DETERMINATION OF THE MUTUAL ARRANGEMENT OF NUCLEON ASSOCIATIONS IN A NUCLEUS BY AID OF THE DIRECTED ORBITAL METHOD

S. MATTHIES, V. G. NEUDACHIN, and Yu. F. SMIRNOV

Institute of Nuclear Physics, Moscow State University

Submitted to JETP editor July 26, 1962; resubmitted March 21, 1963

J. Exptl. Theoret. Phys. (U.S.S.R.) **45**, 107-115 (August, 1963)

Some general rules are formulated, based on the directed orbital method familiar from quantum chemistry, which can be used to determine the geometric arrangement of alpha-clusters in a nucleus if the Nilsson orbit series or shell configuration is given. As an example, some excited states of O^{16} and C^{12} are considered and the geometric arrangement of alpha-clusters in Ca^{40} is determined. In conclusion some critical remarks are made concerning the application of the alpha-model to light nuclei.

1. INTRODUCTION

A very timely question in the theory of light nuclei concerns the limited nature of the shell description. This arises, in particular, in connection with the problem of alpha-clustering of nucleons. From the energy point of view this effect manifests itself, as is well known, in that the difference between the binding energies of the last neutrons in p-shell nuclei with A = 4k and 4k+1 reaches 15—17 MeV. The excitation energy of the lowest level with T = 1 in nuclei with A = 4k and Z = A/2 has the same magnitude. These phenomena are essentially connected with the relatively large weight of the Majorana forces in the nuclear interaction.

As long ago as in the Thirties, an alpha-particle model of the nucleus was proposed to describe the alpha-clustering of nucleons. This is the simplest model that takes alpha-clustering into account. It has turned out, however, that this model is not based on reality, since it stipulates that the nucleus contains alpha particles that are distinctly set apart and are located sufficiently far from one another. This assumption, as will be discussed below, does not hold true, so that interest has attached in recent years to Wheeler's model of resonating groups[1]. It has been developed by many authors and in recent papers it is called most frequently the cluster model. For the sake of brevity we shall designate it CM. In the CM the wave function of the nucleus with A = 4k and Z = A/2 is written in the form of an antisymmetrized product of the k wave functions φ_α that describe the internal motion of each alpha-cluster by the wave function Φ of their relative motion. For the simplest nucleus of this type, namely Be^8, this function has the form

$$\Psi_{Be^8} = \hat{A}\varphi_{\alpha_1}\varphi_{\alpha_2}\Phi_L(\mathbf{R}_{\alpha_1} - \mathbf{R}_{\alpha_2}), \tag{1}$$

where L is the total orbital momentum and $\hat{A}$ is the antisymmetrization operator.

In specific calculations[2-4] these functions are chosen in the form

$$\varphi_{\alpha_1} = \exp\left[-\frac{a}{2}\sum_{i=1}^{4}(\mathbf{r}_i - \mathbf{R}_{\alpha_1})^2\right], \qquad \mathbf{R}_{\alpha_1} = \frac{1}{4}\sum_{i=1}^{4}\mathbf{r}_i,$$
$$\Phi_{LM} = R^n Y_{LM}(\theta_R, \phi_R)\exp[-bR^2], \qquad \mathbf{R} = \mathbf{R}_{\alpha_1} - \mathbf{R}_{\alpha_2}, \tag{2}$$

n = 4 for the lowest levels with L = 0, 2, 4.

From the form of the functions (1) and (2) it is clear that if the parameter a (characterizing the internal dimensions of the alpha-clusters) is much larger than the parameter b (which determines the distance between clusters) then the role of nucleon exchange between clusters, due to the antisymmetry of the wave function, is quite small, and we deal with a function

$$\Psi_{Be^8} \approx \varphi_{\alpha_1}\varphi_{\alpha_2}\Phi_L(\mathbf{R}_{\alpha_1} - \mathbf{R}_{\alpha_2}),$$

that is, essentially with the wave function of the simple alpha-model.

