FEASIBILITY OF ZERO-SOUND OSCILLATIONS IN METALS

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Zero-sound electron oscillations in an anisotropic metal are studied on the basis of the theory of Fermi liquids. Spin as well as non-spin oscillations are possible. The latter apparently exist in any type of metal and possess a linear dispersion law throughout the whole frequency range. Spinless waves can exist if some restrictions are imposed on the magnitude of the Fermi-liquid interaction; these restrictions can be appreciably relaxed for symmetric directions in the crystal. The non-spin oscillations have two linear dispersion regions in the radio-frequency and infrared ranges. The possibility of observing zero sound in metals is discussed.

ONE of the most important results of the Landau theory of the Fermi liquid [1] was the existence of the so-called "zero" sound oscillations in He^3. It has turned out that ordinary acoustic oscillations of the Fermi liquid attenuate at sufficiently low temperatures. Instead, waves with a linear dispersion law and with a speed higher than that of ordinary sound can propagate in He^3 at $T = 0$. The existence of zero-sound oscillations is connected with the self-consistent interaction of the excitations.

According to [1], the energy change $\delta\epsilon(\mathbf{p})$ of an excitation with momentum $\mathbf{p}$, resulting from variation of the distribution function $\delta n(\mathbf{p}')$, is equal to

$$\delta\varepsilon(\mathbf{p}) = \int f(\mathbf{p}, \mathbf{p}')\, \delta n(\mathbf{p}')\, d\tau'. \qquad (1)$$

The function f, which is connected with the amplitude of the mutual zero-angle scattering of the excitations [2], determines the propagation velocity of the zero-sound oscillations.

Owing to experimental difficulties, zero sound has not yet been observed in He^3. In the present note we discuss the possibility of producing zero-sound oscillations in metals whose conduction electrons also form a sort of Fermi liquid. In addition to general physical interest that attaches to this phenomenon, the study of zero sound in metals is attractive also because one can attempt to excite it by radio-frequency and optical methods.

A specific feature of the electron Fermi liquid in metals is the presence of Coulomb interactions between the electrons and between the electrons and the ions. This creates a situation wherein there are no low-excitation-energy oscillations, accompanied by electron-density oscillations in the electron liquid. The expected zero-sound oscillations in He^3 (see also [3]) are accompanied by changes of the density and therefore cannot exist in a metal. Zero sound in a degenerate isotropic electron plasma was investigated by Silin [4]. Unfortunately, Silin's result cannot be ascribed to a real metal, since the presence of anisotropy, as will be shown below, leads to a whole series of essential singularities. We shall also discuss the possibility of observing zero sound in metals.

Let us consider the kinetic equation for electronic excitations in a metal:

$$\frac{\partial n}{\partial t} + \frac{\partial \varepsilon}{\partial \mathbf{p}} \frac{\partial n}{\partial \mathbf{r}} - \frac{\partial \varepsilon}{\partial \mathbf{r}} \frac{\partial n}{\partial \mathbf{p}} = J_{st},$$

which we write for small deviations δn from the equilibrium state:

$$\frac{\partial \delta n}{\partial t} + \mathbf{v}\frac{\partial \delta n}{\partial \mathbf{r}} - \frac{\partial n_0}{\partial \varepsilon}\mathbf{v}\frac{\partial \delta\varepsilon}{\partial \mathbf{r}} + e\mathbf{E}\mathbf{v}\frac{\partial n_0}{\partial \varepsilon} = 0. \qquad (2)$$

We have neglected in (2) the collision integral, assuming henceforth that the range of the excitations, due at low temperatures to the scattering of the electrons by the impurities or by the phonons, is large compared with the wavelength of the zero-sound oscillations.

The term with $\partial\delta\epsilon/\partial\mathbf{r}$, first introduced by Landau, where $\delta\epsilon$ is given by (1), is the result of the interaction of the excitations in the Fermi liquid. For simplicity we assume for the time being that the function $f(\mathbf{p}, \mathbf{p}')$ does not contain a dependence on the electron spins.

