SOVIET PHYSICS JETP VOLUME 17, NUMBER 6 DECEMBER, 1963

THE CONTRIBUTION OF STRONG INTERACTIONS TO THE PHOTON GREEN'S FUNCTION

V. G. PIVOVAROV

Institute of Mathematics and Computation Center, Siberian Section, Academy of Sciences, U.S.S.R.

Submitted to JETP editor January 11, 1963

J. Exptl. Theoret. Phys. (U.S.S.R.) **44**, 2023-2028 (June, 1963)

The problem considered is that of the change of the photon Green's function which comes from inclusion of the strong interactions between virtual π mesons, and of the associated change of the total (differential) cross section for processes which go via virtual photons.

In order to get information about the character of the interactions of elementary particles and about the structure of space-time at small distances, it is necessary to conduct experiments on the scattering of high-energy particles. The most interesting experiments are the electrodynamic ones, because there exists a quantitative theory which describes these experiments (electrodynamics), whereas there is no such theory for the strongly interacting particles.

Until very recently, however, the maximum electron energies attained (of the order of several MeV in the center-of-mass system) have still been far from enough to allow us to speak of a test of electrodynamics. Larger energies ($\sim 10^6$ MeV) can be attained in collisions of electrons and positrons having oppositely directed momenta. Now that there is no doubt that electron-positron collisions in clashing beams can be realized experimentally, it is especially important to calculate the total cross sections for scattering of electrons by electrons and by positrons with great accuracy.

There have been many papers on improvements in the accuracy of the total cross sections for these processes. Along with this higher accuracy it becomes necessary to take into account new effects arising at great energies. One possible effect is a change of the photon Green's function owing to strong interactions.

1. A number of calculations by perturbation-theory methods for processes that can occur at high energies have been made in papers by Baĭer and Kheĭfets,[1] and also in a paper by Cabibbo and Gatto.[2] Since in this case the squared momentum transfer $k^2 = t$ can reach values of the order of 1 $(\mathrm{BeV})^2$, radiative corrections to the main process of electron-positron scattering via a virtual photon become important.

2. At these energies, along with the ordinary electron-positron scattering, it is possible for the electron-positron pair to be converted into other particles, in particular into strongly interacting particles. In all of these processes it is important to know the Green's function of the photon

$$D_{mn}(t) = -\frac{d(t)}{t}\left(g^{mn} - \frac{k_m k_n}{t}\right),$$

$$d(t) = 1 + \frac{\alpha}{\pi}\left(I_e(t) + I_\mu(t) + I_\pi(t) + \ldots\right),$$

$$d(0) = 1, \quad \alpha = {}^1/_{137}, \tag{1}$$

where $I_e(t)$, $I_\mu(t)$, $I_\pi(t)$, and so on correspond to electron, muon, and pion loops, and so on, each of which describes the virtual decay of the photon into the corresponding pair of particles.

If we take into account only electromagnetic interactions, then at the energies in question $I_e(t)$ is much larger than all of the other $I_i(t)$. The picture may change, however, when strong interactions are taken into account.

We shall take the strong $\pi\pi$ interaction into account in the two-particle (two-pion) approximation, neglecting, as usual, the contributions of other intermediate states. To find $I_\pi(t)$ we consider the diagram shown in Fig. 1. This diagram represents the unitarity condition for the scattering amplitude. Using this condition in the two-particle approximation, we have for the imaginary part of the amplitude

$$\mathrm{Im}\, U_{ab}^{mn}(t) = \frac{e^2}{64\pi^2}\sqrt{\frac{t-4\mu^2}{t}}\,|F_\pi(t)|^2 \int do\,(k-2q)_m\,(k-2q)_n, \tag{2}$$

where μ is the mass of the pion and $F_\pi(t)$ is its

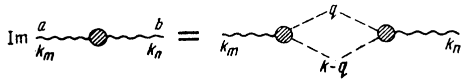

FIG. 1

electromagnetic form-factor, Im U_{ab}^{mn} is the imaginary part of the amplitude which corresponds to this diagram, k, q are the four-momenta of the virtual photon and π meson, and do is an element of solid angle in the center-of-mass system of the emerging π mesons.

