ON THE NUMBER OF "FREE CARRIERS" IN BISMUTH-TYPE METALS AT HIGH TEMPERATURES

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The number of free carriers in bismuth-type metals is investigated on the basis of the electron spectrum obtained in [1] for such metals. It is shown that as a result of the appearance of open energy surfaces for deep energy levels the number of carriers at high temperatures may be similar to that in semiconductors of intrinsic conductivity. Consequences of this are discussed in relation to the conductivity and optical properties.

THE electron spectrum of metals with bismuth-type lattice has been obtained earlier.[1] An interesting property of this spectrum is the fact that the energy surfaces can be closed or open.

The principal properties of metals are usually determined by the neighborhood of the Fermi surface. Numerous experimental data indicate that in the case of bismuth[2] and antimony[3] the Fermi surfaces are closed and have volumes corresponding to about 10^{-5} electrons per atom. This number is in agreement with theory.[1] The case of arsenic is not clear[4] and it may have open surfaces. The parameters which represent the arsenic lattice should (according to [1]) correspond to about 10^{-2} electrons per atom.

However, in contrast to the normal "good" metals, in the case of bismuth-type metals we are interested not only in the neighborhood of the Fermi surface. The scale of energies in these metals (for example the Fermi energy taken from the bottom of the conduction band, or the separation of the bottom of the conduction band from the upper edge of the nearest band, etc.) extends only over several hundredths of an electron-volt. This means that at temperatures of the order of hundreds of degrees the properties of these metals are governed by the whole energy spectrum and not only by the neighborhood of the Fermi surface.[1)] Most important is the fact that by varying the energy we can go over from closed energy surfaces to open ones.[2)]

In the present work we shall consider what influence this effect has on the number of "free carriers" in bismuth-type metals. We shall deal only with metals in which the Fermi surfaces are closed.

First of all we shall establish precisely the conditions for the appearance of open energy surfaces. This can be done most simply for the vicinity of the point $\mathbf{k} = 0$. According to [1] in the vicinity of this point, which has a rhombohedral (trigonal) small group, we obtain four functions $\epsilon_i^{(0)}(\mathbf{k})$ corresponding to four bands:

$$\varepsilon_i^{(0)} = f \pm [\Delta^2 + p^2 + \gamma^2 + q^2 \pm 2(\gamma^2\Delta^2 + p^2q^2 + \Delta^2p^2)^{1/2}]^{1/2}, \tag{1}$$

where $p = ak_z$; $q = b\sqrt{k_x^2 + k_y^2}$; z is the direction along the rhombohedral axis; and f, Δ, γ, a, and b are constants. Two of these functions ($\epsilon_1^{(0)}$, $\epsilon_2^{(0)}$) decrease as $|p| \to \infty$, $q \to \infty$, while the other two ($\epsilon_3^{(0)}$, $\epsilon_4^{(0)}$) increase.

There are two different cases. If $|\gamma| > |\Delta|$ then the minima of the increasing and maxima of the decreasing functions, or in other words the band edges, correspond to the point $\mathbf{k} = 0$. We then obtain

$$\varepsilon_{1max}^{(0)} = f - |\gamma| - |\Delta|, \quad \varepsilon_{2max}^{(0)} = f - |\gamma| + |\Delta|,$$
$$\varepsilon_{3min}^{(0)} = f + |\gamma| - |\Delta|, \quad \varepsilon_{4min}^{(0)} = f + |\gamma| + |\Delta|. \tag{2}$$

If, however, $|\gamma| < |\Delta|$ then the upper edge of the lowest band $\epsilon_1^{(0)}$ and the lower edge of the highest band $\epsilon_4^{(0)}$ correspond to the point $\mathbf{k} = 0$, while the edges of the middle bands correspond to points $q = 0$, $p = \pm\sqrt{\Delta^2 - \gamma^2}$. Then we have

$$\varepsilon_{1max}^{(0)} = f - |\gamma| - |\Delta|, \quad \varepsilon_{2max}^{(0)} = f,$$
$$\varepsilon_{3min}^{(0)} = f, \quad \varepsilon_{4min}^{(0)} = f + |\gamma| + |\Delta|. \tag{3}$$

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1) Arguments for the applicability of the quasi-particle model proposed here will be given in a later communication dealing with the behavior of bismuth-type metals under the action of infrared radiation.

2) The possible types of surfaces near $\mathbf{k} = 0$ are listed in [1].

