DETERMINATION OF THE SIZE OF NUCLEI FROM THE DIFFERENTIAL π^0-MESON PHOTOPRODUCTION CROSS SECTIONS

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Angular distributions of π^0 mesons produced by photons of 182 MeV mean energy are presented. The results, combined with those of the previous work of the authors on the photoproduction at 154 MeV,[3,5] are analyzed with the aim to obtain data on the distribution of nucleons in the Be, C, Al, Cu, Cd, Ta, and Pb nuclei.

IN the experiments on the π^0-meson photoproduction on nuclei[1,5] it has been shown that the main contribution to the cross section is due to the coherent production. The angular and energy dependence of the coherent photoproduction on a nucleus with mass number A in the impulse and Born approximations are given by the relation

$$\left(\frac{d\sigma}{d\Omega}\right)_A = A^2\frac{\sigma_t}{4\pi}\left|\int_\tau \rho(\tau)\exp\left(\frac{i}{\hbar}\mathbf{qr}\right)d\tau\right|^2\sin^2\theta, \qquad (1)$$

where σ_t is the π^0 meson photoproduction cross section on a nucleon (which is assumed to be independent of the isotopic spin), $\rho(\tau)$ is the normalized nucleon density in the nucleus, $\mathbf{q}$ is the momentum of the recoil nucleus, and θ is the angle of emission of the meson.

In deducing Eq. (1), the inelastic production and the effects involving the spin have been neglected, and it has also been assumed that the spin-independent part of the photoproduction cross section on a nucleon amounts to $^2/_3$ of the total cross section. Equation (1) is in a good agreement with the experimental results over a wide range of values of A and θ for photon energy up to 250 MeV.[2]

In the following, we present the results on the π^0 meson photoproduction for the mean energy of primary photons E = 182 MeV. These data, as well as the angular distributions of π^0 mesons produced at E = 154 MeV obtained in earlier experiments [3,5] are analyzed in order to obtain some parameters of the nucleon distribution in nuclei.

The experimental arrangement and the instrumentation used for the detection of the π^0 mesons have been described earlier.[5] For the study of the photoproduction at E = 182 MeV, the maximum bremsstrahlung energy from the synchrotron was increased to 200 MeV, and the angle between the scintillation counters which detected the photons from the $\pi^0 \to \gamma + \gamma$ decay was decreased to 100°, which resulted in an increase in the minimum energy of detected mesons to 162 MeV. The measurements were carried out for seven elements (Be, C, Al, Cu, Cd, Ta, and Pb) at nine angles whose mean values of the cosine were 0.953, 0.825, 0.614, 0.328, 0.006, −0.328, −0.614, −0.825, and −0.953. The angular (Fig. 1) and energy resolution (Fig. 2) of the system were calculated by the Monte-Carlo method.[6] The differential photoproduction cross sections of π^0 mesons on various nuclei at E = 182 MeV are shown in the l.s. in Figs. 3—9.

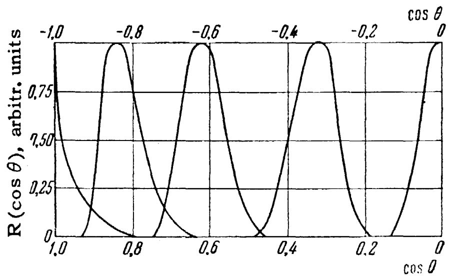

FIG. 1. Angular resolution of the apparatus; θ – angle between the plane of the telescopes and the direction of the primary photon beam.

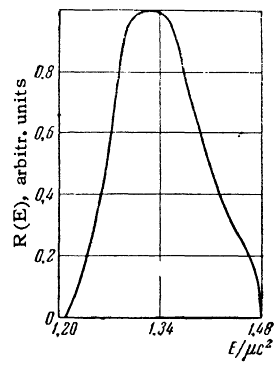

FIG. 2. Energy resolution of the apparatus (energy expressed in units of the π^0-meson rest mass).

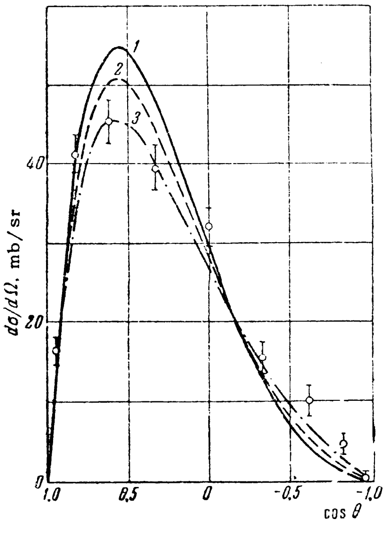

FIG. 3. Differential cross section for the π^0 meson photoproduction on Be.

