + 2\mu)\,\partial^2\mathbf{V}/\partial t^2 = -\operatorname{grad}(\partial p/\partial t) + \partial\mathbf{f}/\partial t. \qquad (10)$$

Applying the operator ∇ to Eq. (7′) and also using the relations (6′) and (8) we can rewrite Eq. (10) in the form

$$\operatorname{grad}\operatorname{div}\mathbf{V} - \frac{(1 + 2\mu + 3\mu^2)}{c_{s0}^2}\frac{\partial^2\mathbf{V}}{\partial t^2} - 3\operatorname{grad}(\mu^2)\operatorname{div}\mathbf{V}$$

$$= -\frac{\partial\mathbf{f}_0}{\partial t}(1 + \mu)\frac{1}{c_{s0}^2\rho_{00}},$$

$$\mathbf{f}_0 = \Gamma S\rho_{00}c_{s0}\mathbf{V}_0\delta(\mathbf{r} - \mathbf{V}_0 t). \qquad (11)$$

Here we have neglected all quantities whose order exceeds μ^2, assuming the quantity μ to be small ($\mu \ll 1$).

The velocity V we write in the form

$$\mathbf{V} = \bar{\mathbf{V}} + \mathbf{V}', \qquad (12)$$

where $\bar{\mathbf{V}}$ is the velocity averaged over the ensemble of inhomogeneities, and $\mathbf{V}'$ is its fluctuating part, proportional to μ. Here Eq. (4) is written in the following way:

$$\partial W/\partial t = \operatorname{Re}(\mathbf{F}_0\bar{\mathbf{V}} + \mathbf{F}_0\overline{\mu\mathbf{V}'}). \qquad (4')$$

After averaging Eq. (11) and neglecting terms of the order μ^3, we have

$$\operatorname{grad}\operatorname{div}\bar{\mathbf{V}} - \frac{(1 + 3\overline{\mu^2})}{c_{s0}^2}\frac{\partial^2\bar{\mathbf{V}}}{\partial t^2} - \frac{2}{c_{s0}^2}\overline{\mu\frac{\partial^2\mathbf{V}'}{\partial t^2}} = -\frac{1}{c_{s0}^2\rho_{00}}\frac{\partial\mathbf{f}_0}{dt}. \qquad (13)$$

Equation (13) is obtained under the assumption that $\overline{\mu^2}$ does not depend on the coordinates. Subtracting Eq. (13) from (11), and neglecting the terms

$$\frac{3}{c_{s0}^2}(\mu^2 - \overline{\mu^2})\frac{\partial^2\bar{\mathbf{V}}}{\partial t^2}, \quad \frac{2}{c_{s0}^2}\left(\mu\frac{\partial^2\mathbf{V}'}{\partial t^2} - \overline{\mu\frac{\partial^2\mathbf{V}'}{\partial t^2}}\right),$$

we obtain the following equation for the determination of $\mathbf{V}'$:

$$\operatorname{grad}\operatorname{div}\mathbf{V}' - \frac{1}{c_{s0}^2}\frac{\partial^2\mathbf{V}'}{\partial t^2} = -\frac{\partial\mathbf{f}_0}{\partial t}\frac{\mu}{c_{s0}^2\rho_{00}} + \frac{2\mu}{c_{s0}^2}\frac{\partial^2\bar{\mathbf{V}}}{\partial t^2}. \qquad (14)$$

3. To solve Eq. (14) we extend the left and right side in Fourier integrals in ω and k. Here it is convenient to represent the right side by means of the δ-function $\delta(\mathbf{r} - \mathbf{r}')$ in the form of an integral over the volume. As a result, we get the relation

$$-\mathbf{k}(\mathbf{k}\mathbf{V}'_{\omega,k}) + k_0^2\mathbf{V}'_{\omega,k} = -\frac{2k_0^2}{(2\pi)^3}\int e^{-i\mathbf{k}\mathbf{r}'}\mu(r')\,\bar{\mathbf{V}}_\omega(r')\,d^3r'$$

$$+ \frac{i\omega F_0\mathbf{z}_0}{(2\pi)^4 V_0 c_{s0}^2\rho_{00}}\int\exp\left\{-ik_z z' + i\frac{\omega}{V_0}z'\right\}\mu(0, 0, z')\,dz'. \qquad (15)$$

Here $\mathbf{z}_0$ is a unit vector along the z axis which is chosen in the direction of the moving body, $k_0 = \omega/c_{s0}$.