Irrespective of the relationship between $|\Delta|$ and $|\gamma|$, open energy surfaces appear only for the functions $\epsilon_2^{(0)}$, $\epsilon_3^{(0)}$ on condition that

$$|\varepsilon_i^{(0)} - f| > |\gamma|. \tag{4}$$

This means that if, for example, we go from the upper edge of the band $\epsilon_2^{(0)}$ downward then we necessarily obtain first a series of closed energy surfaces, which then become open and finally the band $\epsilon_1^{(0)}$ is included which gives only closed surfaces. The same will occur when the energy is increased beginning from the lower edge of the band $\epsilon_3^{(0)}$.

According to experimental data,[2,3] there are holes in the vicinity of $\mathbf{k} = 0$ in bismuth and antimony, and the Fermi surface is closed. This means that $f > 0$ and the Fermi energy $\epsilon = \mu(0)$ is in the range

$$\varepsilon_{2max}^{(0)} > \mu(0) > f - |\gamma|. \tag{5}$$

The electron parts of the Fermi surface lie in the region of the points $\mathbf{k} = \mathbf{k}_i$, which have the small group C_{2h}. According to the experimental data,[2,3] each such part of the Fermi surface is close to an ellipsoid with its center at a point $\mathbf{k}_i$.

From the theory we again have, as in the vicinity of $\mathbf{k} = 0$, two alternatives: either the maxima of the bands ϵ_1, ϵ_2 and the minima of ϵ_3, ϵ_4 correspond to $\mathbf{k} = \mathbf{k}_i$, or the extremal points of ϵ_2 and ϵ_3 are displaced. According to the experimental data the first case is realized. In view of this, substituting into the general formula (10) in [1] $\kappa_x = \kappa_y = \kappa_z = 0$ and $\gamma^{(1)} = -\gamma/3$, $\Omega^{(1)} = -f/3 - \epsilon$, we obtain

$$\varepsilon_{1max}^{(1)} = -\tfrac{1}{3}f - \tfrac{1}{3}|\gamma| - \sqrt{[\beta + \delta \operatorname{sign}\gamma]^2 + \Delta^2},$$

$$\varepsilon_{2max}^{(1)} = -\tfrac{1}{3}f - \tfrac{1}{3}|\gamma| + \sqrt{[\beta + \delta \operatorname{sign}\gamma]^2 + \Delta^2},$$

$$\varepsilon_{3min}^{(1)} = -\tfrac{1}{3}f + \tfrac{1}{3}|\gamma| - \sqrt{[\beta - \delta \operatorname{sign}\gamma]^2 + \Delta^2},$$

$$\varepsilon_{4min}^{(1)} = -\tfrac{1}{3}f + \tfrac{1}{3}|\gamma| + \sqrt{[\beta - \delta \operatorname{sign}\gamma]^2 + \Delta^2}. \tag{6}$$

This applies when the condition $\epsilon_{2max}^{(1)} < \epsilon_{3min}^{(1)}$ is satisfied, i.e.,

$$\tfrac{2}{3}|\gamma| > \sqrt{(\beta + \delta)^2 + \Delta^2} + \sqrt{(\beta - \delta)^2 + \Delta^2}. \tag{7}$$

In the earlier work[1] conditions were obtained [formula (7)] for the energy surface to be closed. Substituting $\Omega^{(1)} = -\epsilon - f/3$, $\gamma^{(1)} = -\gamma/3$, we obtain a range corresponding to closed surfaces:

$$-\tfrac{1}{3}f + \tfrac{1}{3}|\gamma| - |\beta - \delta \operatorname{sign}\gamma| > \varepsilon > -\tfrac{1}{3}f - \tfrac{1}{3}|\gamma| + |\beta + \delta \operatorname{sign}\gamma|. \tag{8}$$

From this we see that the order of appearance of surfaces of various topological types is completely analogous to the situation in the vicinity of $\mathbf{k} = 0$.

However, open surfaces appear in the vicinity of $\mathbf{k} = \mathbf{k}_i$ in a different way than near $\mathbf{k} = 0$. In the latter case for $\epsilon = f \pm |\gamma|$ open surfaces appear along directions which cover a complete conical surface. In the vicinity of $\mathbf{k} = \mathbf{k}_i$ the energy surface becomes open along one direction only [cf. formula (16) in [1]]. Away from the boundary the range of open surfaces becomes finite and proportional to $\epsilon - \epsilon_c$, where ϵ_c is the threshold energy.

