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CONCERNING ONE MODEL OF ππ INTERACTION

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Pion interaction via an intermediate vector meson is considered. The model describes $\pi\pi$ resonance and yields asymptotically a constant total cross section and the diffractional angular distribution. Other properties of the model are likewise not in contradiction with experiment.

1. INTRODUCTION

A model of the $\pi\pi$ interaction resulting from exchange of a vector particle—the ρ meson—has already been mentioned in the literature (see [1] and the references cited there). An advantage of this model is the simplicity with which the existence of $\pi\pi$ interaction is deduced from it.

If only $M_\rho > 2\mu$ the interaction

$$\langle \rho \,|\, \pi\pi \rangle = \gamma e_\mu (p_1 - p_2)_\mu \tag{1}$$

leads to the decay $\rho \to 2\pi$ with probability

$$w = \frac{2}{3}\left(\frac{\gamma^2}{4\pi}\right)\frac{q^3}{\rho^2} \tag{2}$$

(ρ = ρ-meson mass, $q = \sqrt{\rho^2/4 - \mu^2}$ = pion momentum). The transition $\pi\pi \to \rho \to \pi\pi$ is registered as a resonance pion interaction. It is interesting to note that owing to Bose statistics of the pions, the state $\pi\pi\rangle$ with $J = 1$ has an isospin $I = 1$, i.e., the ρ meson is an isovector particle.

The mass and the coupling constant of the ρ meson with the pions are determined by the position and the width of the $\pi\pi$ resonance[2]:

$$W_r = 750\ \text{MeV}, \qquad \Gamma = 150\ \text{MeV} \tag{3}$$

(it must be recognized that $\Gamma = 2w$). We assume

$$\rho^2 = 30\mu^2, \qquad \gamma^2/4\pi = 1.50. \tag{4}$$

These parameters (together with the quantum numbers $I = J = 1$) enable us to determine the amplitude of the $\pi\pi$ scattering (see Sec. 2) and gain an idea of the energy dependence of the cross section, the angular distribution, etc.

2. PARTIAL AMPLITUDES

The pole part of the amplitude $\mathfrak{M} = \langle \pi\pi \,|\, \pi\pi \rangle$ is due to the ρ-meson exchange and can be represented in the form

$$\mathfrak{M} = \gamma^2\left(\frac{t-s}{\rho^2-u} e_{r\alpha\gamma} e_{r\beta\delta} + \frac{u-s}{\rho^2-t} e_{r\beta\gamma} e_{r\alpha\delta} + \frac{t-u}{\rho^2-s} e_{r\delta\gamma} e_{r\beta\alpha}\right) \tag{5}$$

(the notation is the same as in [3]). From this we can obtain the scattering amplitudes in states with specified isospin (cf. [1]):

$$\mathfrak{M}^{(0)} = 2\gamma^2\left(\frac{t-s}{\rho^2-u} + \frac{u-s}{\rho^2-t}\right),$$

$$\mathfrak{M}^{(1)} = \gamma^2\left(2\frac{t-u}{\rho^2-s} + \frac{t-s}{\rho^2-u} - \frac{u-s}{\rho^2-t}\right), \qquad \mathfrak{M}^{(2)} = -\tfrac{1}{2}\mathfrak{M}^{(0)}. \tag{6}$$

Since

$$s = W^2 = 4(q^2 + \mu^2), \qquad t = -2q^2(1 - \cos\theta),$$
$$u = -2q^2(1 + \cos\theta), \tag{7}$$

$\mathfrak{M}^{(0,2)}$ and $\mathfrak{M}^{(1)}$ are even and odd functions of $\cos\theta$, respectively (this calls for pion Bose statistics). Therefore the expansion into partial waves

$$\mathfrak{M} = \sum_l (2l+1) P_l(\cos\theta)\, \mathfrak{M}_l \tag{8}$$

contains only even P_l when $I = 0$ and 2 and only odd ones when $I = 1$. The corresponding partial amplitudes are expressed by the formulas

$$\mathfrak{M}_l^{(0)} = 2\gamma^2\left(2\delta_{l0} - \left(8 + \frac{\rho^2 + 4\mu^2}{q^2}\right) Q_l\left(1 + \frac{\rho^2}{2q^2}\right)\right),$$

$$\mathfrak{M}_l^{(2)} = -\tfrac{1}{2}\mathfrak{M}_l^{(0)},$$

$$\mathfrak{M}_l^{(1)} = \gamma^2\left\{\frac{8}{3}\frac{q^2}{\rho^2 - s}\delta_{l1} + \left(8 + \frac{\rho^2 + 4\mu^2}{q^2}\right) Q_l\left(1 + \frac{\rho^2}{2q^2}\right)\right\} \tag{9}$$

(Q_l are Legendre functions of the second kind).