The scalar product $(\mathbf{k}\cdot\mathbf{V}'_{\omega,k})$ can be found by multiplying both sides of Eq. (15) scalarly by k. As a result we get the following expression for $\mathbf{V}'_{\omega,k}$:

$$\mathbf{V}'_{\omega,k} = -\frac{2}{(2\pi)^3}\int e^{-i\mathbf{k}\mathbf{r}'}\mu(r')\,\bar{\mathbf{V}}_\omega(r')\,d^3r'$$

$$+ \frac{i\omega F_0\mathbf{z}_0}{(2\pi)^4 V_0 c_{s0}^2\rho_{00}k_0^2}\int\exp\left\{-i\left(k_z - \frac{\omega}{V_0}\right)z'\right\}\mu(0, 0, z')\,dz'$$

$$- \frac{2}{(2\pi)^3(k_0^2 - k^2)}\int e^{-i\mathbf{k}\mathbf{r}'}\mathbf{k}(\mathbf{k}\bar{\mathbf{V}}_\omega(r'))\,d^3r'$$

$$+ \frac{i\omega F_0\mathbf{k}(\mathbf{k}\mathbf{z}_0)}{(2\pi)^4 V_0 c_{s0}^2 k_0^2(k_0^2 - k^2)\rho_{00}}\int\exp\left\{-i\left(k_z - \frac{\omega}{V_0}\right)z'\right\}$$

$$\times\mu(0, 0, z')\,dz'. \qquad (16)$$

[3)]We assume that the distribution of the quantity μ depends only on the coordinates and not on the time. This assumption is valid if it is taken into account that the small thermal conductivity of the gas leads only to a very slow change in the fluctuations with time in comparison both with the period T of the sound wave and with the other characteristic times of the problem, for example l/V_0.

We can calculate $V'_\omega(\mathbf{r})$ by taking the inverse Fourier transform of (16):

$$\mathbf{V}'_\omega(\mathbf{r}) = \int \mathbf{V}'_{\omega,k} e^{i\mathbf{kr}}\, d\mathbf{k}. \tag{17}$$

The result of the calculation yields

$$\mathbf{V}'_\omega(\mathbf{r}) = -2\mu\bar{\mathbf{V}}_\omega(\mathbf{r}) + \frac{i\omega F_0 \mathbf{z}_0}{2\pi V_0 c_{s0}^2 \rho_{00} k_0^2}\,\delta(x)\,\delta(y)\, e^{i\omega z/V_0}\mu(0,0,z)$$
$$-2\,\mathrm{grad}\,\mathrm{div}_{\mathbf{r}}\int \mu(r')\bar{\mathbf{V}}_\omega(r')\frac{e^{ik_0|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\,d^3r'$$
$$+\frac{i\omega F_0}{2\pi V_0 c_{s0}^2\rho_{00}k_0^2}\,\mathrm{grad}\,\mathrm{div}_{\mathbf{r}}\int \mathbf{z}_0\mu(0,0,z')$$
$$\times\frac{\exp\{ik_0\sqrt{x^2+y^2+(z-z')^2}+i\omega z'/V_0\}}{\sqrt{x^2+y^2+(z-z')^2}}\,dz'. \tag{18}$$