In order to satisfy the experimental data the Fermi energy $\epsilon = \mu(0)$ should be in the range

$$-\tfrac{1}{3}f + \tfrac{1}{3}|\gamma| - \sqrt{[\beta - \delta \operatorname{sign}\gamma]^2 + \Delta^2} < \mu(0) < -\tfrac{1}{3}f + \tfrac{1}{3}|\gamma| - |\beta - \delta \operatorname{sign}\gamma|. \tag{9}$$

The condition of Eq. (7) obviously means that in the vicinity of $\mathbf{k} = 0$ we also have the case when all the extrema of $\epsilon_i^{(0)}(\mathbf{k})$ are at one point $\mathbf{k} = 0$. Then the expressions in Eq. (2) apply and, according to Eq. (5)

$$f - |\gamma| < \mu(0) < f - |\gamma| + \Delta. \tag{10}$$

The compatibility of conditions (9) and (10) requires that f should be in the range

$$|\gamma| - \tfrac{3}{4}|\beta - \delta \operatorname{sign}\gamma| > f > |\gamma| - \tfrac{3}{4}(\Delta + \sqrt{\Delta^2 + [\beta - \delta \operatorname{sign}\gamma]^2}). \tag{11}$$

Obviously under real conditions all the parameters f, $|\gamma|$, $|\beta|$, $|\delta|$, and $|\Delta|$ are of the same order of magnitude. At temperatures $T \ll |\gamma|$ the number of holes and electrons is governed by the volumes of the electron and hole parts of the Fermi surface, which are equal. These numbers will change as the temperature is increased. At first this leads to the usual corrections of the order of $(T/\epsilon_F)^2$, but at higher temperatures there may be more important corrections due to deep levels with open energy surfaces. On further increase of temperature we can have the situation when the number of carriers is completely governed by these levels.

Let us determine the additional number of carriers due to such levels. Since open energy surfaces appear only in the second and third bands, the number of additional electrons is

$$\Delta N_e = 3\int d\varepsilon \frac{dZ_3^{(1)}/d\varepsilon}{e^{(\varepsilon-\mu)/T} + 1} + \int d\varepsilon \frac{dZ_3^{(0)}/d\varepsilon}{e^{(\varepsilon-\mu)/T} + 1}, \tag{12}$$

where, as before, the subscripts 0 and 1 in the integrals indicate respectively the vicinities of the points $k = 0$ and $k = k_i$, and $dZ_3/d\epsilon$ is the density of states in the range of energies in the third band. The number of additional holes is

$$\Delta N_h = 3\int d\varepsilon \frac{|dZ_2^{(1)}/d\varepsilon|}{e^{(\mu-\varepsilon)/T}+1} + \int d\varepsilon \frac{|dZ_2^{(0)}/d\varepsilon|}{e^{(\mu-\varepsilon)/T}+1}. \quad (13)$$

We shall assume that the exponential terms in the denominators of Eqs. (12) and (13) are large compared with unity. In view of the rapid decrease of the integrands the main contribution comes only from the close vicinity of the values of ϵ_i at which open energy surfaces first appear. Since the corresponding value of $\epsilon_3^{(0)}$ is greater than $\epsilon_3^{(1)}$, the second integral in Eq. (12) can be neglected. The relationship between the thresholds for the two integrals in Eq. (13) is less definite. The difference between the threshold for the vicinity of $k = k_i$ and that of $k = 0$ is

$$\tfrac{4}{3}f - \tfrac{2}{3}|\gamma| - |\beta + \delta\,\mathrm{sign}\,\gamma|.$$

According to Eq. (11) this difference is greater than

$$\tfrac{2}{3}|\gamma| - \Delta - |\beta + \delta\,\mathrm{sign}\,\gamma| - \sqrt{\Delta^2 + [\beta - \delta\,\mathrm{sign}\,\gamma]^2}.$$

The sign of this quantity is, strictly speaking, indeterminate although the inequality (7) gives us grounds for assuming that it is most probably positive. In view of this we shall leave only the second of the integrals in Eq. (13).

