THEORETICAL DETERMINATION OF THE SIGN OF THE MASS DIFFERENCE BETWEEN THE K_1^0 AND K_2^0 MESONS

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Submitted to JETP editor January 4, 1963

J. Exptl. Theoret. Phys. (U.S.S.R.) 44, 2011-2015 (June, 1963)

The passage of a beam of K_2^0 mesons through a system of many plates is considered from the standpoint of determining the sign of the K_1^0 and K_2^0 meson mass difference. The use of a large number of closely-spaced thin plates results in a much larger yield of regenerated K_1^0 mesons than from a pair of thin plates, yet the formulas used to determine the sign of the mass difference Δm remain simple. In light of the possible violation of the $\Delta S = \Delta Q$ rule, a method of determining the sign of Δm by means of lepton decays[3] is considered. It is also proposed to use the charge exchange of K^0 and $\bar{K}^0$ mesons for the determination of the sign of Δm.

1. The question posed in 1960 by Kobzarev and Okun'[1] "Which is heavier, the K_1^0 meson or the K_2^0 meson?" has been recently attracting more attention both from the point of view of finding the most effective experimental methods[2-7] and from the point of view of attempts at a theoretical prediction of the sign of the mass difference ($\Delta m = m_1 - m_2$) of the K_1^0 and K_2^0 mesons[8-11].

The methods proposed are based on the idea of using the interference produced when a beam of K_2^0 mesons passes either through two plates having different nuclear properties[1,2,7] or through one plate, using different analyzers of K^0 and $\bar{K}^0$ mesons (lepton decay, hyperon production, etc.)[3]

Each of these two methods has its shortcomings and advantages. In the two-plate method it is necessary to know beforehand the sign of the phase difference $\Delta\varphi$ of the interfering K_1^0 waves produced in each of the plates. The one-plate method[3] calls for very large statistics, in view of the long lifetime of the K^0 mesons relative to lepton decays. At the same time, the first method has the advantage of the fast K_1^0 decay and the possibility of its reliable identification. A favorable property of the second method is the absence of uncertainties connected with the sign and magnitude of $\Delta\varphi$.

Great interest attaches in this connection to the paper of Good and Pauli[7], who calculated $\Delta\varphi$ for a specific pair of substances and showed that the sign of $\Delta\varphi$ is independent of the model of the nucleus, owing to the relative transparency of the nuclear matter for regeneration. The result obtained in [7] is promising with respect to the use of a pair of plates made of different substances to determine the sign of Δm. In this connection, further investigation of this method seems desirable.

In the present article we consider interference phenomena that occur when a beam of K_2^0 mesons passes througn n pairs of thin plates with different nuclear properties. We shall see that the use of a large number of very thin plates placed at short distances from one another increases appreciably the yield of the regenerated K_1^0 mesons, compared with the yield from a single pair of thin plates, and at the same time the same simple expressions for the sign of Δm apply as in the case of one pair of thin plates[1].

2. Assume that a monochromatic beam of K_2^0 mesons with momentum k ($\hbar = c = 1$) is incident on a system of n pairs of thin plates of different substances (a_j, b_j; $j = 1, 2, \ldots, n$) spaced, for the sake of simplicity, an equal distance x_0 apart. Assuming the plates to be thin ($x_{a_j}, x_{b_j} \ll \gamma\tau_1 v$, τ_1 is the lifetime of the K_1^0 meson, v the velocity of the K_2^0 beam, and $\gamma = 1/\sqrt{1 - v_2}$), i.e., neglecting the decays and the absorption in each of the plates, we obtain for the amplitude $\alpha_1(a_j(b_j))$ of the undeflected K_1^0 wave coherently regenerated in plate $a_j(b_j)$, on the basis of formula (6) of the author's earlier paper[2],

$$\alpha_1(a_j(b_j)) = ix_{a_j(b_j)} N_{a_j(b_j)} \lambda f_{12}^{a_j(b_j)}(0), \tag{1}$$

where $N_{a_j(b_j)}$ is the number of atoms per cm^3 of plate matter, λ is the wavelength of the K_2^0 mesons, $f_{12}^{a_j(b_j)}(0) = \frac{1}{2}(f^{a_j(b_j)}(0) - \bar{f}^{a_j(b_j)}(0))$ is the amplitude of coherent $K_2^0 \longleftrightarrow K_1^0$ regeneration on each individual nucleus of the substance of plate $a_j(b_j)$, and $f(0)$ and $\bar{f}(0)$ are the amplitudes of the forward elastic scattering of the K^0 and $\bar{K}^0$ mesons on the nucleus, respectively.

For the sake of convenience we write α_1 in the form

$$\alpha_1(a_j(b_j)) = ir_{a_j(b_j)} \exp\{i\varphi_{a_j(b_j)}\}, \tag{2}$$

where $r_{a_j(b_j)}$ and $\varphi_{a_jb_j}$ are expressed in terms of $\bar{f}(0)$ and $f(0)$ in complete analogy with formulas (8) and (9) of the paper of Kobzarev and Okun'[1].

