PION PRODUCTION BY HIGH-ENERGY MUONS IN THE FIELD OF THE NUCLEUS

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The production of π mesons on nuclei by high-energy μ mesons in the reaction $\mu + A \rightarrow \pi + \nu + A$ by the Coulomb and diffraction mechanisms is considered. It is shown that the diffraction mechanism yields a reaction cross section which exceeds that of the Coulomb interaction by two orders of magnitude.

1. At high energies it is interesting to study the reactions of the type

$$\mu^{\pm} + A \rightarrow \pi^{\pm} + \nu\,(\bar{\nu}) + A, \tag{1}$$

occurring in the field of the nucleus, since it is possible to calculate the cross section from pole diagrams describing the interaction at low momentum transfer. Measurements of the cross section for the process (1) can be used to test our ideas on weak interactions and yield information on the form factor of the $\pi \rightarrow \mu + \nu$ decay. In the present article we discuss the Coulomb mechanism of π meson production by μ mesons in the reaction (1), and also the diffraction mechanism proposed by Nikitin[1] for the $\nu + A \rightarrow \pi + \mu + A$ process.

2. If the μ meson energy is sufficiently high, and only small transferred momenta are important, we can use the Weizsäcker-Williams method[2] to calculate the cross section for the process (1) occurring in the Coulomb field of the nucleus. The cross section is

$$\sigma_k = \frac{Z^2\alpha}{\pi}\int F^2(t)\,\frac{dt}{t^2}\left[t - \frac{(s-m_\pi^2)^2}{4E_\mu^2}\right]\frac{\sigma_{\text{ph}}(s)\,ds}{s}, \tag{2}$$

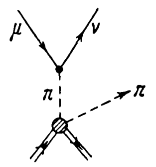
where Z is the nuclear charge, $\alpha = 1/137$, $s = (p_\pi + p_\nu)^2$, t is the square of the momentum transferred to the nucleus, E_μ is the energy of the incident μ meson in the laboratory system (l.s.), $F(t)$ is the electromagnetic nuclear form factor, and $\sigma_{\text{ph}}(s)$ is the cross section for the process $\gamma + \mu^{\pm} \rightarrow \pi^{\pm} + \nu\,(\bar{\nu})$.

In calculating the cross section for such processes, the Lagrangian of the $(\pi\mu\nu)$ interaction is taken in the form[3]

$$L = \frac{G}{\sqrt{2}} f_\pi m_\mu \bar{\psi}_\nu (1 + \gamma_5)\,\psi_\mu \varphi_\pi.$$

In this expression, G is the weak-interaction constant, and $f_\pi \approx m_\pi$ on the mass surface. The form factor $F(t)$ was taken in the form

$$F(t) = (1 + tA^{2/3}/6m_\pi^2)^{-1}$$



where A is the atomic mass number.

It should be noted that, because of the nuclear form factor, the effective values of t are of the order of $t_{\text{eff}} \lesssim 6m_\pi^2 A^{-2/3}$. A kinematical analysis of the process (1) shows that S varies then within the limits $m_\pi^2 \leq s \lesssim m_\mu^2 + 2E\mu\sqrt{t_{\text{eff}}}$. Numerical results of the integration of (2) over t and s in the limits given above are given in Table I.

3. Let us consider now the diffraction mechanism of π-meson production in reaction (1). The diagram corresponding to the process under consideration is shown in the figure. The diffraction mechanism occurs if the component of the momentum $q_{\parallel}$ transferred to the nucleus parallel to the initial direction of the beam satisfies the inequality $q_{\parallel} \lesssim 1/R$[4] (where R is the nuclear radius), and the perpendicular component $q_{\perp} \lesssim m_\pi$.[4] The first condition corresponds to the fact that the interaction occurs at large distances from the nucleus and the details of the strong interaction between the π mesons and the nucleus become insignificant. The second condition corresponds to the fact that the whole nucleus participates in the interaction. The differential cross section for the process (1) was calculated according to the rule given in [5]

$$d\sigma_d = \frac{G^2 f_\pi^2 m_\mu^2 R^2 J_1^2(q_\perp R)\,d\mathbf{q}_\perp\,ds}{8\,(2\pi)^3 E_\mu^3 q_\perp^2}$$

$$\times\int\frac{[(p_\pi P)^2/M^2 - m_\pi^2]\,(p_\mu p_\nu)}{[p_\mu p_\nu + (m_\pi^2 - m_\mu^2)/2]^2}\,\frac{d^3p_\pi}{E_\pi}\,\frac{d^3p_\nu}{E_\nu}\,\delta(g - p_\pi - p_\nu). \tag{3}$$

where $g^2 = s = (p_\pi + p_\nu)^2$, P is the four-momentum of the nucleus, p_μ, p_ν, and p_π are four-momenta of the corresponding particles, $J_1(q_\perp R)$

Table I

E_μ, BeV	$\sigma_{k\,\mathrm{Fe}}\cdot 10^{42}\,\mathrm{cm}^2$	$\sigma_{k\,\mathrm{Pb}}\cdot 10^{42}\,\mathrm{cm}^2$
1	0.268	1.73
2	0.507	3.63
3	0.681	5.06
5	0.924	7.12
10	1.298	10.30
20	1.688	13.80
50	1.950	18.70

is the Bessel function, and E_μ is the μ meson energy in the l.s. It follows from Eq. (3) that the effective angle of emission of the produced π mesons in the l.s. is $\theta \sim m_\pi/E_\mu$. If $q_{\|} \lesssim 1/R$, then the maximum angle of emission in the l.s. is $\theta_{\max} \lesssim A^{-1/3}$ for $E_\mu \gtrsim m_\pi A^{1/3}/4$.

Integrating (3) over all variables excepting s, we obtain the distribution of the particles with respect to the square of their total mass s:

$$d\sigma_d = \frac{G^2 f_\pi^2 m_\mu^2 R^2}{32\pi} \frac{(s + m_\mu^2)(s - m_\pi^2)^2}{s^2 (s - m_\mu^2)^2} ds. \tag{4}$$

The quantity s varies within the limits $m_\pi^2 \le s \lesssim m_\mu^2 + 2E_\mu/R$. In the integration it was taken into account that for heavy nuclei $m_\pi R \gg 1$. The final result after integrating over s is

$$\sigma_d = \frac{G^2 f_\pi^2 m_\mu^2 R^2}{32\pi}\left[\ln\frac{s_{max}}{2m_\pi^2} + \frac{0{,}28 m_\pi^2}{s_{max}}\right]$$

$$\approx \frac{G^2 f_\pi^2 m_\mu^2 R^2}{32\pi}\ln\frac{E_\mu}{m_\pi A^{1/3}} \quad \text{for } s_{max} \gg m_\pi^2.$$

The numerical values of σ_d for Fe and Pb are given in Table II.

Table II

E_μ, BeV	$\sigma_{d\,\mathrm{Fe}}\cdot 10^{40}\,\mathrm{cm}^2$	$\sigma_{d\,\mathrm{Pb}}\cdot 10^{40}\,\mathrm{cm}^2$
1	0.636	1.00
2	1.156	2.01
3	1.465	2.70
5	1.870	3.60
10	2.440	4.87
20	3.00	6.17
50	3.740	7.93

We find thus that the diffraction mechanism gives a cross section greater by approximately two orders of magnitude than for the case of π meson production in the Coulomb field of the nucleus.

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