Substituting Eq. (18) in Eq. (13), we get

$$\mathrm{grad}\,\mathrm{div}\,\bar{\mathbf{V}}_\omega + k_0^2(1+3\overline{\mu^2})\bar{\mathbf{V}}_\omega - 4k_0^2\overline{\mu^2}\,\bar{\mathbf{V}}_\omega$$
$$-\overline{4k_0^2\mu(\mathbf{r})\,\mathrm{grad}\,\mathrm{div}_{\mathbf{r}}\int\mu(\mathbf{r}')\bar{\mathbf{V}}_\omega(r')\frac{e^{ik_0|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\,d^3r'}$$
$$=\frac{i\omega F_0\mathbf{z}_0}{2\pi V_0 c_{s0}^2\rho_{00}}\,\delta(x)\,\delta(y)\,e^{i\omega z/V_0}(1-2\overline{\mu^2}w(x,y,0))$$
$$-\frac{2i\omega F_0}{2\pi V_0 c_{s0}^2\rho_{00}}\,\overline{\mu(\mathbf{r})\,\mathrm{grad}\,\mathrm{div}_{\mathbf{r}}\int\mathbf{z}_0\mu(0,0,z')}$$
$$\times\frac{\exp\{i(k_0\sqrt{x^2+y^2+(z-z')^2}+\omega z'/V_0)\}}{\sqrt{x^2+y^2+(z-z')^2}}\,dz', \tag{19}$$

where $w(\mathbf{r}-\mathbf{r}')$ is the correlation coefficient of fluctuations of the quantity $\mu(\mathbf{r})$. We assume that the field of random fluctuations of the quantity $\mu(\mathbf{r})$ is homogeneous.

The integro-differential equation (19) is materially simplified and reduces to a differential equation if one assumes that the distance $R \approx V_0/\omega$ over which the quantity $\bar{V}_\omega(\mathbf{r})$ changes significantly is much greater than the correlation radius of the fluctuations of μ. In this case, Eq. (19) takes the form

$$\mathrm{grad}\,\mathrm{div}\,\bar{\mathbf{V}}_\omega + k_0^2(1-\overline{\mu^2})\bar{\mathbf{V}}_\omega - 4k_0^2\overline{\mu^2}\,\mathbf{e}_j^0\bar{V}_{\omega i}(\mathbf{r})\int\frac{\partial^2 w(\rho)}{\partial\rho_i\partial\rho_j}\frac{d^3\rho}{|\rho|}$$
$$+2k_0^4\overline{\mu^2}\,\mathbf{e}_j^0\bar{V}_{\omega i}(\mathbf{r})\int\frac{\partial^2 w(\rho)}{\partial\rho_i\partial\rho_j}|\rho|\,d^3\rho$$
$$+i\frac{2}{3}k_0^5\overline{\mu^2}\,\mathbf{e}_j^0\bar{V}_{\omega i}(\mathbf{r})\int\frac{\partial^2 w(\rho)}{\partial\rho_i\partial\rho_j}|\rho|^2\,d^3\rho$$
$$=\frac{i\omega F_0\mathbf{z}_0}{2\pi V_0 c_{s0}^2\rho_{00}}\,\delta(x)\,\delta(y)\,e^{i\omega z/V_0}(1-2\overline{\mu^2})$$
$$-\frac{i\omega F_0\overline{\mu^2}}{\pi V_0 c_{s0}^2\rho_{00}}\,e^{i\omega z/V_0}\int\delta(x-x_1)\,\delta(y-y_1)\,e^{-i\omega\xi/V_0}w(\mathbf{r}_1)$$
$$\times\,\mathrm{grad}_{\mathbf{r}_1}\frac{\partial}{\partial\xi}\frac{e^{ik_0|r_1|}}{|r_1|}\,d^3r_1, \tag{20}$$

where $\mathbf{e}_j^0$ is the unit vector in the direction of the j-th axis and $\mathbf{r}_1 = \{x_1, y_1, \xi\}$.

A further simplification of Eq. (20) lies in the assumption of the isotropic character of the field of random fluctuations of the quantity μ. In this case, the correlation coefficient depends only on the modulus of the vector ρ:

$$w(\rho) = w(|\rho|). \tag{21}$$

Then Eq. (20) is written in the following way:

$$\mathrm{grad}\,\mathrm{div}\,\bar{\mathbf{V}}_\omega + k_0^2\varepsilon_{\mathrm{eff}}\bar{\mathbf{V}}_\omega$$
$$=\frac{i\omega F_0\mathbf{z}_0}{2\pi V_0 c_{s0}^2\rho_{00}}\,\delta(x)\,\delta(y)\,e^{i\omega z/V_0}(1-2\overline{\mu^2})$$
$$-\frac{i\omega F_0\overline{\mu^2}}{\pi V_0 c_{s0}^2\rho_{00}}\,e^{i\omega z/V_0}\int\delta(x-x_1)\,\delta(y-y_1)\,e^{-i\omega\xi/V_0}w(|\mathbf{r}_1|)$$
$$\times\,\mathrm{grad}_{\mathbf{r}_1}\frac{\partial}{\partial\xi}\frac{\exp\{ik_0|r_1|\}}{|r_1|}\,d^3r_1. \tag{22}$$