E is the electric field, which accompanies the oscillations of the electron liquid in a metal. As already mentioned, the variation of the electron density should be equal to zero, for otherwise the oscillation frequency would be of the order of the plasma frequency ω_0 ($\omega_0 \sim 1$ eV in metals). In this region of frequencies, (2) is no longer valid. We shall determine the longitudinal part of the field **E** in (2) from the condition that the charge-density oscillations be equal to zero. The transverse part of the field **E** is connected with the transverse-current oscillations, which generally speaking cannot vanish, as will be shown presently. Let us change over to Fourier components in (2) and represent δn in the form

$$\delta n(\mathbf{p}) = \nu \partial n_0/\partial \varepsilon,$$

where ν depends only on the position of **p** directly on the Fermi surface itself. For ν we have

$$\omega\nu - \mathbf{kv}\hat{L}(\nu) + ie\mathbf{Ev} = 0, \tag{3}$$

where $\hat{L}(\nu) = \nu + \int f(\mathbf{p},\mathbf{p}')\nu(\mathbf{p}')dS$, and dS is the Fermi "surface element":

$$2d^3p/(2\pi)^3 = d\varepsilon\, dS. \tag{4}$$

The expressions for the charge density ρ and for the current **j** are obviously

$$\rho = -e\int \nu dS, \qquad \mathbf{j} = -e\int \mathbf{v}\hat{L}(\nu)\, dS. \tag{5}$$

Let us eliminate **E** from (3), expressing the transverse field $\mathbf{E}_\perp$ in terms of the current with the aid of Maxwell's equations, and using the condition $\rho = 0$ for the longitudinal field $\mathbf{E}_{||}$. For this purpose we introduce

$$\omega\nu + ie\mathbf{Ev} = \omega X. \tag{6}$$

We then have in place of (3)

$$\omega X = \mathbf{kv}\hat{L}(\nu). \tag{7}$$

We apply to (6) the operator $\hat{L}$, multiply by **v**, and integrate over the Fermi surface. As a result we obtain

$$-\omega j_i + ie^2 E_k \int (v_i \hat{L}(v_k))\, dS = \omega e \int v_i \hat{L}(X)\, dS.$$

For the longitudinal field $\mathbf{E}_{||}$ we obtain directly from the condition of vanishing of the longitudinal current

$$ieE_{||} = \omega \int v_{||}\hat{L}(X)\, dS \Big/ \int v_{||}\hat{L}(v_{||})\, dS$$
$$= \omega \int v_{||}\hat{L}(X)\, dS \Big/ \frac{1}{3}\int \mathbf{v}\hat{L}(\mathbf{v})\, dS. \tag{8}$$

(To be specific, we confine ourselves to a crystal of cubic symmetry.)

From Maxwell's equations it follows that

$$\mathbf{j}_\perp = -i\,(c^2k^2/4\pi\omega)\,\mathbf{E}_\perp, \tag{9}$$

hence

$$ie\mathbf{E}_\perp \left[\frac{c^2k^2}{4\pi} + \frac{e^2}{3}\int (\mathbf{v}\hat{L}(\mathbf{v}))\, dS\right] = \omega e^2 \int \mathbf{v}_\perp \hat{L}(X)\, dS. \tag{10}$$

Thus

$$\nu = X - \frac{v_{||}\int v_{||}\hat{L}(X)\, dS}{{}^1/_3\int \mathbf{v}\hat{L}(\mathbf{v})\, dS} - \frac{4\pi e^2 \mathbf{v}_\perp \int \mathbf{v}_\perp \hat{L}(X)\, dS}{\omega_0^2 + c^2k^2}, \tag{11}$$

where

$$\omega_0^2 = \frac{4\pi e^2}{3}\int (\mathbf{v}\hat{L}(\mathbf{v}))\, dS$$

is of the order of the square of the plasma frequency.

Substituting (11) in (7), we arrive ultimately at the following equation:

$$\omega X - \mathbf{kv}\hat{L}(X) + \mathbf{kv}\left[\frac{\hat{L}(v_{||})\int v_{||}\hat{L}(X)\, dS}{{}^1/_3\int (\mathbf{v}\hat{L}(\mathbf{v}))\, dS} + \frac{4\pi e^2 (\hat{L}(\mathbf{v}_\perp)\int \mathbf{v}_\perp)\hat{L}(X)\, dS}{\omega_0^2 + c^2k^2}\right] = 0. \tag{12}$$