The density of states $|dZ_2^{(0)}/d\epsilon|$ can be easily determined by means of Eq. (1) rewritten with q as a function of p and $\epsilon_2^{(0)}$. We then have

$$\left|\frac{dZ_2^{(0)}}{d\varepsilon}\right| = \frac{1}{2\pi^2 ab^2}\int dp\, \frac{dq}{d\varepsilon_2^{(0)}}.$$

Here we must allow for the fact that $q^2(p, \epsilon_2^{(0)})$ is a two-valued function and the above integral is in fact the difference of two integrals with different $q^2(p, \epsilon_2^{(0)})$. This doubles the result. Another multiplier of 2 appears because p can have different signs. As a result of this we find that

$$\left|\frac{dZ_2^{(0)}}{d\varepsilon}\right| = \frac{2}{\pi^2 ab^2}\int \frac{(f-\varepsilon)\,p^2\,dp}{\{[(f-\varepsilon)^2-\gamma^2]\,p^2+\gamma^2\Delta^2\}^{1/2}}.$$

Beyond the threshold, i.e., when $\epsilon < f - |\gamma|$, large momenta, of the order of the period of the reciprocal lattice, i.e., values of p of the order of 1 eV,[3)] become important. Then we obtain finally[4)]

$$\left|\frac{dZ_2^{(0)}}{d\varepsilon}\right| = A\,\frac{(f-\varepsilon)}{\sqrt{(f-\varepsilon)^2-\gamma^2}} \quad (\varepsilon < f - |\gamma|), \quad (14)$$

where

$$A = \frac{1}{\pi^2 ab^2}\int 2p\,dp \sim \frac{E_0^2}{\pi^2 v^3}$$

$$(E_0 \sim 1\text{ eV},\ v \sim 10^8\text{ cm/sec}). \quad (15)$$

To find the density of states for electrons we shall proceed in an analogous way. Since we are interested in large momenta, we shall use formula (15) from [1]. We have

$$\frac{dZ_3^{(1)}}{d\varepsilon} = \frac{1}{4\pi^3 ab^2}\int dp\,dq_y\,\frac{dq_x}{d\varepsilon_3^{(1)}} = \frac{2}{\pi^3 ab^2}\int_{p>0} dp\,dq_y$$

$$\times \frac{[(^1/_3 f+\varepsilon)\,p - \beta q_y p]}{\{[(^1/_3 f+\varepsilon)\,p - \beta q_y]^2 - (^1/_3\gamma p - \delta q_y)^2\}^{1/2}\sqrt{p^2-q_y^2}}$$

$$= \frac{A}{\pi}\int ds\,\frac{^1/_3 f + \varepsilon - \beta s}{\{(^1/_3 f+\varepsilon-\beta s)^2 - (^1/_3\gamma - \delta s)^2\}^{1/2}\sqrt{1-s^2}},$$

where $s = q_y/p$ and allowance is made for the fact that we are interested in large momenta.

The limits of the integral with respect to s are automatically determined by the condition for the surface $\epsilon_3^{(1)} = \mathrm{const}$ to be open. According to [1], these conditions reduce to the requirement that both roots in the denominator of the last integral should be real. It was shown in [1] that open surfaces appear first along the directions $q_x = 0$, $q_y = p$, or $q_y = -p$, depending on the values of the coefficients. When ϵ exceeds the threshold there is a range of permissible values of s. We shall not describe all the possibilities or give all the calculations. In general the result is a complete elliptical integral of the first kind, but we need only the limiting value at $\epsilon \to \epsilon_c$. This value is

$$\frac{dZ_3^{(1)}}{d\varepsilon} \to \frac{A}{2}\left\{\frac{^1/_3|\gamma| + \delta\,\mathrm{sign}\,(\beta - \delta\,\mathrm{sign}\,\gamma)}{|\beta - \delta\,\mathrm{sign}\,\gamma|}\right\}^{1/2}$$

$$\left(\varepsilon \to -\frac{f}{3} + \frac{|\gamma|}{3} - |\beta - \delta\,\mathrm{sign}\gamma|\right). \quad (16)$$

Substituting Eqs. (15), (16) into Eqs. (13) and (12) we obtain

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[3)]Strictly speaking in the case of large momenta we cannot use the formulas obtained in [1]. However, the error is probably small because of two circumstances. First, the spectrum considered has the property that when deformation vanishes the conduction and valence band touch along the energy surface (cf. [1]). It is assumed that the intrinsic spectrum has this property at all values of momenta. Second, the maximum possible values of the momentum for "open" surfaces do not exceed 1/3 of the reciprocal lattice period, i.e., they may be considered relatively "small."

