SOME RULES FOR A UNIFIED CLASSIFICATION OF ELEMENTARY PARTICLES

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It is shown that one can include the leptons consistently in a unified classification of elementary particles, if one assumes that muons and muonic neutrinos have unit hypercharge. In this classification the place of each elementary particle is determined by its spin and by three elementary numbers each of which can only take the values 0, ±1. The proposed classification is compatible with conservation of hypercharge in all leptonic decays.

AFTER the discovery of the second kind of neutrinos [1] there have appeared new possibilities for the construction of a unified classification of elementary particles which includes the leptons, too. A unified classification assumes evidently that all particles are characterized by the same elementary numbers (quantum numbers), including strangeness or hypercharge. Consequently, the leptons too must be regarded as particles bearing a certain hypercharge.

Let us regard the muons and muonic neutrinos as "strange" particles and the electrons and usual neutrinos as nonstrange particles. In other words let us assume that electrons and electronic neutrinos have zero hypercharge and that muons and muonic neutrinos have unity hypercharge. In this case all known elementary particles will be described by their spin and three elementary numbers: Q, Y, and n, i.e., the electric charge, the hypercharge, and the baryon number. For all known particles these numbers can only have the values −1, 0, and +1, while the spin can only have the values 0, ½, and 1. Some of the "resonances" may possibly be exceptions to this rule, but it is more correct to consider such resonances as groups of particles "stuck" together rather than as separate elementary particles.

Isospin is another important characteristic by which to group elementary particles. The I_Z component of the isospin of baryons and mesons is connected with Q and Y through the well-known relation:

$$I_z = Q - Y/2. \tag{1}$$

For all known baryons and mesons, the isospin can only have the values 0, ½, 1. If the hypercharge for the leptons is chosen as indicated above, then according to Eq. (1) the value of $|I_Z|$ for leptons will also take only the values 0, ½, and 1. It is important to note, however, that for leptons the quantity I_Z can only formally be called a component of isospin, since in distinction from baryons and mesons the leptons do not form groups of particles with closely spaced masses, which would possess properties describable by isospin space. Taking into account this reservation, we will nevertheless call the quantity I_Z, defined by Eq. (1) for leptons, the isospin projection.

Thus, if one attributes hypercharge to the leptons one can make the assertion that for all known elementary particles the admissible values of spin and isospin are 0, ½, and 1, and that the quantum numbers Q, Y, and n can only be equal to −1, 0, and +1.

This hypothesis is compatible also with the simple rules for the decay processes which can be realized in practice. These rules are the following: 1) in the pionic decays of hyperons and K-mesons the hypercharge changed by ±1 or

$$\Delta Y = -2\Delta I_z = \pm 1, \tag{2}$$

2) in all leptonic decays the hypercharge does not change, i.e.,

$$\Delta Y = -2\Delta I_z = 0. \tag{3}$$

It is easy to see that these rules can be true only if two kinds of neutrinos exist, one having hypercharge, the other not.

One can very simply convince oneself of the truth of the above assertions if instead of hypercharge one uses neutronic charge [2-4] for the characterization of particles, the neutronic charge ϵ being connected with the quantum numbers Q, Y, n, I_Z and with the strangeness S by the following relations:

$$S + n = Y = Q + \varepsilon, \; I_z = (Q - \varepsilon)/2, \tag{4}$$

Usual symbols	Rational symbols	Spin	Hypercharge $Y=Q+\varepsilon$	Isospin projection I_z	Isospin
γ	γ	1	0	0	0
$\nu\ \bar{\nu}$	$\vec{\nu}\ \overleftarrow{\nu}$	$1/2$	0 0	0 0	1
$e^-\ e^+$	$e^-_+\ e^+_-$		0 0	-1 $+1$	1
$\nu'\ \bar{\nu}'$	$\nu_-\ \nu_+$		-1 $+1$	$+1/2$ $-1/2$	$1/2$
$\mu^-\ \mu^+$	$\mu^-\ \mu^+$		-1 $+1$	$-1/2$ $+1/2$	$1/2$
π^0	π	0	0	0	1
$\pi^+\ \pi^-$	$\pi^+_-\ \pi^-_+$		0 0	$+1$ -1	
$K^0\ \bar{K}^0$	$K_+\ K_-$		$+1$ -1	$-1/2$ $+1/2$	$1/2$
$K^+\ K^-$	$K^+\ K^-$		$+1$ -1	$+1/2$ $-1/2$	
$p\ \bar{p}$	${}_1p^+\ {}_{-1}p^-$	$1/2$	$+1$ -1	$+1/2$ $-1/2$	$1/2$
$n\ \bar{n}$	${}_1n_+\ {}_{-1}n_-$		$+1$ -1	$-1/2$ $+1/2$	
$\Lambda^0\ \bar{\Lambda}^0$	${}_1\Lambda\ {}_{-1}\Lambda$		0 0	0 0	0
$\Sigma^+\ \bar{\Sigma}^+$	${}_1\Sigma^+_-\ {}_{-1}\Sigma^-_+$		0 0	$+1$ -1	1
$\Sigma^0\ \bar{\Sigma}^0$	${}_1\Sigma\ {}_{-1}\Sigma$		0 0	0 0	
$\Sigma^-\ \bar{\Sigma}^-$	${}_1\Sigma^-_+\ {}_{-1}\Sigma^+_-$		0 0	-1 $+1$	
$\Xi^0\ \bar{\Xi}^0$	${}_1\Xi_-\ {}_{-1}\Xi_+$		-1 $+1$	$+1/2$ $-1/2$	$1/2$
$\Xi^-\ \Xi^+$	${}_1\Xi^-\ {}_{-1}\Xi^+$		-1 $+1$	$-1/2$ $+1/2$	

and if one uses furthermore the notation for elementary particles proposed in [4], according to which the electric charge is denoted as a right superscript, the neutronic charge as a right subscript, and the baryon number as a left subscript of the symbol used for the particle, thus:

$$ {}_nA^Q_\varepsilon, \qquad (5)$$

where A is the symbol for the particle. In doing so it is convenient not to use any signs for vanishing values of Q, ϵ, or n, to denote positive and negative values of the electric and neutronic charge by the signs + and − respectively, and to denote nonvanishing values of the baryon number by 1 and −1. Furthermore, for the usual (electronic) neutrino, for which all elementary numbers vanish, it is convenient to denote right and left polarization by means of the orientation of an arrow above the symbol.