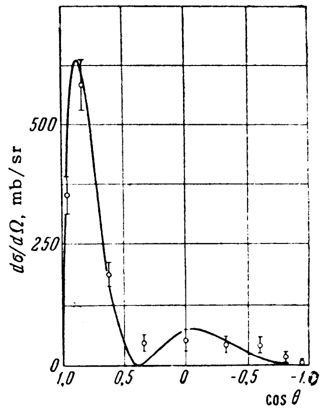

FIG. 6. π^0-meson photoproduction cross section on Cu.

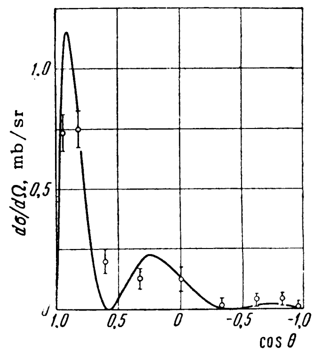

FIG. 7. π^0-meson photoproduction cross section on Cd.

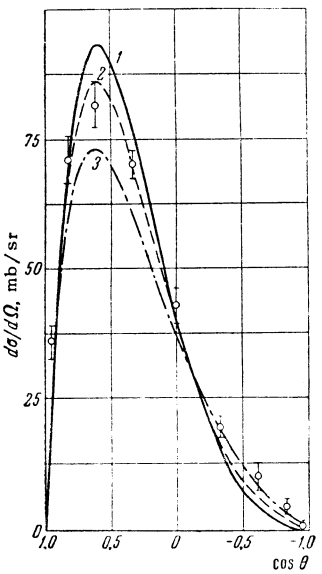

FIG. 4. π^0-meson photoproduction cross section on C.

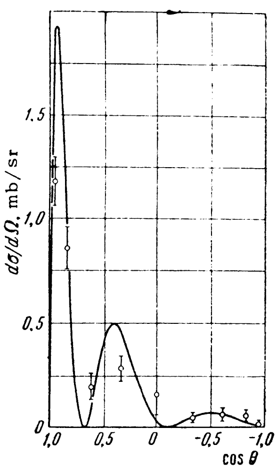

FIG. 8. π^0-meson photoproduction cross section on Ta.

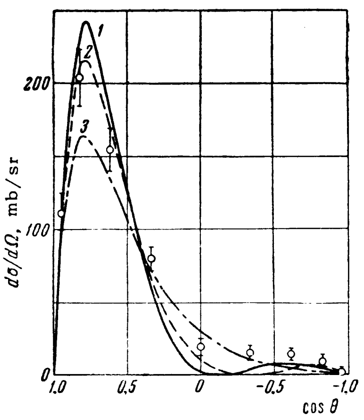

FIG. 5. π^0-meson photoproduction cross section on Al.

The size of the nuclei was determined by comparing the experimental data with calculations according to Eq. (1). The function $\rho(\tau)$ was approximated by spherically symmetrical models of nucleon density distribution in nuclei: uniform for all nuclei, modified exponential and shell-model for Be, C, and Al, and trapezoidal for Cu, Cd, Ta, and Pb. The uniform distribution is characterized by a radius R, the modified and shell-model distributions by a root mean square radius a, while the trapezoidal distribution is described by two parameters: c —the distance at which the density decreases by a factor of two as compared with the density at the center, and t —the thickness of the boundary layer (in this region the density falls from 0.9 to 0.1 of the maximum value). These distributions were used to study the results of the experiments on scattering of high-energy electrons by Hofstadter et al. and were described in the re-

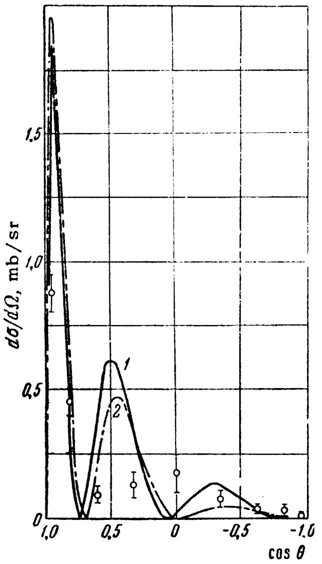

FIG. 9. π^0-meson photoproduction cross section on Pb.

view article by Hofstadter [7] (the trapezoidal distribution is similar, for medium and heavy nuclei, to the Fermi distribution discussed in that review).