At energies close to 750 MeV, at which the ρ-meson resonance is observed in the $\pi\pi$ scattering amplitude in the p state, we can expect the largest changes of the quantity $I_\pi(t)$ as compared with perturbation theory.

Using the spectral properties of the function d, by means of Eq. (2) we get

$$I_\pi(t) = -\frac{t}{12}\int\limits_{4\mu^2}^{\infty}\left(\frac{t'-4\mu^2}{t'}\right)^{3/2}\frac{|F_\pi(t')|^2}{t'(t'-t)}\,dt'. \qquad (3)$$

We now use for the calculation of $I_\pi(t)$ the form-factors obtained in papers by Grashin[3] and by Isaev and Meshcheryakov,[4] and also use the notation of these papers for these quantities. The results of calculations of the integral (3), which have been made with the electronic computing machine of the Siberian Section of the U.S.S.R. Academy of Sciences, are given in the tables.

Table I. Values of $(\alpha/\pi)I_\pi^{(i)}(t)$ corresponding to $|F_\pi(t)|^2 \approx 1$

Correction terms	E, MeV								
	280	400	485	560	625	685	740	840	890
$(\alpha/\pi)I_\pi^{(1)}$	−0.14	−0.19	−1.35	−0.07	−0.02	−0.05	0.05	0,09	$0.60\cdot10^{-3}$
$(\alpha/\pi)I_\pi^{(2)}$	0.00	−0.35	−0.33	−0.28	−0.19	−0.05	0.10	0.51	$1.10\cdot10^{-3}$
$(\alpha/\pi)I_\pi^{(3)}$	0.00	−0.19	−0.12	−0.07	0.00	0.07	0.23	0.04	$0.55\cdot10^{-3}$

Table II. Dependence of $(\alpha/\pi)I_\pi^{(4)}(t)$ on $\gamma = a(x_0 - x_r)/(x_0 + x_r)$; $x_0 = 7$, $x_r = 6.25$

γ	E, MeV												
	280	625	740	755	760	765	770	775	780	785	790	805	890
0,2	−0,00	−0,03	−0,16	−0,09	−0,01	0,07	0,12	0.15	0,15	0,15	0.13	0,11	0,01
0,4	−0,01	−0,12	−0,52	−0,67	−0,69	−0,64	0.47	0.17	0,07	0,47	0,63	0,68	0,06
0,6	−0,06	−0,73	−3,00	−4,35	−5.00	−5,65	−6.30	−4.55	−0,61	3.76	6,50	5.60	3,20

Table III. Dependence of $(\alpha/\pi)I_\pi^{(4)}(t)$ on x_0; $\gamma = 0.4$, $x_r = 6.25$

x_0	E, MeV												
	280	625	740	755	760	765	770	775	780	785	790	805	890
6,625	−0,12	−9,00	−17,0	−21,0	−17,0	5,00	22,0	20,5	17,5	13,5	9,80	7,70	0,20
7,00	−0,01	−0,12	− 0,52	− 0,67	− 0,69	−0,64	− 0,47	− 0.17	0,07	0,47	0,63	0,68	0.06
7,25	0,00	−0,05	− 0,17	− 0,21	− 0,20	−0,18	− 0.14	− 0,07	0,00	0,08	0,13	0,19	0,03
7,50	0,00	−0,02	− 0,09	− 0,09	− 0,08	−0,08	− 0,06	− 0,03	0,00	0,02	0,04	0,07	0,02
8,00	−0.01	−0,11	− 0,34	− 0,33	− 0,30	−0,25	− 0,19	− 0,12	− 0,03	0,03	0,10	0,20	0,013
12,00	−0,02	−0,09	− 0,34	− 0,32	− 0,30	−0,26	− 0,23	− 0.19	− 0,15	− 0,11	−0,07	0.02	0,0021