We see from this that with increasing energy ($q \to \infty$) all the amplitudes increase logarithmically:

$$\mathfrak{M}_l \sim \ln(4q^2/\rho^2), \tag{10}$$

in contradiction to the unitarity condition. This cannot be excluded in the study of the pole part—the unitarity condition is imposed only on the cut.

The amplitude

$$M_l = \frac{\mathfrak{M}_l/8\pi W}{1 - iq\mathfrak{M}_l/8\pi W} \tag{11}$$

has cuts $q^2 \geq 0$ and $q^2 \leq -\mu^2$ and is analytic in the remainder of the complex q^2 plane[4] On the right-hand cut

$$\mathrm{Im}\, M_l = q\,|M_l|^2, \tag{12}$$

i.e., the unitarity condition is satisfied. It is possible to verify with the aid of crossing symmetry that the unitarity is satisfied on the left cut, too. $M_l \sim q^{2l+1}$ for small q and satisfies all the requirements imposed on the scattering amplitude in dispersion theory. From the uniqueness of the analytic functions we can conclude (within the framework of the two-particle unitarity condition) that M_l is the sought amplitude.

We can now obtain with the aid of the equation

$$\mathrm{tg}\delta_l = \frac{q}{8\pi W}\mathfrak{M}_l \tag{13)*}$$

the phase shifts and the partial cross sections

$$\sigma_l = 4\pi\,(2l+1)\,|M_l|^2 = 4\pi\,(2l+1)\,q^{-2}\sin^2\delta_l. \tag{14}$$

As $q \rightarrow \infty$ we have $\delta_l \rightarrow \pi/2$, ($l \neq 1$), i.e., each cross section goes to its unitary limit. This leads to the constancy of the total cross section and to the Pomeranchuk theorem.

3. ENERGY DEPENDENCE OF THE CROSS SECTIONS

Figures 1 and 2 show plots of $\sigma(q)$ calculated with the aid of (9), (13), and (14).

When I = 0 and 2, the total cross section is due to the S wave only up to $q = (3-4)\mu$. Only at higher energies do the remaining waves begin to contribute and to raise the cross section to a plateau near 7 mb. The plateau is the same in both states, owing to the Pomeranchuk theorem. The

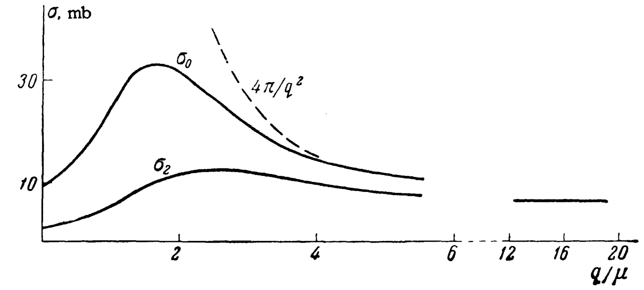

FIG. 1. σ_0 – scattering cross section in state with isospin T = 0, σ_2 – the same for T = 2, dashed line – unitary limit for the S wave.

*tg = tan.

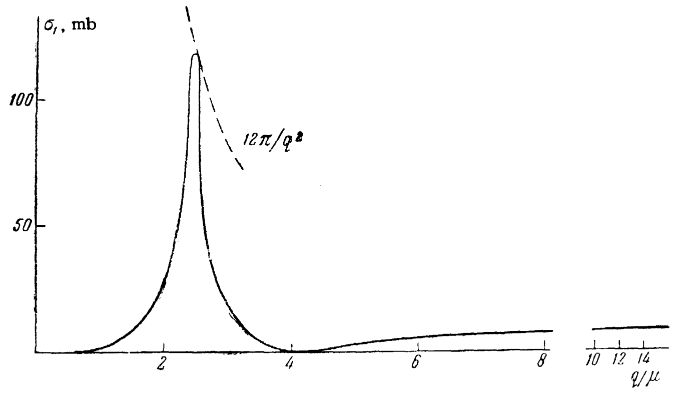

FIG. 2. Scattering cross section for T = 1, dashed curve – unitary limit for P wave.

maximum of $\sigma(q)$ in the vicinity of $q \sim \mu$ is due to the kinematic factor in formula (13).

When I = 1 the cross section depends in more complicated fashion on the energy. This is due essentially to the behavior of the P wave, which dominates up to $q \sim 6\mu$. It passes through resonance at $q = 2.55\mu$ ($s = 30\,\mu^2$), and the resonance curve is almost symmetrical about the maximum ($\Gamma \sim \mu$). This is due to the interference between the resonant and nonresonant parts of the amplitude.