It is seen from (12) that if the equation has a solution, then the spectrum $\omega = \omega(\mathbf{k})$ has two linear parts at low and high frequencies, respectively. In fact, if $c^2k^2 \ll \omega_0^2$, (12) is transformed into

$$\omega X - \mathbf{kv}\hat{L}(X) + (\mathbf{kv})\left(\hat{L}(\mathbf{v})\int \mathbf{v}\right)\hat{L}(X)\, dS \Big/ \frac{1}{3}\int (\mathbf{v}\hat{L}(\mathbf{v}))\, dS = 0, \tag{13}$$

that is, it has the form of an ordinary equation for zero-sound oscillations in a Fermi liquid:

$$\omega X - \mathbf{kv}\, X - \mathbf{kv}\int F(\mathbf{p},\mathbf{p}')\, X(\mathbf{p}')\, dS = 0,$$

where, however, the function $F(\mathbf{p},\mathbf{p}')$ is connected with $f(\mathbf{p},\mathbf{p}')$ in the following manner:

$$F(\mathbf{p},\mathbf{p}') = f(\mathbf{p},\mathbf{p}') - (\hat{L}(\mathbf{v})\hat{L}'(\mathbf{v})) \Big/ \frac{1}{3}\int (\mathbf{v}\hat{L}(\mathbf{v}))\, dS. \tag{14}$$

In this case, according to (9) and (10), $\mathbf{j}_\perp$ is proportional to k^2, and consequently, as $k \to 0$ the transverse current vanishes, like the longitudinal one. The condition $c^2k^2 \ll \omega_0^2$ can be written in the form

$$\omega^2 \ll v^2\omega_0^2/c^2.$$

Together with the condition $kl \gg 1$ ($\omega\tau \gg 1$), it determines the region of radio frequencies in which the anomalous skin effect occurs in metals[5,6].

In the opposite limiting case $c^2k^2 \gg \omega_0^2$, the transverse current is not small, but, in accordance with (9) and (10), the transverse field is small. We can neglect in (12) the last term

$$\omega X - \mathbf{kv}\hat{L}(X) + (\mathbf{kv})\hat{L}(v_{||})\int v_{||}\hat{L}(X)\, dS \Big/ \frac{1}{3}\int (\mathbf{v}\hat{L}(\mathbf{v}))\, dS = 0. \tag{15}$$

This region of frequencies $\omega_0 v/c \ll \omega \ll \omega_0$ corresponds to the infrared part of the spectrum [6].

So far we have simply assumed that Eqs. (13) and (15) have a solution. Let us now discuss the requirements for making this possible. Even in an isotropic Fermi liquid (see, for example, [3]) it is impossible to obtain a solution for zero-sound oscillations in general form. One can see, however, that this solution must exist if the function $f(\mathbf{p}, \mathbf{p}')$ is sufficiently large and positive. The same holds true in the anisotropic case, too.

In the absence of anisotropy, $f(\mathbf{p}, \mathbf{p}')$ depends only on the angle between the vectors $\mathbf{p}$ and $\mathbf{p}'$ and can be expanded in Legendre polynomials. In the general case we can write instead

$$f(\mathbf{p}, \mathbf{p}') = \sum_i \lambda_i f_i(\mathbf{p}, \mathbf{p}'), \qquad (16)$$

where $f_i(\mathbf{p}, \mathbf{p}')$ are functions which transform in each variable in accord with the i-th irreducible representation of the crystal symmetry group. To be specific, we assume that we are dealing with the O_h group. Let one of the components in (16) be large, $\lambda_{i_0} \rightarrow \infty$. For simplicity let $i_0 \neq F_{1u}$ (see [7]), that is, $f_{i_0}(\mathbf{p}, \mathbf{p}')$ and $\mathbf{v}(\mathbf{p})$ transform in accordance with different representations. Then Eq. (13), for example, assumes the form

$$(\omega - \mathbf{kv}) X - (\mathbf{kv}) \left\{ \lambda_{i_0} \int f(\mathbf{p}, \mathbf{p}') X dS - \left(\mathbf{v} \int \mathbf{v}'\right) X dS \Big/ \tfrac{1}{3} \int v^2 dS \right\} = 0. \qquad (17)$$

Making the substitution $X = (\mathbf{kv})\chi/(\omega - \mathbf{kv})$, we see immediately that χ has the form

$$\chi = \chi_{i_0} - \mathbf{vc},$$

where the unknown function χ_{i_0} transforms in accordance with the representation i_0, and $\mathbf{c}$ is

$$\mathbf{c} = \int \mathbf{v} X dS \Big/ \tfrac{1}{3} \int v^2 dS. \qquad (18)$$

If $\lambda_{i_0} \rightarrow \infty$, then it is obvious from (17) that $\omega \gg \mathbf{kv}$. Therefore

$$\frac{\mathbf{c}}{3} \int v^2 dS = \int \frac{\mathbf{v}(\mathbf{kv})}{\omega - (\mathbf{kv})} \chi_{i_0} dS. \qquad (19)$$

For our purposes it is convenient to choose for i_0 a representation such that when $\mathbf{v}(\mathbf{kv})/(\omega - (\mathbf{kv}))$ is expanded in $\mathbf{kv}/\omega$, it appears first only in terms of sufficiently high order in $\mathbf{kv}/\omega$. Then $\mathbf{c}$ will be small. For example, the representation of F_{1g} in the expansion of $\mathbf{v}(\mathbf{kv})/(\omega - (\mathbf{kv}))$ enters, as can be readily verified, with a factor $(\mathbf{kv}/\omega)^3$. Then $c \sim v^{-1}(\mathbf{kv}/\omega)^3 \chi_{F_{1g}}$ and we can neglect the term with $\mathbf{c}$ in (17).