[4)]This expression is not in fact quite accurate. Open surfaces appear not immediately but gradually over the range of energies $|\gamma|(\Delta/E_0)^2$. Consequently $|dZ_2^0/d\epsilon|$ for $\epsilon = f - |\gamma|$ does not become infinite but reaches values of the order of $A(E_0/\Delta)$. However, this point is of little importance in later parts of our treatment.

$$\Delta N_e = \frac{3}{2} A \left\{\frac{{}^1\!/_3\,|\gamma| + \delta\,\mathrm{sign}\,(\beta - \delta\,\mathrm{sign}\,\gamma)}{|\beta - \delta\,\mathrm{sign}\,\gamma|}\right\}^{1/2} T$$
$$\times \exp\left\{\left(\mu + \frac{f}{3} - \frac{|\gamma|}{3} + |\beta - \delta\,\mathrm{sign}\,\gamma|\right)\frac{1}{T}\right\},$$
$$\Delta N_h = A\sqrt{\frac{\pi|\gamma|T}{2}}\exp\left\{\frac{f - |\gamma| - \mu}{T}\right\}. \qquad (17)$$

We shall consider two limiting cases. In the first case we shall assume that ΔN_e and ΔN_h are small compared with the number of carriers at $T = 0$, but at least one of these quantities represents the main part of the correction, i.e. [5]

$$\Delta N_e/N(0) \gg (T/\varepsilon_F)^2 \text{ or } \Delta N_h/N(0) \gg (T/\varepsilon_F)^2.$$

From the condition of equality of the number of electrons and holes we find

$$\Delta\mu = (\Delta N_h - \Delta N_e)\Big/\left[3\left(\frac{dZ_3^{(1)}}{d\varepsilon}\right)_{\mu(0)} + \left|\frac{dZ_2^{(0)}}{d\varepsilon}\right|_{\mu(0)}\right]. \qquad (18)$$

For the total number of electrons and holes we obtain

$$N_e(T) = N_h(T) = N_e(0) + 3\left(\frac{dZ_3^{(1)}}{d\varepsilon}\right)_{\mu(0)}\Delta\mu + \Delta N_e$$
$$= N_e(0) + \left[3\Delta N_h\left(\frac{dZ_3^{(1)}}{d\varepsilon}\right)_{\mu(0)}\right.$$
$$\left.+ \Delta N_e\left|\frac{dZ_2^{(0)}}{d\varepsilon}\right|_{\mu(0)}\right]\Big/\left[3\left(\frac{dZ_3^{(1)}}{d\varepsilon}\right)_{\mu(0)} + \left|\frac{dZ_2^{(0)}}{d\varepsilon}\right|_{\mu(0)}\right]. \qquad (19)$$

The quantities ΔN_e and ΔN_h in the above equation are given by Eq. (17) with $\mu = \mu(0)$.

Now we shall consider the other limiting case when open surfaces are dominant: i.e., $\Delta N_h \approx N_h$ and $\Delta N_e \approx N_e$. From the equality of the numbers of electrons and holes we find

$$\mu = \frac{f - |\gamma|}{3} - \frac{|\beta - \delta\,\mathrm{sign}\,\gamma|}{2}$$
$$+ \frac{T}{2}\ln\left\{\frac{4}{3}\left[\frac{\pi|\gamma|}{2T}\,\frac{|\beta - \delta\,\mathrm{sign}\,\gamma|}{{}^1\!/_3\,|\gamma| + \delta\,\mathrm{sign}\,(\beta - \delta\,\mathrm{sign}\,\gamma)}\right]^{1/2}\right\}, \qquad (20)$$

$$N_e = N_h = A\left\{\sqrt{\frac{3}{4}}\left[\frac{\pi|\gamma|\,[{}^1\!/_3\,|\gamma| + \delta\,\mathrm{sign}\,(\beta - \delta\,\mathrm{sign}\,\gamma)]}{2|\beta - \delta\,\mathrm{sign}\,\gamma|}\right]^{1/4} T^{3/4}\right.$$
$$\left.\times \exp\left\{-\left(\frac{2|\gamma|}{3} - \frac{2f}{3} - \frac{|\beta - \delta\,\mathrm{sign}\,\gamma|}{2}\right)\frac{1}{T}\right\}. \qquad (21)$$

Let us consider in greater detail the possibilities of various laws being applicable. First it should be noted that all the characteristic energies of conduction—the Debye frequency ω_D, the Fermi energy ϵ_F, the powers of the exponential terms in Eqs. (19) and (22)—are of the same order of magnitude. Furthermore the exponential laws represent rapid variation and the estimates of the corresponding quantities require exact values of the powers. Finally the melting points of bismuth (542°K) and antimony (903°K) are of the same order of magnitude as the characteristic energies. All this makes it difficult to predict in which temperature ranges the different laws apply.