At $s = 71\,\mu^2$ both parts of the amplitude cancel each other, and the amplitude vanishes. In dispersion theory this calls for the introduction of the Castillejo-Dalitz-Dyson (CDD) pole[5]. Unfortunately, we do not know as yet how to find the position and the residue of this pole within the framework of the dispersion formalism. There are no CDD poles in any of the remaining amplitudes, so that there are no interfering terms in them.

With increasing q, the role of the resonant amplitude decreases, the higher waves are turned on, and the cross section goes to a plateau of the same magnitude as in the case when I = 0 and 2. It can be calculated by the optical theorem

$$\sigma = \frac{4\pi}{q}\,\mathrm{Im}\,A. \tag{15}$$

Here A is the forward-scattering amplitude:

$$A = \frac{1}{q}\sum_l (2l+1)\,A_l\,(1 - iA_l)^{-1}, \qquad A_l = \frac{q\mathfrak{M}_l}{8\pi W}. \tag{16}$$

The series for A turns into the integral[6]

$$A = \frac{1}{2\pi i q}\int\limits_{(C)} \frac{A_l}{1 - iA_l}\,\frac{\pi}{2}\,\mathrm{tg}\,\frac{\pi l}{2}\cdot(2l+1)\,dl. \tag{17}$$

An expression is given here for I = 2, when the summation in (16) is only over odd l. For I = 0 and 2 it is necessary to replace $\tan(\pi l/2)$ by $\cot(\pi l/2)$. The contour C is shown in Fig. 3.

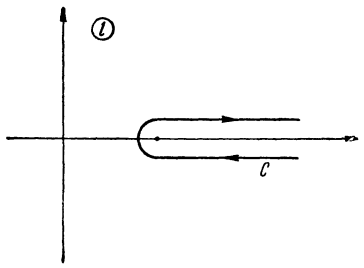

FIG. 3

If the integral (17) is approximated by the contribution from the pole of the partial amplitude $A_l(1 - iA_l)^{-1}$ (Regge pole[7]), it assumes a value

$$A = \frac{1}{q} \frac{A_L}{-idA_L/dL} \frac{\pi}{2} \operatorname{tg} \frac{\pi L}{2} \cdot (2L+1), \qquad (18)$$

where L is the root of the equation $A_L = -i$ or $\cot \delta_L = i$[8].

When $q^2 \gg \rho^2$ we have $A_l \approx (\gamma^2/2\pi) K_0(\rho l/q)$. By solving the equation

$$K_0(z) = -2\pi i/\gamma^2, \qquad (19)$$

we get

$$L = qz/\rho. \qquad (20)$$

Substituting these expressions in (15) and (17) we get

$$\sigma = \frac{4\pi^2}{\rho^2} \operatorname{Im} \frac{zK_0(z)}{K_1(z)} = \text{const}. \qquad (21)$$

The same result is obtained also by direct graphic summation of the partial cross sections.

4. ANGULAR DISTRIBUTION

Since the mesons are identical, the angular distribution is symmetrical about $\theta = \pi/2$. It stays constant up to relatively high energies ($s \sim 40 - 60\mu^2$), remains isotropic in the states I = 0 and 2, and is proportional to $\cos^2\theta$ when I = 1.

However, as the higher waves enter, the distribution becomes more and more peaked for θ equal to 0 and π. It can be asymptotically represented by the formula

$$\sigma(\theta) \sim |AP_L(\cos\theta)|^2, \qquad (22)$$

where A is the forward scattering amplitude (18). The factor $|P_L|^2$ leads to oscillations with damped amplitude, corresponding to a diffraction pattern.

5. CONCLUSION

We have considered a model of pion interaction via an intermediate vector ρ meson, in which it becomes possible to satisfy the main requirements imposed on the scattering amplitude: crossing symmetry (elastic), unitarity in all channels, etc. This model leads to the following results: a) the P wave has a resonance accompanied by a CDD pole, b) the cross section reaches asymptotically a plateau, the Pomeranchuk theorem holds, and the cross sections are independent of the isospin, c) the angular distribution acquires a diffraction character with increasing energy, and d) the P phase has the properties necessary for the description of the electromagnetic form factor of the nucleon (see [9]).

It seems to us, however, that this model must be refined still further. This is indicated by: 1) the low value of $\sigma(\infty)$ (cf. [10]); 2) the smallness of the electromagnetic radius of the pion ($\langle r_\pi^2 \rangle \sim \gamma^2/4\pi\rho^2$); 3) the contradictory data on the $\pi\pi$ resonance obtained from the analysis of the nucleon structure and from the reactions $\pi N \to \pi\pi N$. It is apparently necessary to take into account the low-energy resonances (ζ meson).

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