We obtain for the determination of χ_{i_0} an integral equation of the Fredholm type:

$$\chi_{i_0} = \frac{\lambda_{i_0}}{3} \frac{k^2}{\omega^2} \int f_{i_0} v^2 \chi_{i_0} dS, \qquad (20)$$

which, according to the general theory, has positive eigenvalues $\omega^2 \sim k^2/\lambda_{i_0}$. In fact, when λ_{i_0} is sufficiently large, the variation in energy upon deviation from equilibrium is determined by a term quadratic in the energy

$$\delta E \backsim \lambda_{i_0} \int f_{i_0}(\mathbf{p}, \mathbf{p}') \nu(\mathbf{p}) \nu(\mathbf{p}') dS.$$

The δE should be positive (from stability considerations), from which it follows that the kernel of the integral equation (20) is positive definite. Of course, this analysis applies also to Eq. (15).

Thus, if the function f is sufficiently large, the equation for the zero-sound oscillations must have a solution. The question therefore reduces to whether this quantity is sufficiently large in the given specific metal. Unfortunately, however, no information whatever is available on the function f in metals; it is clear from general considerations that the value of this function is of the order unity.

The foregoing pertains to zero sound with arbitrary direction of propagation. If we choose as the propagation direction one of the preferred directions in a crystal, then zero sound, subject to some limitations on the Fermi surface, can exist also for a positive function $f(\mathbf{p}, \mathbf{p}')$ which is as small as desired. To prove this we turn to the equation for zero-sound oscillations in the form (3):

$$(\omega - \mathbf{kv}) \nu - \mathbf{kv} \int f(\mathbf{p}, \mathbf{p}') \nu dS + \mathbf{cv} = 0. \qquad (21)$$

We consider first the simpler case of frequencies in the infrared region $v\omega_0/c \ll \omega \ll \omega_0$. As was already noted above, in this limit the vector $\mathbf{c}$, which is proportional to the electric field, has only one longitudinal component $c_{\|}$, which should be determined from the condition that the density oscillations be equal to zero. The transverse current is in this case different from zero.

If f is very small, then, as can be seen from (20), ν can be large only at those points, at which $\omega - \mathbf{kv}$ is small. The latter denotes that the propagation velocity $\mathbf{u}$ of the wave is close to the velocity of the electrons at some point on the Fermi surface [1)]. In the vicinity of this point, $\mathbf{kv}$ has a maximum

$$\mathbf{kv} = (\mathbf{kv})_0 \{1 - [a_{11}(\theta - \theta_0)^2 + 2a_{12}(\theta - \theta_0)(\varphi - \varphi_0) + a_{22}(\varphi - \varphi_0)^2]\},$$

where $(\mathbf{kv})_0$ is the value at the point $\mathbf{p}_0$, while the

[1)]Obviously, $|\mathbf{u}|$ is always larger than $|\mathbf{v}|$. Otherwise the excitation corresponding to zero sound would attenuate rapidly because of decay into an electron and a hole.

quantity in the square brackets is a positive-definite quadratic form, with the function ν having near $\mathbf{p}_0$ the form

$$\nu(\mathbf{p}) \approx A/(\omega - \mathbf{kv}). \qquad (22)$$

If the vector $\mathbf{k}$ has an arbitrary direction and does not lie on any of the symmetry elements of the crystal, then, generally speaking, only one maximum of the quantity $\mathbf{kv}$ exists. The condition for the absence of density oscillations immediately yields, with logarithmic accuracy,

$$\delta\rho = -\frac{e\alpha}{h_0 k v_0} LA(\mathbf{p}_0) = 0, \qquad L = \ln\frac{\text{const}}{\omega - kv_0}, \qquad (22a)$$

where $(\mathbf{kv})_0 = kv_0$; h_0 is the Gaussian curvature of the Fermi surface at the point $\mathbf{p}_0$: $dS = h^{-1}d\Omega$; α is a numerical coefficient that depends on the logarithmic integration.