Table IV. Dependence of $(\alpha/\pi)I_\pi^{(5)}(t)$ on the width of the resonance

a	E, MeV											
	280	625	685	715	720	725	730	735	740	775	790	890
0.1	−0,01	−0,06	−0,16	−0,30	−0,08	0,22	0,35	0,33	0,28	0,15	0,10	0,00
0,07	0,00	−0,05	−0,13	−0,26	−0,30	−0,39	−0,42	−0,23	0,18	0,27	0,16	0,00
0.03	−0,01	−0,11	−0.23	−0,42	−0.47	−0,57	−0,70	−0,89	−1,20	1,60	0,56	0,01

A. The simplest case $F_\pi(t) \equiv 1$ corresponds to the absence of strong interactions. Then

$$I_\pi^{(1)}(t) = -\frac{1}{12}\left\{\frac{8}{3} - 8\frac{\mu^2}{t} + \left(1 - \frac{4\mu^2}{t}\right)\ln\left|\frac{1 - \sqrt{(t - 4\mu^2)/t}}{1 + \sqrt{(t - 4\mu^2)/t}}\right|\right\}. \qquad (4)$$

Equation (4) can be obtained by perturbation-theory methods; it corresponds to the diagram of Fig. 2.

At the energies considered $\mu^2 \ll k^2$ and $I_\pi^{(1)}(t) \approx -\frac{1}{12}[\frac{8}{3} - \ln(t/\mu^2)]$. In this same region $I_e(t) \approx \frac{1}{3}\ln(t/m^2)$, where m is the mass of the electron. It is obvious that $I_e(t) \gg I_\pi^{(1)}(t)$.

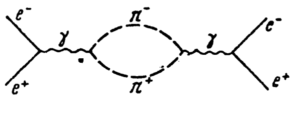

FIG. 2

B. The model of a positive scattering length has been introduced in Grashin's paper.[3] In this case

$$F_\pi(x) = \frac{(1 + \beta + \beta^2)(1 + \beta^2 x)}{(1 + \beta)(1 + \beta^3 x\sqrt{-x})},$$

$$x = \frac{t}{4\mu^2} - 1, \quad \beta^{-3} > 0, \quad \sqrt{-x} = -i\sqrt{x}. \qquad (5)$$

The parameter β characterizes the scattering length. For $\beta = 0.3$, which corresponds to $\langle r^2 \rangle = 0.08\,\mu^{-2}$ ($\langle r^2 \rangle$ is the mean square radius of the π meson[5]), we have $F_\pi(x) \approx 1$. Therefore $I_\pi^{(2)}(t) \ll I_e(t)$.

C. The case of Breit-Wigner (dynamical) resonance[3]:

$$F_\pi(x) = \frac{x_r - x + \gamma}{x_r - x + \gamma\sqrt{-x}},$$

$$\gamma \leqslant 1, \qquad \sqrt{-x} = -i\sqrt{x}, \qquad (6)$$

where x_r is the position of the p-wave resonance in the $\pi\pi$ amplitude. If we take $\gamma = 0.1$ and $x_r = 6.25$, then $|F_\pi(t)|^2 \leq 1$ and $I_\pi^{(3)}(t)$ does not differ much from the values $I_\pi^{(1)}(t)$ and $I_\pi^{(2)}(t)$ which correspond to the first two cases we have considered.

If we use the phases of the $\pi\pi$ amplitude which correspond to the second and third form-factors, it is not possible to get πN amplitudes which agree with the experiments on πN scattering.

Let us consider the form-factors, which have absolute values much greater than unity near the resonance of the p-wave amplitude for $\pi\pi$ scattering. We can get the general form of the corresponding quantities $I_\pi(t)$ if we use the following approximate formula for this kind of form-factors:

$$|F_\pi(t)|^2 = b\delta(t - t_r), \qquad (7)$$

where b is a numerical coefficient and t_r is the position of the resonance. Then

$$I_\pi^{(\delta)}(t) = -\frac{t}{12}\left(\frac{t_r - 4\mu^2}{t_r}\right)^{3/2}\frac{b}{t_r(t_r - t)}. \qquad (8)$$

The shape of the curve is shown in Fig. 3.