We have used the least-squares method to calculate the parameters of the distributions which fit the experimental data best. In order to avoid the errors due to the indeterminacy in the mean angle of meson emission and the mean energy of primary photons (the angular and energy distributions are mainly determined by the kinematics of the $\pi^0 \to \gamma + \gamma$ decay and cannot be made sufficiently narrow without considerably lowering the counting rate), the unknown parameters were determined by minimizing the following sum:

$$\sum_i p_i (N_i - N_i^T)^2 \quad (i = 1, 2, \ldots, m), \tag{2}$$

where N_i are the experimental data, p_i are their statistical weights, m is the number of measured points of the angular distribution, and N_i^T are the theoretical results:

$$N_i^T = n \int\limits_E \int\limits_\theta \frac{d\sigma}{d\Omega} \varepsilon (E, \theta)\, dE\, d\theta. \tag{3}$$

In Eq. (3), n denotes the number of nuclei per 1 cm^2 of the target, and $\epsilon(E, \theta)$ gives the detection probability for the process under consideration [the method of calculating $\epsilon(E, \theta)$ has been described in [6]]. In determining the parameters of the various models by the least-squares method we have taken the systematic errors in the normalization of the bremsstrahlung intensity into account. [5]

The results of the least-squares analysis of the experimental data are given in Table I for light nuclei, and in Table II for medium and heavy nuclei. Two values are given for each parameter: the upper one has been obtained from the analysis of angular distributions at E = 154 MeV, and the lower one at E = 182 MeV (all values of the parameters in the tables and in the following are given in Fermis). In order to choose a model that fits best the experiment, we have used the F distribution with a 5% significance level; the parameters of the models which do not satisfy the experimental data are in the table enclosed in parentheses. Since the distribution parameters at the two energies coincide within the limits of the experimental errors, we give in the following the results for the nuclear radii for the two energy values combined.

Table I

Model	Parameter	Be	C	Al
Uniform	R	(2.76±0.08) (2.64±0.11)	2.91±0.05 (2.80±0.08)	(3.73±0.09) (3.90±0.10)
Modified exponential	a	2.48±0.09 2.44±0.09	2.70±0.08 2.65±0.08	(3.75±0.20) 3.84±0.13
Shell	a	2.41±0.21 (2.19±0.09)	2.37±0.15 2.31±0.08	(3.07±0.23) (3.18±0.10)

Table II

Model	Parameter	Cu	Cd	Ta	Pb
Uniform	R	(4.99±0.12) 5.06±0.09	5.77±0.07 5.97±0.13	6.76±0.08 6.83±0.10	— (7 23±0.16)
Trapezoidal	c	(5.28±0.25) 5.03±0.15	5.73±0.11 5.83±0.18	6.79±0.12 6.68±0.12	— 6.83±0.20
	t	(1.10±0.78) 0.35±0.71	0.46±0.52 0.85±0.42	1.48±1.42 0.82±0.22	— 1.89±0.29

The angular dependence of the π^0 photoproduction on Be and C can be described by Eq. (1) if we chose for Be the modified exponential model with a root mean square radius $a = 2.45 \pm 0.07$, and for C the shell model with $a = 2.32 \pm 0.07$ (the latter result is in good agreement with the value $a = 2.40 \pm 0.05$ obtained for the root mean square radius of the charge distribution in the C nucleus from the study of the high-energy electron scattering.[8]) The experimental results for Al do not agree with the theoretical description by any of the assumed models. This may be due either to the limited choice of models used in the analysis, or to the fact that Eq. (1) is not adequate to describe the π^0 photoproduction on Al.

The theory of the coherent photoproduction of π^0 mesons on medium and heavy nuclei is in good agreement with the experiment if for $\rho(\tau)$ we chose the trapezoidal model. From a statistical analysis of the parameters of the trapezoidal distribution it follows that if we put $c = r_1 A^{1/3}$ and consider t independent of A, then the obtained values of the parameters can be combined and we find $r_1 = 1.20 \pm 0.01$ and $t = 1.07 \pm 0.16$. These values do not agree with the values $r_1 = 1.07$ and $t = 2.4$ found by Hofstadter.[7] The disagreement may be due either to the difference between the distribution of nucleons and that of the electrical charge in the nuclei or to the approximate character of Eq. (1). The assumptions made in deducing Eq. (1) are discussed in [4,5].

