THE PROBABILITY OF ISOMERISM IN THE STATISTICAL MODEL OF THE NUCLEUS

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The probability of the occurrence if isomerism of a nuclear level is estimated on the basis of the statistical model of the nucleus. It is shown that the probability of isomerism is relatively large for states with angular momenta which are close to the maximal possible values for a given excitation. The nature of such states and the conditions under which it might be possible to observe them are analyzed.

THE statistical model of the nucleus predicts a monotonic increase of the average value of the angular momentum of a nucleus with the increase of its excitation energy (cf. e.g. [1]). From the viewpoint of the statistical model, nuclear isomerism is a consequence of the fluctuations of the distribution of angular momenta. It is easy to estimate the probability of such a fluctuation.

Let $P_E(j)\Delta j$ denote the probability that the value of the angular momentum of a level corresponding to an excitation energy E lies between the limits j and $j + \Delta j$. In order that a level with angular momentum j be isomeric it is necessary that the angular momenta of all levels with lower energy be outside the interval $j - \Delta j$, $j + \Delta j$, where Δj must be sufficiently large ($\gtrsim 4$). The probability for this to occur is

$$q_{\Delta j}(E, j) = \prod_{E'<E} (1 - 2P_{E'}(j)\,\Delta j), \tag{1}$$

where the product runs over all nuclear levels below the one under consideration.

Eq. (1) can also be written in the form

$$q_{\Delta j}(E, j) = \exp\left\{\int_0^E dE'\rho_t(E') \ln (1 - 2P_{E'}(j)\,\Delta j)\right\},$$

where $\rho_t(E')$ is the average level density of the nucleus around the energy E′. It is interesting to consider the case when

$$2P_{E'}(j)\,\Delta j \ll 1.$$

In this case

$$q_{\Delta j}(E, j) \approx \exp\left\{-2\Delta j \int_0^E dE'\rho_t(E') P_{E'}(j')\right\},$$

which can also be represented in the form

$$q_{\Delta j}(E, j) \approx \exp\{-2N(E, j)\,\Delta j\}. \tag{2}$$

In Eq. (2) $N(E, j)\Delta j$ is the average number of nuclear levels with the angular momentum inside the interval j, $j + \Delta j$ and with excitation energy $E' < E$. The quantity $N(E, j)\Delta j$ is computed from the statistical model. The level density of a nucleus with spin j and excitation energy E is proportional to

$$\exp\{S[E - (\hbar^2 j^2/2J)]\}, \tag{3}$$

where S denotes the entropy and J is the moment of inertia. In general the function $N(E, j)$ contains the exponentially large factor (3). Under the condition that the "rotation energy"

$$\mathcal{E}_r = \hbar^2 j^2/2J \tag{4}$$

is small compared with the total excitation energy E, the probability for the spin of a given level to be inside the interval j, $j + \Delta j$ is proportional to

$$\exp(-\hbar^2 j^2/2JT), \tag{5}$$

where T is the temperature of the nucleus.

From Eqs. (2) and (3) one can see that the probability for the occurrence of isomerism is in general extraordinarily small. For $T > 0$ isomerism is possible only if the exponential (5) is very small, i.e. for

$$j^2 \gg \langle j^2 \rangle, \tag{6}$$

where the mean square of the angular momentum is

$$\langle j^2 \rangle \approx \hbar^{-2} JT. \tag{7}$$

However, for a given excitation the angular momentum of a nucleus cannot be infinitely large. Its maximal value can be estimated from the condition that the argument of the function S in (3) vanish, i.e., from the vanishing of the "thermal energy":

$$j \approx j_{max} = \hbar^2 JS. \tag{8}$$

Evidently, in this case $S \approx 1$. There are relatively few states for which the condition (8) is satisfied, therefore for these states the probability of isomerism is relatively large.

One can get a rough idea of the nature of these states in the following manner. The mean square angular momentum of the nucleus, for an excitation energy E, is

$$\langle j^2 \rangle \approx (k_F R)^2 \bar{n}, \tag{9}$$

where $\bar{n}$ is the average number of excited particles (generally, in several elementary excitations) and $k_F R$ is the average angular momentum of a particle near the Fermi limit (k_F is the wave number of the particle and R the radius of the nucleus). For a nearly degenerate Fermi system

$$\bar{n} \approx E/T \approx g_F T, \tag{10}$$

where the nuclear temperature T is taken as a measure of the average excitation energy for one particle and g_F is the density of the one-particle states. Substituting (10) into (9) one obtains (up to a numerical factor) the expression (7), where J is the rigid moment of inertia, which is the usual estimate of this quantity. The states for which $j^2 \approx \langle j^2 \rangle$ are the most likely ones for a given excitation energy.