If we introduce a parameter x = b/a, characterizing the degree of separation of the α-clusters, then $x \to 0$ ($a \to \infty$) denotes a transition to the limiting case of the CM, namely the alpha-model of the nucleus, when the alpha clusters do not overlap. As x increases from zero to a value comparable with unity, the role of the antisymmetrization becomes more and more appreciable, since the distance between clusters becomes comparable with their internal dimensions, and the wave func-

tion of the nucleus differs noticeably from the simple product $\varphi_{\alpha_1}\varphi_{\alpha_2}\Phi_L(\mathbf{R}_{\alpha_1}-\mathbf{R}_{\alpha_2})$ in which no antisymmetrization is carried out. It is sufficient to state that if we take $n < 4$ in the function Φ_L, then the antisymmetrized function (1) vanishes identically when $x = 1$, whereas the non-antisymmetrized function differs from zero for all values of x.

The wave function (1), (2) tends to a known limit as $x \to 1$. It is shown in several papers[2] that when $x = 1$ the wave function (1), (2) coincides with the function $|\, s^4p^4[44]L_1S = 0,\ T = 0\rangle$ of the shell model with an oscillator potential [of course, for complete coincidence it is necessary to multiply the function (1) by the function of the zero-point oscillations of the nuclear mass center $\Psi_{00}(\mathbf{R}_{Be^8})$, or else it becomes necessary to compare the function (1) with the functions of the translation-invariant shell model]. Thus in the limit $x = 1$ the CM goes over into the shell model with oscillator wave functions and LS coupling.

What is the actual degree of separation of the clusters, that is, to what extent is x smaller than unity? From the fact that the shell model describes quite successfully the properties of the light nuclei[5], it is to be expected that x is somewhat smaller than unity, and that the physical picture is closer to the shell model than to the alpha model. In particular, such a conclusion can be drawn from the data on the (p, 2p) reaction with fast protons, even for the very loose nucleus Be^9[6]. Were the alpha clusters to be far from one another (alpha-model), then the spectrum of the knock-on protons would have one maximum corresponding to the binding energy of the proton in the alpha cluster. Actually, two peaks are observed, separated by a large energy interval, which can be attributed only to the fact that the function of the nucleus is close to the shell limit, with one peak corresponding to the protons knocked out from the p-shell (weakly bound protons) and the other to protons from the s-shell (strongly bound protons). The question of the possible values of x will be discussed in greater detail in Sec. 3.

From all the foregoing it follows that the shell function of the nucleus can be regarded as a good zero-order approximation.

In the general case, taking into account the mixing of the configurations due to the alpha clustering effect, the wave function of the nucleus in the CM is of the form

$$\hat{A}P(\ldots(\mathbf{R}_k)\ldots)\exp\left\{-\frac{a}{2}\left(\sum_{k,i}^{n/4}(\mathbf{r}_{ik}-\mathbf{R}_k)^2+4x(\mathbf{R}_k^2)\right\}\right\}\chi(\sigma,\tau). \quad (3)$$

It is assumed here that the Young tableau has the form [44...4]. $\hat{A}$ is the antisymmetrization operator, $\mathbf{R}_k$ the radius vector of the k-th alpha cluster, $\chi(\sigma,\tau)$ the spin-isospin function, $\mathbf{r}_{ik}$ the radius vector of the i-th nucleon in the k-th alpha cluster, n the number of alpha-clusters, and $x = b/a$ a parameter characterizing the degree of the separation of the alpha-clusters.

When $x = 1$ ($a = b$) the function (3) goes over into the wave function of a definite shell configuration. When $x \to 0$ ($a \to \infty$), (3) goes over into the wave function of the alpha model. $P(\ldots(\mathbf{R}_k)\ldots)$ is a polynomial independent of x in the vectors of the distances between the alpha clusters, with its degree and coefficients determined by the shell configuration. The polynomial $P(\ldots(\mathbf{R}_k)\ldots)$ determines the spatial arrangement of the alpha clusters (the alpha geometry) and, to the contrary, can be determined if the alpha geometry is known.