Here the following notation has been introduced:

$$\varepsilon_{\mathrm{eff}} = 1 - 5\overline{\mu^2} + 2\overline{\mu^2}k_0^2\overline{l^2} + i\frac{2}{3}\overline{\mu^2}k_0^3\overline{l^3}, \tag{23}$$

$$\overline{l^2} = \int\frac{\partial^2 w(|\rho|)}{\partial\rho_i^2}|\rho|\,d^3\rho, \qquad \overline{l^3} = \int\frac{\partial^2 w(|\rho|)}{\partial\rho_i^2}|\rho|^2\,d^3\rho. \tag{24}$$

Expanding the left and right sides of Eq. (22) in a Fourier integral over $\mathbf{k}$, we get the following expression for $\bar{V}_{\omega k}$:

$$\bar{\mathbf{V}}_{\omega k} = \frac{i\omega F_0\mathbf{z}_0}{(2\pi)^3 V_0 c_{s0}^2\rho_{00}k_0^2\varepsilon_{\mathrm{eff}}}\,\delta\left(\frac{\omega}{V_0}-k_z\right)(1-2\overline{\mu^2})$$
$$-\frac{i\omega F_0\overline{\mu^2}}{4\pi^3 V_0 c_{s0}^2\rho_{00}k_0^2\varepsilon_{\mathrm{eff}}}\,\delta\left(\frac{\omega}{V_0}-k_z\right)$$
$$\times\int\exp\left\{-i\left(k_x x_1 + k_y y_1 + \frac{\omega}{V_0}\xi\right)\right\}w(|\mathbf{r}_1|)$$
$$\times\,\mathrm{grad}\frac{\partial}{\partial\xi}\frac{e^{ik_0|r_1|}}{|r_1|}\,d^3r_1$$
$$+\frac{i\omega F_0\mathbf{k}(\mathbf{k}\mathbf{z}_0)}{(2\pi)^3 V_0 c_{s0}^2\rho_{00}k_0^2\varepsilon_{\mathrm{eff}}(k_0^2\varepsilon_{\mathrm{eff}}-k^2)}\,\delta\left(\frac{\omega}{V_0}-k_z\right)(1-2\overline{\mu^2})$$
$$-\frac{i\omega F_0\overline{\mu^2}\delta(\omega/V_0-k_z)}{4\pi^3 V_0 c_{s0}^2\rho_{00}k_0^2\varepsilon_{\mathrm{eff}}(k_0^2\varepsilon_{\mathrm{eff}}-k^2)}$$
$$\times\int\exp\left\{-i\left(k_x x_1 + k_y y_1 + \frac{\omega}{V_0}\xi\right)\right\}$$
$$\times\, w(|\mathbf{r}_1|)(\mathbf{k}(\mathbf{k}\,\mathrm{grad}_{\mathbf{r}_1}))\frac{\partial}{\partial\xi}\frac{e^{ik_0|r_1|}}{|r_1|}\,d^3r_1. \tag{25}$$

For calculation of the energy of the sound waves emitted by the moving body, it is necessary only to know the z-component of the velocity. It can be found from Eq. (25) by Fourier inversion:

$$\bar{\mathbf{V}}_{\omega z}(\mathbf{r}) = \int\bar{\mathbf{V}}_{\omega z}(\mathbf{k})\,e^{i\mathbf{kr}}\,d\mathbf{k}. \tag{26}$$