All elementary particles (excluding resonances) are represented in the table in order of increasing masses [1)]. In the first column the usual symbols are listed, and the second column lists the above-mentioned corresponding rational symbols. The remaining columns contain the most important characteristics of the particles: spin, hypercharge, and isospin. It can be seen from the second column that the lepton and meson groups of particles are formed by taking all possible combinations of two elementary numbers (Q and ϵ) and the baryon group is formed of all combinations of three elementary numbers (Q, ϵ, and n), neither of these numbers being larger than one in absolute value. Only the combinations for which the hypercharge $Q+\epsilon$ becomes larger than one are excluded. The group of spin-1 particles contains only the photon. However, if one includes among the elementary particles the resonances ρ, ω, and K*, then this photonic group is also generated according to the general rule.

It is easy to see that the rules formulated above are realized if one writes out known reactions using the rational symbols. Thus, for instance, for the well known pionic decays

$$ {}_1\Lambda \to {}_1p^+ + \pi^-_+,\quad {}_1\Xi^- \to {}_1\Lambda + \pi^-_+,\quad K^+ \to \pi^+_- + \pi,$$
$$K_+ \to \pi^+_- + \pi^-_+ + \pi \text{ etc.}$$

one gets indeed $|\Delta\epsilon| = |\Delta Y| = 1$. For the known leptonic decays $n_+ \to {}_1p^+ + e^-_+ + \overleftarrow{\nu}$, $\pi^+_- \to \mu^+ + \nu_-$, $\pi^+_- \to e^+_- + \vec{\nu}$, $\mu^+ \to e^+_- + \vec{\nu} + \nu_+$ one obtains $\Delta\epsilon = \Delta Y = 0$, in agreement with the second rule. For the forbidden decays $\mu^+ \to e^+_- + \gamma$, $\mu^+ \to e^+_- + e^-_+ + e^+_-$ etc. one obtains $|\Delta\epsilon| = |\Delta Y| = 1$.

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1)A possible exception to this rule is the muonic neutrino ν', the mass of which is as yet unknown, but which is conveniently placed between the electron and muon.

In order to maintain hypercharge conservation in the leptonic decays of the K mesons, one has obviously to assume that in this case the electrons are formed together with a muonic neutrino, i.e.,

$$K^+ \to e^{\pm}_{-} + \nu_+, \tag{6}$$

and the muons are formed together with an electronic neutrino, i.e.,

$$K^+ \to \mu^+ + \vec{\nu} \text{ or } K^+ \to \mu^+ + \overleftarrow{\nu}. \tag{7}$$

The second of the last two possibilities is compatible with the assumption that the particles $\overleftarrow{\nu}$, e^-_+, ν_+, and μ^+ have the same orientation of their polarization. This also forbids the decays

$$K_+ \to \mu^+ + e^-_+, \tag{8}$$

since μ^+ and e^-_+ have the same polarization. However this alternative seems to be excluded by experimental evidence[1], although this conclusion still requires additional experimental verification.

The first alternative of the decay modes listed in (7) does not contradict experiment[1], since in this case the K^+-meson decay gives birth to a flux of neutrinos, and not antineutrinos. If one admits this alternative, however, one must assume that the $\vec{\nu}$, e^-_+, ν_-, and μ^- have the same polarizations, since only this choice is compatible with the fact that in the K_{μ_2} and π_{μ_2} decays the muons have the same polarization. But in this case the μ^+ and e^-_+ have opposite polarizations and therefore the reaction (8) is not forbidden.

Thus if one starts from the fact that the decay mode (8) is forbidden and if one considers that the experiments[1] exclude the second alternative of the decay mode (7), one must necessarily arrive at the conclusion that the law of conservation of hypercharge in leptonic decays is not sufficient to explain all known hindrances.

From the table it is also clear that the so-called Kiev symmetry (B-L or GMO-symmetry) between the proton, neutron, and lambda-hyperon on the one hand and the electronic neutrino, the electron, and negative muon on the other hand may be due to the fact that the first group is obtained from the second by adding to the quantum numbers of the second a positive unit of baryon number and a positive unit of electric charge[5]. However, the muonic neutrino must not enter into the Kiev symmetry, since by adding to the latter one positive unit of n and Q one obtains the ${}_1\Sigma^{+}_{-}$-hyperon.

It is interesting to note that the other hyperons are obtained from the antileptons $\overleftarrow{\nu}$, e^+_-, ν_+, and μ^+ by adding to these a positive unit of baryon number and a negative electric charge. Consequently all elementary particles can be considered combinations of the eight leptons with four hypothetical bosons ${}_1B^+$, ${}_{-1}B^-$, ${}_1B^-$, and ${}_{-1}B^+$.

Thus, the second neutrino allows one to attribute hypercharge to the leptons in a consistent manner and to include these particles into a unified classification which is a natural generalization of the Gell-Mann–Nishijima classification.

In conclusion, I use this occasion to express my sincere gratitude to R. Feynman for a valuable discussion on the subject.

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