Recognizing that the K^0 wave oscillates prior to regeneration with frequency m_2/γ and after regeneration with frequency m_1/γ, and taking into consideration the decays in the spaces x_0 between the plates ($x_0 \gg x_{a_j(b_j)}$), we can write for the coherently regenerated K_1^0 wave in the beam passing through the system without deflection

$$\begin{aligned}\alpha_1((2n-1)t_0) = i\sum_{j=1}^{n}\Big\{r_{a_j}\exp\Big[i\varphi_{a_j} - \Big(\frac{im_1}{\gamma} + \frac{1}{2\gamma\tau_1}\Big) \\ \times[2(n-j)+1]\,t_0 - \Big(i\frac{m_2}{\gamma} + \frac{1}{2\gamma\tau_2}\Big)2(j-1)\,t_0\Big] \\ + r_{b_j}\exp\Big[i\varphi_{b_j} - \Big(\frac{im_1}{\gamma} + \frac{1}{2\gamma\tau_1}\Big)2(n-j)\,t_0 \\ - \Big(\frac{im_2}{\gamma} + \frac{1}{2\gamma\tau_2}\Big)(2j-1)\,t_0\Big]\Big\},\end{aligned} \tag{3}$$

where $t_0 = x_0/v$, $\tau_1(\tau_2)$ is the lifetime of the K_1^0 (K_2^0) meson [Eq. (3) is written for the amplitude α_1 near the edge of the last plate).

We consider for simplicity the case when all the plates a are made of the same substance and have the same thickness $x_a = vt_a$. Then

$$r_{a_j} = r_a, \quad \varphi_{a_j} = \varphi_a, \quad x_{a_j} = x_a, \quad j = 1, \ldots, n.$$

Similarly, for the plates b,

$$r_{b_j} = r_b, \quad \varphi_{b_j} = \varphi_b, \quad x_{b_j} = x_b = vt_b, \quad j = 1, \ldots, n.$$

Then, summing (3) and calculating the square of the modulus, we obtain the total number of the K_1^0 mesons leaving the system of plates without experiencing deflection, referred to the initial flux of K_2^0 mesons

$$\begin{aligned}N_1 = \frac{1 + e^{-2nt_0/\gamma\tau_1} - 2e^{-nt_0/\gamma\tau_1}\cos(2nt_0\Delta m/\gamma)}{1 + e^{-2t_0/\gamma\tau_1} - 2e^{-t_0/\gamma\tau_1}\cos(2t_0\Delta m/\gamma)} \\ \times[r_a^2e^{-t_0/\gamma\tau_1} + r_b^2 + 2r_ar_be^{-t_0/2\gamma\tau_1}\cos(\Delta\varphi - t_0\Delta m/\gamma)],\end{aligned} \tag{4}$$

$\Delta\varphi = \varphi_a - \varphi_b$. Here $\tau_2 = \infty$.

The expression in the square brackets is the relative number of K_1^0 mesons regenerated by one pair of plates (a, b)[1]. For n = 1 we obtain Eq. (12) of Kobzarev and Okun'[1]. If account is taken of the absorption in 2n thin plates, formula (4) must be multiplied by

$$\exp\{-n[N_a(\sigma_a + \bar{\sigma}_a)x_a + N_b(\sigma_b + \bar{\sigma}_b)x_b]\},$$

where $\sigma_i(\bar{\sigma}_i)$ is the total cross section for the interaction of the $K^0(\bar{K}^0)$ mesons with the nuclei of the plates i (i = 1, 2). The role of this factor is insignificant—the exponent contains a number of order $10^{-2}\,nx_i$, and x_i is very small.

3. What are the most favorable conditions for the determination of the sign of Δm? Calculation[7] shows that $\Delta\varphi$ is small ($\sim 0.1 - 0.2$). Taking $|\Delta m|t_0/\gamma = \pi/2$ and taking into account the fact that $|\Delta m| \approx 1/\tau_1$, we obtain

$$\begin{aligned}N_1 &\approx (r_a^2e^{-t_0/\gamma\tau_1} + r_b^2)(1 \pm \delta\Delta\varphi); \\ \delta &= 2r_ar_be^{-t_0/2\gamma\tau_1}/(r_a^2e^{-t_0/\gamma\tau_1} + r_b^2),\end{aligned} \tag{5}$$