As follows from Tables I and II, the uniform model, which evidently is a very rough approximation for the distribution of nucleons in light nuclei, agrees well with the experiment for Cu, Cd, and Ta. A statistical analysis has shown that the values of the radii of the uniform model can be well approximated by the following relation:

$$R = (0.37 \pm 0.07) + (1.14 \pm 0.02)A^{1/3}. \tag{4}$$

The presence in Eq. (4) of a term independent of A indicates a tighter packing of nucleons in heavy nuclei. Equation (4) and the values of R, averaged over the energy, from Tables I and II are illustrated in Fig. 10

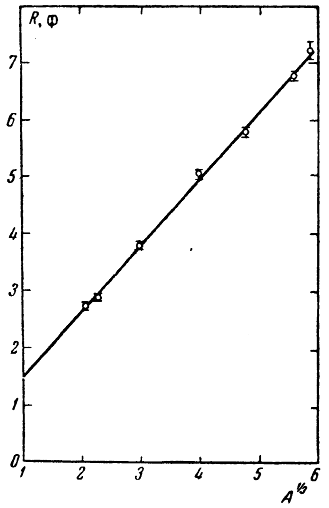

FIG. 10. Dependence of the nuclear radius on the mass number.

Calculations show that the coherent photoproduction is very sensitive to the value of the nucleon distribution radius (or root-mean-square radius), since the contribution to the cross section from non-coherent processes (~ 10—15%) practically does not affect the values of R and a. We can therefore regard the estimates of the radii given in Tables I and II as satisfactory. Equation (1) is less sensitive to the higher moments of the distribution of the nuclear matter and, consequently, the estimates of the parameters c and t are not as reliable as the estimates of R and a.

The cross sections calculated according to Eq. (1) using the values of the parameters determined by the least-squares method from the angular distribution of the π^0 meson photoproduction at $E = 182$ MeV are given in Figs. 3—9, together with the experimental points. Curve 1 represents the cross section according to the uniform model, curve 2 according to the shell model (Figs. 3 and 5) and to the trapezoidal model (Fig. 9), and curve 3 according to the modified exponential model. For Cu, Cd, and Ta, the results for the trapezoidal and uniform models coincide.

It should be noted that the values of nuclear radii obtained in the present experiment from the angular distribution of the π^0-meson photoproduction at $E = 154$ MeV and $E = 182$ MeV primary photon energy coincide, within the limits of experimental errors, with the results obtained from the angular distribution of π^0-meson production at $E = 165$ MeV.[4]

In calculating the above-given parameters of the nucleon density distribution in nuclei we have used for σ_t the values given in [9]. In analyzing the experimental results by the least-squares method we could, however, consider σ_t as an additional parameter and from the data on the photoproduction on heavy nuclei determine the quantities characterizing the π-meson photoproduction on nucleons.

For nuclei with spin zero it is only necessary to account for the spin-independent part of the π^0-meson photoproduction cross section on nucleons and in that case σ_t in Eq. (1) can be expressed by the invariant amplitude F_2[9] ($\hbar = c = \mu = 1$):

$$\sigma_t = (4\pi p/k)\ |F_2|^2, \tag{5}$$

where p and k are the momenta of the meson and photon in the c.m.s., respectively. Thus, a study of the elastic photoproduction of neutral mesons on such nuclei enables us to determine experimentally the amplitude F_2 for the π^0-meson photoproduction on nucleons.

It is not easy, however, to obtain the amplitude F_2 from the data on the meson photoproduction on protons. In order to determine F_2 it is necessary to carry out a full phase shift analysis which, in turn, requires a measurement of the polarization of recoil protons.

On the other hand it should be noted that the form of the dispersion relations for the amplitude F_2 is such[9] that the latter is practically fully determined by the dispersion integral. An experimental determination of the F_2 amplitude can therefore be used to check the adequacy of an estimate of the dispersion integrals for the π^0-meson photoproduction. This is important in view of the fact that the present discrepancies between calculations based on the dispersion theory and the results of experiments on the π^0 photoproduction on nucleons may be interpreted as due to a contribution from resonance meson states. The same result, however, can be also ascribed to the errors in estimating the dispersion integrals.

To calculate the amplitude F_2 we have used our data on the π^0-meson photoproduction on the C^{12} nucleus (spin zero) at 154 and 182 MeV. The results of the calculation are given in Fig. 11. In the same figure is also shown the curve from [9], obtained by solving the one-dimensional relativistic dispersion relations for the π^0-meson photoproduction on nucleons. In spite of the fact that the dispersion integrals were calculated in [9] in the pole approximation for the $M_{1+}^{3/2}$ resonance amplitude, we can consider that there is an agreement between the experimental and calculated values of F_2.

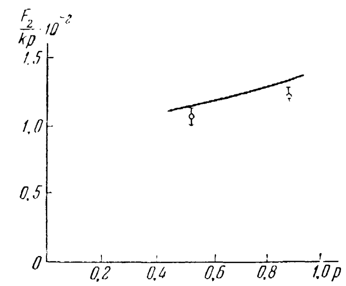

FIG. 11. Dependence of the amplitude F_2 on the meson momentum.

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