This raises the general problem of determining the alpha geometry for a given state of the nucleus with a Young tableau [44...4]. In the general formulation this problem has not been discussed heretofore. In view of the difficulty of the computations in the CM, connected essentially with the antisymmetrization of the wave function (in [16] we consider some possibilities of simplifying these calculations), only the lightest nuclei have been analyzed ($A < 12$[6,7]), and for the heavier nuclei there are only ineffective attempts to consider them as a system consisting of two clusters, one light (triton, alpha particle) and one heavy (the remainder of the nucleus), with the heavy cluster taken in a fixed excited state and with the exchange effects disregarded. Such approximations are not justified at all. A correct formulation of the problem makes knowledge of the alpha geometry obligatory.

The polynomial P, which determines the alpha geometry, does not depend on x and it is therefore clear that in order to determine the alpha geometry we can use the wave functions of the shell model.

Wheeler[1] has noted that the alpha geometry can be determined by the directed orbital method (DOM) which is known from quantum chemistry[8], and indicated that the wave function s^4p^{12} of O^{16} corresponds to an alpha-cluster arrangement in the form of a regular tetrahedron. An analogous analysis of the states of the configurations s^4p^4 (Be^8 nucleus) and s^4p^8 (C^{12} nucleus) shows that Be^8 is a dumbbell made up of two overlapping alpha clusters and C^{12} is an equilateral triangle. All these results coincide with the usual alpha-model predictions relative to the geometry of these nu-

clei. However, for heavier nuclei there is usually no such agreement between the alpha geometry in the shell model (or the unified model) and the alpha model. In addition, the alpha model actually cannot tell anything about the alpha geometry of the excited states of light nuclei, where this geometry may be different than in the ground state.

The solution of all these problems constitutes the topic of the next section, in which we determine the geometry of the alpha clusters from zeroth-approximation functions with the aid of a modified DOM.

2. DETERMINATION OF ALPHA GEOMETRY BY THE DIRECTED ORBITAL METHOD

In nuclear theory, the DOM is used somewhat differently than in quantum chemistry. Whereas in the theory of molecular structure we are interested only in the directions of the chemical bonds, that is, only in the angular coordinates of the maxima of the electron wave function, in nuclear theory we deal not only with angular but also with radial coordinates and with the maxima of the wave functions of the nucleons constituting the alpha clusters. The point is that the alpha clusters can produce in the nucleus several belts, that is, there can be several maxima in a given direction, each described by its own directed or, more accurately speaking, localized orbital. In addition, in some configurations the alpha cluster may be located at the center of the nucleus, that is, at the origin, and such cases are not considered in quantum chemistry. Finally, another feature of the application of the DOM to nuclear theory is that the concept "strength of the bond" is missing here, and the choice of the most suitable alpha-figures from the energy point of view is based on the number of short-range couplings between alpha clusters (four bonds in a square, etc.).

It is assumed, as in general in the CM, that the total orbital angular momentum L, the total spin S, and the Young tableau are "good" quantum numbers, which is approximately satisfied for many nuclei in the 1p and 2s-1d shells. We shall refer only to nuclei with N = Z = 2m.

By way of the first example let us consider the nucleus O^{16}, where groups of four nucleons occupy the orbits $|1s\rangle$, $|1p_x\rangle$, $|1p_y\rangle$, $|1p_z\rangle$, ($|1p_x\rangle \sim x \exp(-\nu r^2/2)$ [8], etc.). Determining from the usual rules of the theory of finite point groups the possible types of the alpha-figures, we find that the only possible figure is a tetrahedron, with the wave functions of the hybrid orbits having the form

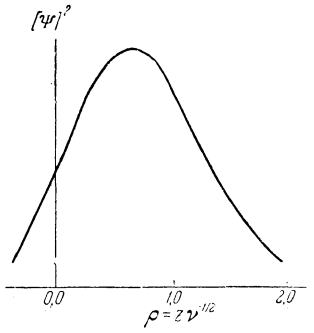
$$\Psi_i = \tfrac{1}{2}|1s\rangle + \alpha_i|\mathbf{r}\rangle, \qquad i = 1, 2, 3, 4; \quad \alpha_i\alpha_j = \delta_{ij} - \tfrac{1}{4},$$

where α_i are three-dimensional numerical vectors and $|\mathbf{r}\rangle$ is a three-dimensional vector with components $|1p_x\rangle$, $|1p_y\rangle$, and $|1p_z\rangle$. The vectors α_i are directed toward the vertices of the tetrahedron, and it is clear from symmetry considerations that the radius vectors $\mathbf{e}_i^0$ corresponding to the maxima of the functions are parallel to the vectors α_i.