[5] At present numerical values of the parameters are not available and, therefore, we cannot say which of the thresholds for the appearance of open surfaces is closer to $\mu(0)$.

APPENDIX (received April 5, 1963)

Let us consider in which physical phenomena these effects may appear. It is very natural to assume that they may appear in conduction. However, this is not so. For a considerable number of carriers to appear at levels with open surfaces we need temperatures slightly less than the activation energies which appear in the powers of terms in Eqs. (17) and (21). These energies ($\sim|\gamma|$) are most probably of the order of several hundredths or tenths of an electron-volt, i.e., they are larger than the Debye energy of phonons (117°K). At these temperatures the resistance is due to the interaction between electrons and phonons, the momenta of the latter being $k \sim \pi/a$, where a is the lattice constant. On absorption of such a phonon electrons at levels with closed surfaces can undergo a transition only to another closed surface. This limits greatly the region of solid angles in the integral for phonon momenta and leads to the appearance of a multiplier of the order of $(\epsilon_F/E_0)^2$ in the probability for scattering and consequently to a corresponding rise of conductivity. There is no such limitation for electrons at levels with open surfaces and consequently there is no such multiplier in the scattering probability. It can easily be seen that in the conductivity this compensates exactly the gain in the pre-exponential multiplier in Eqs. (17) and (21) compared with the number of electrons at $T = 0$. Since the exponential term is assumed to be small, the contribution to the conductivity will consequently be small.

The presence of additional carriers obviously affects the optical properties of bismuth-type metals at low frequencies $\omega < |\gamma|$ in the region of temperatures under consideration. Since the carriers considered here obey the Boltzmann distribution law for which $\partial n/\partial\epsilon = -n/T$ and $\partial\epsilon/\partial p \sim v \sim 10^8$ cm/sec, it can easily be shown that in the case $\omega\tau \gg 1$, where τ is the time between collisions, the permittivity tensor ϵ_{ik} should have an additional term of the order of $-e^2v^2\Delta N/T\omega^2$, where ΔN is given by Eqs. (17) or (21). In the opposite case of $\omega\tau \ll 1$ the additional term is imaginary and of the order of $e^2v^2\Delta N\tau_i/\omega T$.

For the electrons considered the value of τ is of the order of $1/T$ so that in principle we can have both cases. In the first case the ratio of the additional term to $-N(0)e^2/m^*\omega^2$, which occurs in ϵ_{ik} at $T = 0$, is of the order of $(E_0^2/\gamma T)e^{-U/T}$ where $U \sim \gamma$, $E_0 \sim 1$ eV. In the second case we have an imaginary term $\Delta\epsilon_{ik} \sim i(E_0^2\gamma/T^2\omega)e^{-U/T}$.

For electrons at levels with closed surfaces $\tau \sim E_0^2/\gamma^2 T$, as pointed out above. If we assume that $\omega\tau \gg 1$ ($E_0 \sim 1$ eV), then the imaginary addition to ϵ_{ik} due to these electrons is of the order of $\max(E_0^2/\gamma^2, \gamma^2/\omega^2)i/\omega\tau$ (this will be proved in a later communication). Consequently the ratio of this imaginary addition to the existing term is of the order of $\min(E_0^2\gamma/T^3, E_0^4\omega^2/\gamma^3T^3)e^{-U/T}$. According to the available experimental data $E_0^2/\gamma^2 \sim 100$.

Consequently the effects considered may become quite noticeable, and even dominate the effect of electrons at the Fermi surface, at temperatures $T \sim |\gamma|$. It must be noted, however, that the exponential laws represent rapid variation and, therefore, the actual magnitude of the effect depends on the exact values of the activation energies or, in other words, the parameters of the spectrum. This is particularly important in view of the fact that the melting points of the metals under consideration are of the same order of magnitude (542°K for Bi and 903°K for Sb).

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