Substituting (18), (25), and (26) in Eq. (4′), and neglecting all terms proportional to $(\overline{\mu^2})^2$, we find the following for the spectral energy density:

$$W_\omega = \frac{F_0^2\omega\overline{\mu^2}k_0^3\overline{l^3}}{6\pi^3 V_0 c_{s0}^2\rho_{00}k_0^2}\left\{\frac{5\pi}{3}\int_0^{\varkappa_{max}}\varkappa\,d\varkappa - \frac{\pi^2}{\overline{l^3}}\int\frac{\partial^2 w(|\xi|)}{\partial\xi^2}|\xi|^2 d\xi - \frac{5\pi}{3}\frac{\omega^2}{V_0^2} \times \mathrm{Re}\int_0^{\varkappa_{max}}\frac{\varkappa\,d\varkappa}{\varkappa^2+\omega^2V_0^{-2}(1-\varepsilon_{\mathrm{eff}}\beta^2)} + \frac{3\omega^2\pi}{V_0^2\overline{\mu^2}k_0^3\overline{l^3}}\mathrm{Im}\int_0^{\varkappa_{max}}\frac{\varkappa\,d\varkappa}{\varkappa^2+\omega^2V_0^{-2}(1-\varepsilon_{\mathrm{eff}}\beta^2)} + \frac{\pi\omega^2}{V_0^2\overline{l^3}}\mathrm{Re}\int K_0\left(\frac{\omega}{V_0}\sqrt{1-\varepsilon_{\mathrm{eff}}\beta^2}\rho\right)\frac{\partial^2 w(|\mathbf{r}_1|)}{\partial\xi^2}|\mathbf{r}_1|^2 d^3 r_1\right\}. \tag{27}$$

Here $K_0(x)$ is the MacDonald function of zero order. The following relations exist:

$$\varkappa_{max}\approx\frac{2\pi}{\overline{l}}, \qquad \int\frac{\partial^2 w}{\partial\xi^2}|\xi|^2 d\xi = \frac{1}{3}\overline{l}.$$

As a result of integration of (27), we get the following expression for the spectral density of the radiation of sound waves:

$$W_\omega = \frac{\omega^4\Gamma^2 S^2\rho_{00}\beta\overline{\mu^2}\,\overline{l^3}}{9\pi^2 V_0^2}\left\{\frac{\gamma}{\omega^2V_0^{-2}\overline{l^2}} + \ln\left|\varkappa_{max}^{-2}\left\{\left[\frac{\omega^2}{V_0^2}(1-a\beta^2)\right]^2 + \left(\frac{2}{3}\overline{\mu^2}k_0^5\overline{l^3}\right)^2\right\}^{1/2}\right| + \tan^{-1}\left[\frac{2\beta^2}{3}\,\frac{\overline{\mu^2}k_0^3\overline{l^3}}{1-a\beta^2}\right]\frac{9}{2\overline{\mu^2}\,k_0^3\,\overline{l^3}}\right\}. \tag{28}$$

Here γ is a coefficient whose magnitude is of the order of unity, $a = 1-5\overline{\mu^2}+2\overline{\mu^2}k_0^2\overline{l^2}$. Upon satisfaction of the condition

$$(1-a\beta^2)\gg\tfrac{2}{3}\overline{\mu^2}\beta^2k_0^3\overline{l^3} \tag{29}$$

Equation (28) simplifies and takes the form

$$W_\omega = \frac{\omega^4\Gamma^2S^2\rho_{00}\beta\overline{\mu^2}\overline{l^3}}{9\pi^2V_0^2}\left\{\frac{\gamma}{\omega^2V_0^{-2}\overline{l^2}} + 2\ln\left|\frac{\omega}{V_0}\frac{\sqrt{1-\beta^2}}{\varkappa_{max}}\right| + \frac{3\beta^2}{1-\beta^2}\right\}. \tag{30}$$

The principal feature of the radiation of sound waves by an object moving in a medium with inhomogeneities is the sharp growth of radiation as one approaches the Cerenkov threshold of radiation of sound waves according to the law $1/(1-\beta^2)$. Of course, it is impossible to use the formula (30) at the threshold itself $(\beta\approx 1/a)$. In this case, the inequality (29) is transformed into its inverse:

$$(1-a\beta^2)\ll\frac{2}{3}\overline{\mu^2}\beta^2k_0^3\overline{l^3}. \tag{31}$$