From (21) it follows, with the same accuracy, that

$$A(\mathbf{p}_0) = \alpha L h_0^{-1} f(\mathbf{p}_0, \mathbf{p}_0) A(\mathbf{p}_0) - cv_0.$$

We thus obtain $A(\mathbf{p}_0) = c = 0$, and there are no oscillations for arbitrarily small f.

Oscillations with arbitrarily small f are possible only if $\mathbf{k}$ lies on some one of the symmetry elements of the crystal (or close to it). Then Eq. (21) itself becomes invariant relative to the symmetry group of the vector $\mathbf{k}$. In this case, as is well known, the equation has several solutions, each of which transforms in accordance with some representations of its symmetry group. Let us consider for simplicity the symmetry plane (C_s group). It has two representations, A′ and A″ (see [7] for the notation). The components of the vector lying in the plane transform in accordance with the first representation, and the perpendicular component transforms in accordance with the second.

Let us denote the two solutions by $\nu_{A'}$ and $\nu_{A''}$. The equation for these solutions has the form

$$(\omega - \mathbf{kv})\,\nu_{A'} - \mathbf{kv}\int f_{A'}\nu_{A'}dS = cv_{\parallel},$$

$$(\omega - \mathbf{kv})\,\nu_{A''} - \mathbf{kv}\int f_{A''}\nu_{A''}dS = 0.$$

The first equation must be solved with the condition $\delta\rho = 0$, while the solution of the second equation satisfies this condition by definition. However, neither equation has solutions with arbitrarily small f if the maximum of $\mathbf{kv}$ is itself in the symmetry plane. In the former case this is connected with the need for satisfying the additional condition (22a), and in the latter case with the fact, which can be readily verified, that any function f_i which transforms in accordance with a non-unitary representation vanishes at a point lying on the symmetry element.

Solutions will exist if the maximum of $\mathbf{kv}$ does not lie on the symmetry plane. Then $\mathbf{kv}$ has two identical maximum values at the points $\mathbf{p}_1$ and $\mathbf{p}_2$. In the vicinity of these points $\nu_{A''}$ has the form

$$\nu_{A''}(\mathbf{p}_1) = \frac{A_1}{\omega - \mathbf{kv}}, \qquad \nu_{A''}(\mathbf{p}_2) = \frac{A_2}{\omega - \mathbf{kv}},$$

and $A_2 = -A_1$ by virtue of the symmetry. Substituting this in (21), we obtain with logarithmic accuracy

$$A_1 = \alpha h_0^{-1} L\,(f_{11}^{A''} - f_{12}^{A''})\,A_1,$$

where $\hat{f}_{ik} = f(\mathbf{p}_i, \mathbf{p}_k)$. (We recall that by virtue of the symmetry $f_{22} = f_{11}$ and $f_{12} = f_{21}$.) A solution of this equation exists when $f_{11}^{A''} - f_{12}^{A''} > 0$, and has the form

$$\omega = kv_0 + \text{const}\cdot k \exp\{-h_0/\alpha\,(f_{11}^{A''} - f_{12}^{A''})\}.$$

It is easy to verify in exactly the same way that a solution of the type A′ with the additional condition $\delta\rho = 0$ exists when $f_{11}^{A'} - f_{12}^{A'} > 0$.

An analogous situation remains at infrared frequencies for the vectors $\mathbf{k}$ that lie on any symmetry element. Thus, in a crystal with arbitrarily small f the zero-sound of infrared frequency can propagate along directions that are close to all the symmetry elements, provided that the maximum of $\mathbf{kv}$ does not lie on these elements.

In the case of radio frequencies it is necessary to take into account not one but three supplementary conditions ($\mathbf{j} = 0$). Without repeating the perfectly similar considerations, we cite the results. Oscillations at arbitrarily small f are possible in this case, too, if $\mathbf{k}$ lies near one of the symmetry elements, and the maximum of $\mathbf{kv}$ does not lie on any of the symmetry elements. In this case oscillations of all types are possible along the three-fold and four-fold axes (corresponding to the representations of the C_{3v} and C_{4v} groups). Only oscillations of type A_2 are possible along the two-fold axis (group C_{2v}).

The foregoing analysis shows that zero sound must be sought predominantly along directions lying on the symmetry elements of the crystal. The probability of finding it is, roughly speaking, 50 per cent.