Similarly, the maximal value of the mean square angular momentum for a given excitation equals

$$\langle j^2 \rangle_{max} \approx (k_F R)^2 n_{max}, \tag{11}$$

where n_{max} is the maximal number of excited particles:

$$n_{max} \approx g_F E. \tag{12}$$

The equalities (11) and (12) are equivalent to (8) if g_F in (12) is taken to mean the total density of one-particle states. In this case

$$\langle j^2 \rangle_{max} / \langle j^2 \rangle \approx E/T. \tag{13}$$

The states with angular momenta close to the limiting value (11) could be called "aligned," since the angular momenta of all excited particles point in the mean to the same direction. The number of such states is relatively small and from the thermodynamic point of view they are nonequilibrium states, in distinction from the states with $j^2 \approx \langle j^2 \rangle$, which would correspond to an equilibrium state within a heat reservoir. For a more rigorous estimate of $\langle j^2 \rangle_{max}$ in (12) one should replace g_F by the density of one-particle states with maximal angular momentum

$$g_{l_{max}} \approx g_F (k_F R)^{-1}. \tag{14}$$

Then

$$\langle j^2 \rangle_{max} \approx (k_F R)^{-1} J E. \tag{15}$$

Comparing (4) and (15) one can say that such states correspond to the rotation of a part of the nucleons in the nucleus. For these the effective moment of inertia is $k_F R$ times smaller (as an order of magnitude only) than the rigid-body value, and for a given angular momentum the angular velocity (rotation frequency) is $k_F R$ times larger.[1)] The estimates (9) — (15) remain valid near the ground state of a nucleus for which isomerism is encountered relatively often. In this case

$$\bar{n} \approx n_{max} \approx 1, \qquad \langle j^2 \rangle \approx \langle j^2 \rangle_{max} \approx (k_F R)^2.$$

From the equality (8) or (15) and Eq. (2) one obtains for "aligned" states

$$q_{\Delta j}(E, j) \approx \exp(-2\Delta j). \tag{16}$$

Although the number of "aligned" states is relatively low, they are obtained with large probabilities in reactions with heavy ions or in other reactions where the angular momentum transfer to the nucleus is large. The large initial value of the angular momentum does not change much in the course of emission of light particles. Therefore, if the angular momentum was large at the beginning, sooner or later the equality (8) will become true at the end of the cascade. The energy of the final "aligned" state can be estimated from the equalities (8) or (15). For heavy nuclei and $j \sim 30$ one obtains in this manner an energy of the order of several MeV. The angular momentum must decrease via the emission of a relatively large number of photons. Their average energy is of the order of the rotation frequency (angular velocity), i.e., approximately $k_F R \approx A^{1/3}$ times the rotation frequency of the nucleus as a whole.

The average number of quanta is $\mathfrak{N}_\gamma \sim j$, therefore the average probability of the formation of an isomer per capture in very large angular momentum reactions will be of the order

$$\mathfrak{N}_\gamma q_{\Delta j}(E, j) \sim j e^{-2\Delta j}. \tag{17}$$

The "isomeric" quanta must be strongly anisotropic with respect to the beam particles, owing to the orientation of the angular momentum of the nucleus. For $\Delta j > 2$ the maximum of the radiation intensity will be at an angle $\theta = 90°$ with respect to the beam (cf. also [2]).

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1)We deal here with the average properties of a group of nuclear levels ("microstates"). The collective rotation appears here as a result of such an averaging and has little in common with the collective "microstates" which are described by the wave function. An example of states of the latter kind are the rotational states of deformed nuclei.

The general method described above apparently agrees qualitatively with the presently known experimental data on the emission of photons by nuclei with large angular momenta[3].

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[2]V. M. Strutinskiĭ, JETP **37**, 861 (1959), Soviet Phys. JETP **10**, 613 (1960).

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