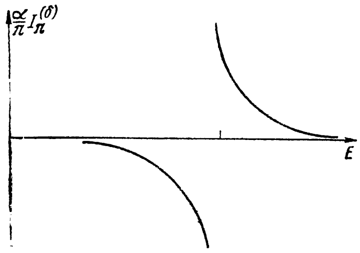

FIG. 3

D. The kinematic-resonance model (ρ-meson effect).[3] In this case

$$F_\pi(x) = \frac{(\xi - x)(x - x_1)}{(x_r - x)(x_0 + x) + a(x_0 - x)\sqrt{-x}},$$

$$\sqrt{-x} = -i\sqrt{x}. \qquad (9)$$

Here x_0 is zero and x_r is the resonance of the p-wave amplitude for $\pi\pi$ scattering. The quantity a characterizes the width of the ρ-meson resonance, x_1 is the root of the equation $(x_r - x)(x_0 - x) + a(-x)^{1/2}(x_0 - x) = 0$ such that $\mathrm{Re}(-x)^{1/2} \geq 0$, and, finally,

$$\xi = -(x_r x_0 + x_0 - x_r + x_1 + a + a x_0)(1 + x_1)^{-1}.$$

It follows from Eq. (9) that $|F_\pi(t)| \gg 1$ near $x = x_r$, which assures that $I_\pi^{(4)}(t)$ is large. We must call attention to the fact that the value of x_0 is unknown, and moreover we do not even know whether or not there exists a zero in the p-wave amplitude. At the same time $I_\pi^{(4)}(t)$ depends strongly on the value of x_0 (see Table III).

E. The form-factor obtained by Isaev and Meshcheryakov[4] has as a common feature with that considered in Case D the fact that $|F_\pi(t)| \gg 1$ over a wide range of the variable t. This form-factor is[1)]

[1)]Equation (10) differs from the formula given in the paper of Isaev and Meshcheryakov,[4] since the latter contains a misprint which Meshcheryakov has pointed out. The writer is grateful to him for this information.

$$F_\pi(x) = \frac{\sqrt{-x} - \sqrt{x_r}/(u+v)}{x_r - x - \sqrt{-x}\sqrt{x_r}(u+v)/(u^2+v^2+{}^1/_3)}$$

$$\times \frac{1 + x_r - \sqrt{x_r}(u+v)/(u^2+v^2+{}^1/_3)}{1 - \sqrt{x_r}/(u+v)};$$

$$\sqrt{-x} = -i\sqrt{x}, \qquad u = (-\varepsilon/2 + \sqrt{{}^1/_{27} + \varepsilon^2/4})^{1/3},$$

$$v = -(\varepsilon/2 + \sqrt{{}^1/_{27} + \varepsilon^2/4})^{1/3}, \quad \varepsilon = a\sqrt{x_r}, \tag{10}$$

where a is a parameter which characterizes the width of the resonance.

2. After one has found the change of $d(t)$ from one pion loop it is easy to find the change of the photon Green's function when all diagrams with arbitrary numbers of such loops are taken into account. Let us consider form-factors such that $\alpha I_\pi(t)/\pi < 1$. Then

$$d(t) = (1 - \alpha I_\pi(t)/\pi)^{-1}.$$

There is then no difficulty in tracing how the total (differential) cross section will vary for a process with an intermediate photon. In fact, $\sigma \sim M^2$ and $M \sim d(t)$, where σ is the total cross section and M is the amplitude for the process. If the cross section has been calculated with $d(t) \equiv 1$, then it is necessary to multiply this result by the function $\psi(t)$, where $\psi(t) = [1 - (\alpha/\pi) I_\pi(t)]^{-2}$. The shapes of the function ψ are shown in Fig. 5 for the two functions $(\alpha/\pi) I_\pi(t)$ shown in Fig. 4.