Taking this fact into account, and leaving in (28) only a single term which can be shown to be larger than all the other terms entering into (28) we get for the spectral intensity of the sound radiation:

$$W_\omega\approx\omega\Gamma^2S^2\rho_{00}c_{s0}/4\pi. \tag{32}$$

As is seen from (32), the energy radiated by the sound waves in this case does not depend on the fluctuations in the index of refraction of the medium. Estimates also show that the method of small perturbations is not violated here; this method was used in the calculation of the mean intensity of the scattered radiation of the sound waves. Thus one can confirm the fact that the expression (32) is actually valid at the threshold $(\beta\approx 1/a)$ for these conditions where one can neglect the viscosity of the medium, inasmuch as the sound absorption in the gas was not into account in the derivation of all the formulas. The condition for neglect of absorption in the expressions (28), (30), and (32) is clearly described in the following manner:

$$\nu/\omega\ll\frac{2}{3}\overline{\mu^2}k_0^3\overline{l^3}, \tag{33}$$

where ν is the damping decrement of the sound waves in time due to the presence of viscosity.

One can similarly obtain formulas for the radiation of electromagnetic waves by charged particles moving in a medium with inhomogeneities. The energy losses of the charge per unit path length are determined by the equation

$$W = \mathrm{Re}\, e\overline{\mathbf{E}}\mathbf{V}_0/|\mathbf{V}_0|,$$

where $\mathbf{V}_0$ is the speed of the moving charged particle and $\overline{\mathbf{E}}$ is the electric field created by the charge at the point where this charge was located and computed from the Maxwell equation under averaging over the ensemble of inhomogeneities of the dielectric constant.

However, we shall make use of already well-known formulas (see, for example, the work of Silin and Rukhadze[15]), setting in them

$$\varepsilon^l(\omega,k)=\varepsilon^{tr}(\omega,k)=\varepsilon_{\mathrm{eff}}(\omega). \tag{34}$$

Then the formula for the loss of energy by the charged particle is described in the form

$$W_\omega = -\frac{2\omega e^2}{\pi V_0^2}\mathrm{Im}\int\frac{(1-\varepsilon_{\mathrm{eff}}\beta^2)\varkappa d\varkappa}{\varepsilon_{\mathrm{eff}}[\varkappa^2+\omega^2V_0^{-2}(1-\varepsilon_{\mathrm{eff}}\beta^2)]}. \tag{35}$$

The expression for $\epsilon_{\mathrm{eff}}(\omega)$ under the condition $k_0 l \ll 1$ has the form (see, for example, the work of Bass et al[16])

$$\varepsilon_{\mathrm{eff}}\approx\varepsilon_0-\tfrac{1}{3}\overline{(\Delta\varepsilon)^2}\,(1-2ik_0^3\,\overline{l^3}\varepsilon_0^{3/2}). \tag{36}$$

Substituting (36) in (35) and computing the integral, we get

$$W_\omega = \frac{\omega^4e^2\,\overline{\Delta\varepsilon^2}\,\overline{l^3}}{6\pi^2c^3V_0^2\,\varepsilon_0^{3/2}}\left\{2\ln\left|\frac{\varkappa_{max}}{\omega V_0^{-1}\sqrt{1-\varepsilon_0\beta^2}}\right|-\varepsilon_0\beta^2\right\}. \tag{37}$$

Thus we obtain an expression for the energy loss by the method of the radiation reaction of a charge in a randomly inhomogeneous medium; this expression is identical with the formula obtained by Kapitza[9] by calculation of the flux of the scattered radiation.

It is interesting to note that one can also consider the radiation of a charge by a similar method, for example in an inhomogeneous magnetoactive plasma and in an inhomogeneous medium with spatial dispersion. For this, it is necessary to know only the corresponding tensor of the effective dielectric constant for the media.

The authors express their gratitude to Professor V. L. Ginzburg and N. G. Denisov for valuable discussion of the results of the research.

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Translated by R. T. Beyer
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