$\delta \approx 1$ for $r_a\exp(-t_0/2\gamma\tau_1) \approx r_b$, and the $\pm$ signs correspond to $\Delta m \gtrless 0$. In this case the situation is quantitatively the same as for the case of a pair of thin plates, i.e., the yield of K_1^0 mesons is small. Thus, if plate b is of lead and the K_2^0-meson energy is 500 MeV we get, using the data of Good et al[12] for f_{12}^0,

$$N_1 \approx r_b^2 \approx 0.35\,x_{Pb}^2 \cdot 10^{-4}.$$

In spite of the need for very large statistics, formula (5) is convenient for the determination of the sign of Δm if interchange of the plates is employed[7]. In the case of thick plates the K_1^0 yield increases appreciably[2], but a formula such as (5) does not apply in general. In this case when $|\Delta m|(t_0 + t_b)/\gamma = \pi/2$ the quantity $\pm\Delta\varphi$ is replaced in the corresponding formula derived from (11) and (12) of [2] by the quantity $\pm[C(t_a, t_b)\Delta\varphi + S(t_a, t_b)]/x_ax_b$, where the functions $C(t_a, t_b)$ and $S(t_a, t_b)$ are given by Eqs. (12) of the same paper. Obviously, in the case of a system of n pairs of thin plates the K_1^0-meson yield will increase appreciably if we dispense with the condition $|\Delta m|t_0/\gamma = \pi/2$. Taking $t_{a,b} \ll t_0 \ll \gamma\tau_1$, we obtain ($\Delta\varphi \approx |\Delta m|t_0/\gamma \ll 1$)

$$N_1 \approx (\gamma\tau_1/t_0)^2P(2nt_0)(r_a + r_b)^2(1 \pm \delta'\Delta\varphi), \tag{6}$$

where

$$P(2nt_0) = \frac{1 + e^{-2nt_0/\gamma\tau_1} - 2e^{-nt_0/\gamma\tau_1}\cos(2nt_0\Delta m/\gamma)}{1 + (2\Delta m\tau_1)^2}, \tag{6'}$$

$$\delta' = \frac{2r_ar_b}{(r_a + r_b)^2}\frac{|\Delta m|}{\gamma}t_0; \tag{6''}$$

the ± signs correspond again to $\Delta m \gtrless 0$.

We have omitted from (6) small terms of order $(\Delta\varphi)^2$ and $(t_0\Delta m/\gamma)^2$, which do not change sign when the plates are rearranged. This rearrangement is necessary to determine the effect due to the sign of Δm.

It is seen from (6) that in such a plate configuration ($t \ll \gamma\tau_1$, $n \gtrsim \gamma\tau_1$) a noticeable gain in the number of K_1^0 mesons is obtained. The function $P(2nt_0)$ has a maximum at $2nt_0/\gamma\tau_1 \approx 2.55$. Taking this value of n, we obtain

$$N_1 \approx n^2(r_a + r_b)^2 (1 \pm \delta'\Delta\varphi), \quad n \gg 1. \tag{7}$$

For example, taking for the distance between plates $x_0 = 0.5$ cm, we get for an approximate K_2^0-meson energy of 500 MeV ($\gamma\tau_1 v = 5.2$ cm) n = 13.

In fact this reduces to a situation wherein two effective plates of thickness $L_{a,b} = \gamma\tau_1 v x_{a,b}/x_0$, practically in contact with each other, are "in operation."

It must be borne in mind, however, that by attaining a definite increase in the K_1^0-meson yield through the choice of $\gamma\tau_1 \gg t_0$, we experience a loss in δ', and this is unfavorable from the point of view of determining the sign of Δm. In this connection, the results obtained are more interesting from the point of view of the dependence on the thin-plate configuration which is due entirely to the interference of the K_1^0 waves produced in a large number of plates.

4. In conclusion we make two remarks concerning the determination of the sign of Δm. The method proposed by the author in [3] for determining the sign of Δm from the lepton decays is based on the selection rule $\Delta S = \Delta Q$, the satisfaction of which is at present subject to some doubt[13]. It is easy to see, however, that if we take into account the contribution made to lepton decays by transitions with $\Delta S = -\Delta Q$, formula (4) of [3] is modified only by a factor $(1 - x^2)$, where x is the relative contribution of the amplitude with $\Delta S = -\Delta Q$. The experimental data yield[13] $x \approx 0.55$, so that if these data are correct, the effect indicated in [3] can again be used for the measurement of the sign of Δm.

The second remark also pertains to the possibility of measuring the sign of Δm by the one-plate method. We can attempt here to analyze the K^0 and $\bar{K}^0$ mesons by their charge exchange in a light substance with Z = N. Then the dependence of the difference of the number of produced K^+ and K^- mesons on Δm will be determined by the same relation as for the difference in the number of lepton decays into positively and negatively charged leptons[3]. The statistics necessary here are approximately the same as for the lepton decays. In view of the long life of the $K^\pm$ mesons, the phenomena occurring in the direct vicinity of the plate are transferred in experiments of this type to larger distances from the plate.

The author is grateful to É. V. Gedalin and O. V. Kancheli for useful discussions.

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Translated by J. G. Adashko
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