The energy dependence of $\psi(t)$ is as follows:

E, MeV :	280	625	740	755	760	765	770	775	780	785	790	805	890
$\psi_1(t)$:	1.00	0.81	0.35	0.36	0.36	0.37	0.48	0.74	1.00	3.7	4.4	9	1.00
E, MeV :	280	625	685	715	720	725	730	735	740	775	790	890	
$\psi_2(t)$:	1.00	0.90	0.8	0.64	0.60	0.52	0.49	0.64	1.44	1.96	1.44	1.00	

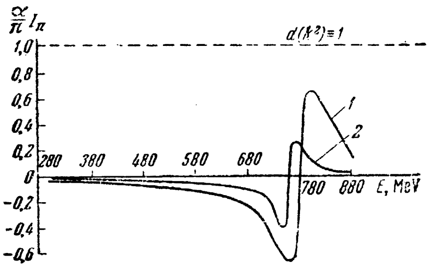

FIG. 4. Curve 1: the function $(\alpha/\pi)I_\pi^{(4)}(t)$ for $x_r = 6.25$, $x_0 = 7$, $\gamma = 0.4$; curve 2: the function $(\alpha/\pi)I_\pi^{(5)}(t)$ for $x_r = 6.255$, $a = 0.07$.

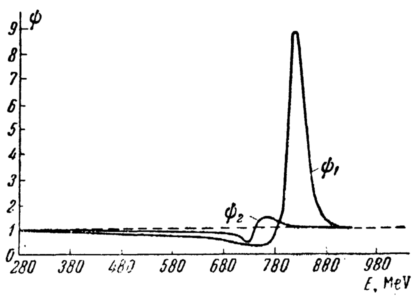

FIG. 5

CONCLUSION

The calculations we have made show that for certain values of the parameters the quantities $(\alpha/\pi) I_\pi^{(4)}(t)$ and $(\alpha/\pi) I_\pi^{(5)}(t)$ amount to some tens of percent of the unperturbed Green's function. Beginning at $t \approx 4\mu^2$ the corrections given by these functions must be taken into account.

Unfortunately, the experiments on $\pi\pi$ scattering do not allow a unique choice of the phases, and consequently of the electromagnetic form-factor of the π meson. The agreement between calculations with the last two form-factors and experiment does, however, allow us to think that form-factors of this kind give a qualitatively correct picture of the processes in question.

In the calculation of the functions $I_\pi(t)$ we have used the two-meson approximation. The electromagnetic form-factors of the meson have been found in this same approximation. Therefore at the energies considered we get only a qualitative idea of the processes which occur. Although we cannot include the contributions from the higher intermediate states, it seems reasonable that the appearance of additional strongly interacting particles in the intermediate state can only increase the contribution to the photon Green's function. The difference between our results for the form-factors of cases D and E and the usual radiative corrections is so large that a test in experiments with colliding beams should not be very difficult.

If our assumptions are not confirmed by experiment, the question arises as to the correctness of the calculation of the πN amplitudes in the papers of Isaev and Meshcheryakov [4] and of Galanin and Grashin. [6] If, on the other hand, our assumptions are confirmed, the question of the verification of electrodynamics at small distances will remain an open one as before.

The author is grateful to I. F. Ginzburg, N. B. Pivovarova, V. V. Serebryakov, and D. V. Shirkov for discussions and for constant interest in this work.

[1] V. N. Baĭer and S. A. Kheĭfets, JETP **40**, 613, 715 (1961), Soviet Phys. JETP **13**, 428, 500 (1961).

[2] N. Cabibbo and R. Gatto, Nuovo cimento **20**, 185 (1961).

[3] A. F. Grashin, JETP **43**, 277 (1962), Soviet Phys. JETP **16**, 198 (1963).

[4] P. S. Isaev and V. A. Meshcheryakov, Preprint R-938, Joint Inst. for Nuclear Research, 1962.

[5] Federbush, Goldberger, and Treiman, Phys. Rev. **112**, 642 (1958).

[6] A. D. Galanin and A. F. Grashin, JETP **41**, 633 (1961), Soviet Phys. JETP **14**, 454 (1962).

Translated by W. H. Furry
319