So far we have left out everywhere the spin-dependent terms. Assume now that we have in place of (1)

$$\delta\hat{\varepsilon} = \int \hat{f}(\mathbf{p}, \mathbf{p}')\,\delta\hat{n}\,\frac{d^3\mathbf{p}'}{(2\pi)^3}, \qquad (1')$$

where the function

$$\hat{f}(\mathbf{p}, \mathbf{p}') = f(\mathbf{p}, \mathbf{p}') + \zeta(\mathbf{p}, \mathbf{p}')\,\sigma\sigma' \qquad (23)$$

has an exchange part $\zeta(\mathbf{p}, \mathbf{p}')$. (Here σ are Pauli matrices, while $\delta\hat{\epsilon}$ and $\delta\hat{n}$ in (1) are two-row matrices with spin variables.)

In place of equation (3) we obtain

$$\omega\hat{\nu} + ie\mathbf{E}\mathbf{v} - \gamma(\mathbf{k}\mathbf{v})(\mathbf{H}\hat{\sigma}) - \mathbf{k}\mathbf{v}\left\{\hat{\nu} + \tfrac{1}{2}\mathrm{Sp}_{\sigma'}\int f_{\sigma\sigma'}(\mathbf{p}, \mathbf{p}')\,\hat{\nu}'dS\right\} = 0 \quad (3')$$

(in a magnetic field $\delta\hat{\epsilon} = \gamma\mathbf{H}\sigma$), which can be written in the form of two equations for the two components of the function $\hat{\nu}$:

$$\hat{\nu} = \nu_0 + \nu_1\sigma.$$

We have

$$\omega\nu_0 + ie\mathbf{E}\mathbf{v} - \mathbf{k}\mathbf{v}\,[\nu_0 + \int f(\mathbf{p}, \mathbf{p}')\nu_0'\,dS] = 0,$$

$$\omega\nu_1 - \gamma(\mathbf{k}\mathbf{v})\,\mathbf{H} - (\mathbf{k}\mathbf{v})\left[\nu_1 + \int\zeta(\mathbf{p}, \mathbf{p}')\,\nu_1'dS\right] = 0. \quad (24)$$

The oscillations ν_0 and ν_1 are generally speaking coupled with each other, since the fields $\mathbf{E}$ and $\mathbf{H}$, in accordance with Maxwell's equations, are expressed in terms of a current $\mathbf{j}$, equal to

$$\mathbf{j} = -e\int\mathbf{v}\hat{L}(\nu_0)\,dS - i\mu c\int[\mathbf{k}\nu_1]\,dS. \quad (25)^*$$

(Here and below we use the symbols $L(\nu_0)$ and $\hat{\hat{L}}(\nu_1)$ for the operators in the square brackets in the first and second equations of (24), respectively.) We shall now show that this coupling is very weak and therefore zero sound not accompanied by spin oscillations, and spin waves of a special kind, can propagate in a metal independently[1,3].

For this purpose we transform (24) in the same manner as in the derivation of (12) above. We put

$$\omega\nu_0 + ie\mathbf{v}\mathbf{E} = \omega X, \qquad \omega\nu_1 - \gamma(\mathbf{k}\mathbf{v})\,\mathbf{H} = \omega\mathbf{Y}. \quad (26)$$

Then we have in place of (24)

$$\omega X = \mathbf{k}\mathbf{v}\hat{L}(\nu_0), \qquad \omega\mathbf{Y} = (\mathbf{k}\mathbf{v})\,\hat{\hat{L}}(\nu_1). \quad (27)$$

With the aid of (26), using the definition (25) for the current and Maxwell's equations, we can express the intensities of the fields $\mathbf{E}$ and $\mathbf{H}$ in terms of integrals of X and $\mathbf{Y}$. For the longitudinal electric field $E_{\parallel}$ the previous expression (8) holds true.

In place of (10) we obtain

$$ie\mathbf{E}_{\perp} = \frac{4\pi\omega}{\omega_0^2 + c^2k^2}\int\{e^2\mathbf{v}_{\perp}\hat{L}(X) + i\mu ce\,[\mathbf{k}\mathbf{Y}]\}\,dS. \quad (28)$$

In addition

$$\mathbf{H} = (c/\omega)\,[\mathbf{k}\mathbf{E}]. \quad (29)$$

After substituting (29), (28), and (8) in (26) we obtain

$$\nu_0 = X - v_{\parallel}\int v_{\parallel}\hat{L}(X)dS \Big/ \tfrac{1}{3}\int\mathbf{v}\hat{L}(\mathbf{v})\,dS - \frac{4\pi}{\omega_0^2 + c^2k^2}\left\{e^2(\mathbf{v}_{\perp}\int\mathbf{v}_{\perp}')\,\hat{L}(X)\,dS + i\mu ec\,(\mathbf{v}_{\perp}\int[\mathbf{k}\mathbf{Y}]\,dS\right\},$$

$$\nu_1 = \mathbf{Y} - i\,\frac{4\pi c\gamma(\mathbf{k}\mathbf{v})}{\omega(\omega_0^2 + c^2k^2)}\int\{e\,[\mathbf{k}\mathbf{v}]\,\hat{L}(X) + i\mu c\,[\mathbf{k}\,[\mathbf{k}\mathbf{Y}]]\}\,dS. \quad (30)$$

Estimates show immediately that $\nu_1 \approx \mathbf{Y}$, and that the cross terms can be neglected, since their order is $k\omega_0^2/mv(\omega_0^2 + c^2k^2)$. Thus, the equation for the spin oscillations has in the entire frequency interval the usual form:

$$\omega\nu_1 - (\mathbf{k}\mathbf{v})\,\hat{\hat{L}}(\nu_1) = 0. \quad (31)$$

An analysis similar to that of the spinless oscillations above, shows that (31) has a solution both for very large values of $\zeta(\mathbf{p}, \mathbf{p}')$ and for small ones. Moreover, in the latter case the solutions exist also for arbitrary direction of $\mathbf{k}$. Indeed, since unlike (21) no additional conditions are imposed on the solution of (31), in order to obtain a solution for small ζ it is sufficient to have only one point $\mathbf{p}_0$ at which the function $\mathbf{k}\mathbf{v}$ has a maximum.

Substituting in (31) the function ν_1 in the form (22), we obtain, with logarithmic accuracy,

$$\omega - \mathbf{k}\mathbf{v}_0 \sim kv_0\exp\{-\alpha/\zeta_0\},$$

where $\zeta_0 = \zeta(\mathbf{p}_0, \mathbf{p}_0) > 0$ should have the antiferromagnetic sign. Therefore, in our opinion, spin zero-sound oscillations should exist in practically any metal.

Without citing the corresponding formulas, we note that in a weak external magnetic field the law governing the dispersion of both spin and spinless oscillations begins with a certain constant frequency, proportional to the magnetic field and dependent, generally speaking, in a rather complicated manner on the function f and on the direction of the vector $\mathbf{k}$. Unlike the isotropic case[4], it is impossible to obtain simple formulas here.

Let us proceed now to the question of the possibility of observing zero sound. These oscillations are connected with the appearance of electromagnetic waves propagating in a metal, so that it is most natural to attempt to excite them with the aid of electromagnetic radiation. The excitation mechanism will be essentially different for the radio and for the infrared frequency intervals. The case of low frequencies is less convenient in this sense. We have already mentioned that the current in a zero-sound wave is very small in the case of long waves, and therefore these waves are weakly excited by the electromagnetic field. If we deal with the excitation

*$[\mathbf{k}\,\nu_1] = \mathbf{k} \times \nu_1$

of a spinless wave, then in the radio-frequency region the current in such a wave is proportional to the square of the frequency.

In a spin wave (31), the current is proportional to k for small $k \to 0$. It must be borne in mind that in this case a contribution is made to the current (25) not only by the part with ν_1, but also by the spinless part ν_0, which accompanies, in accordance with (30), the ν_1 oscillations. Indeed, although ν_0 is small in such a wave,

$$\nu_0 \sim \frac{k}{mv} \frac{\omega_0^2}{\omega_0^2 + c^2k^2} \nu_1$$

it makes in the region $\omega_0^2 \gg c^2k^2$ a contribution of the same order of magnitude to the current.

Let us consider normal incidence of radiation on a semi-infinite metal. In order to determine exactly the amplitude of the zero-sound wave penetrating into the metal, it is necessary to solve a rather difficult electromagnetic problem, which is further complicated by the presence of anisotropy. One can, however, estimate rather crudely the order of magnitude of the transmitted radiation. For this purpose we note that the zero-sound wave is formed at distances on the order of the wavelength $\lambda \sim v/\omega$. In the radio-frequency region, reflection of radiation from the surface of a pure metal is accompanied by anomalous skin-effect, where the field inside the metal decreases rapidly with distance from the surface. The characteristic depth of penetration is $\delta \ll \lambda \sim v/\omega$. The amplitude of the transmitted wave can be estimated from the condition that the field in the wave becomes comparable with the field of the skin layer at distances on the order of λ. Inasmuch as at distances $r \gg \delta$ the field in the skin layer decreases as $H \sim H_0(\delta/r)$ where H_0 is the field on the surface, we have

$$H_\lambda \sim H_0 (\delta/\lambda)^2 \sim H_0 (c/v)^2 (\delta/\lambda_0)^2,$$

where λ is the wavelength of zero sound and λ_0 is the length of the corresponding electromagnetic wave in vacuum. This, of course, is a very small quantity.

It is therefore much more convenient to excite zero sound in the region of the infrared spectrum. Even from the preceding estimate we see that if $\lambda \sim \delta$ the field in the wave becomes of the order of the field on the surface. Let us discuss this case in greater detail. At these frequencies the field in the zero-sound wave is small in the sense that its influence on the particle-number oscillations can be neglected. In this case, however, the current does not vanish. For example, in the spin wave,*

$$\mathbf{j} = -\mu c \,\mathrm{rot} \int \nu_1 dS, \quad \mathbf{H} = -4\pi\mu \int \nu_1 dS.$$

ν_1 itself satisfies the equation

$$-i\omega\nu_1 + v_z d\hat{L}(\nu_1)/dz = 0. \qquad (32)$$

In infinite space this equation has sinusoidal solutions with a dispersion law $\omega = uk$. The solution of (32) in a bounded metal depends on the boundary conditions. In the case of specular electron reflection from the metal surface the sinusoidal solution can be used directly, after which it is easy to determine the transmission coefficient

$$D = E_0/H_0 = \omega/ck \sim v/c.$$

Of course, reflection from a metal surface is always close to diffuse[5,6]. The solution of (32) goes over therefore into a sinusoidal wave only at distances on the order of the wavelength, but the transmission coefficient, naturally, has the same order of magnitude. The same pertains to the spinless oscillations described by equation (15).

We note, however, that with zero-sound oscillations impossible in the infrared region the field attenuates at distances on the order of $\delta \sim c/\omega_0 \gg v/\omega$ ($\delta \sim 10^{-5}$) and the reflection coefficient R is very close to unity[6], with

$$1 - R \sim v/c.$$

To verify the existence of zero-sound waves, it would be most convenient to study indeed the passage of radiation through a plate of thickness d $\gg c/\omega_0$. In the infrared region, zero-sound waves would lead to transparency of such plates. The fraction of the transmitted radiation would be of the order of $v^2/c^2 \sim 10^{-3}$—10^{-4}, and would be independent of the thickness of the plate. Unfortunately, this is true, of course, only so long as the dimension of the plate d is small compared with the mean free path of the electrons in the metal. Actually, two mean free paths enter into the problem: the damping of the spinless oscillations is characterized by the ordinary electron mean free path l, which is involved in the conductivity in a constant field; in addition there is the mean free path l_S needed to flip the electron spin.

At infrared frequencies $l \sim v/\Theta \sim 10^{-5}$—$10^{-6}$, where Θ is the Debye temperature. To observe spinless zero sound it is therefore necessary to satisfy the condition $v/\Theta \gtrsim d \gg c/\omega_0$. This is a very difficult condition even for a substance with a relatively low Debye temperature.

For spin zero sound, the conditions of observation are much more favorable. One can expect that in this case the non-spin mean free path $l_S = \alpha l$

*rot = curl.

(with $\alpha \gg 1$) would be important here, with α determined by the smaller of the two quantities, Z (the charge of the impurity) or $(c/v)^2$ (the latter is connected with the fact that the spin flip can be caused by two factors: exchange interaction, which is of the order of Z^{-1} compared with the non-exchange interaction, and the spin-orbit interaction, which is of order $(v/c)^2$). The corresponding limitation on the plate thickness are then weaker

$$\alpha v/\Theta \gtrsim d \gg c/\omega_0.$$

In the region of lower frequencies, the limitations connected with the mean free path are much weaker, but the transmission coefficient decreases much more significantly. From the foregoing we can estimate that the coefficient of transmission through the plate has in the radio-frequency region an order of[2)] $(v/c)^2(\delta/\lambda)^4 \sim (c/v)^2(\delta/\lambda_0)^4$. At the same time the mean free path in this region is proportional to $l \sim v\Theta/\omega^3$ [8], if the scattering by the impurities can be neglected.

Zero-sound oscillations could also manifest themselves in many other effects, such as the line width in electron diffraction, the characteristic losses of charged particles upon passage through a metal plate in a magnetic field, etc. It seems to us that it would be very interesting to observe this phenomenon in metals.

[2)]It might appear that, on the other side of the plate, the fraction of the transmitted radiation would be reduced by another factor $(\delta/\lambda)^4$. This does not occur, since there is no solution to the problem of the anomalous skin effect for field that increases with increasing depth in the metal.

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