By way of an example, the figure shows the z-dependence of the square of the modulus of one of the possible hybrid wave functions

$$\overline{\Psi} = \tfrac{1}{2}|1s\rangle + \tfrac{1}{2}\sqrt{3}|z\rangle.$$

We see that the cluster localization admitted by the shell wave functions is not sufficiently well pronounced to be able to neglect the overlap of the clusters.



We now choose for O^{16} the configuration (No. 1)^4 (No. 2)^4 (No. 3)^4 (No. 5—7)^4, corresponding to oblate deformation of the nucleus. The linear combination of the Nilsson orbits No. 5 and No. 7 should be chosen such that the end result is an LS coupling. This example is not merely academic. Recent measurements of the intensity of the E2-transition 6.92 MeV (2^+) → 6.05 MeV (0^+) [9] gave a reduced probability of the same value as for the rotational transition 4.42 MeV (2^+) → 0.00 MeV (0^+) in C^{12}. This shows that the levels 0^+(6.05 MeV), 2^+(6.92 MeV) and 4^+(10.36 MeV) form, in confirmation of Morinaga's assumptions [10], a rotational series, and the moment of inertia turns out to be very large so that the nuclear deformation is also very large.

Namely, in four-nucleon excitation, which was proposed by Morinaga [10] and by others [11], orbit No. 4 which stabilizes the spherical form of the nucleus, turns out to be completely empty, and this leads to very large deformation. The large energy gain in a transition to the strongly deformed nuclear form is apparently the principal reason why the level can drop so low in the four-nucleon excitation. That O^{16} is oblate in the

0^+(6.05 MeV) state and has furthermore the configuration indicated above is evident by the presence of the 6.37 MeV ($\frac{1}{2}^+$) level in O^{17}, which can be interpreted as a nucleon on orbit No. 6 "over" an excited O^{16} core of oblate form. The binding energy of this nucleon with the excited core is close to zero (+0.3 MeV), as is also the binding energy of the nucleon in orbit No. 6 over the oblate core in C^{13}[12] (−1.8 MeV). In this energy region O^{17} should also have an orbit No. 4 ($\frac{1}{2}^-$). In the case of a prolate core, O^{17} would have in this energy region only the Nilsson orbit No. 4, and orbit No. 6 would be filled.

Thus, we have a configuration such that in each Nilsson orbit $n_\perp = N$, that is, we have within the limits of the four-particle excitation $1s^4 1p^8 1b^4$ the maximum possible deformation, corresponding to an oblate nuclear form. The set of orbitals $1s1p^2 1d$, as can be directly verified from Table 8 of [8], corresponds to a symmetry D_{4h}, that is, a flat square made up of four alpha clusters. This geometry is closest to a tetrahedron in the number of bonds. It is interesting to note that the configuration (No. 1)^4 (No. 2)^4 (No. 3)^4 (No. 6)^4, which is close in energy to the configuration just considered and has negative deformation ($\epsilon < 0$), corresponds to an entirely different alpha geometry, namely a flat three-pronged star with an alpha cluster at the center (three bonds). This can be simply verified by recognizing that the orbital part of the wave function of orbit No. 6 has a preexponential factor $(x^2 + y^2 - 2)$, that is, it is a two-dimensional scalar, just as orbit No. 1. Therefore, leaving the cluster $|s^4\rangle$ in the center, we construct from the states $|x\rangle$, $|y\rangle$, and $|x^2 + y^2 - 2\rangle$ hybrid orbits which give an equilateral triangle. But since the function $|x^2 + y^2 - 2\rangle$ has a maximum shifted to the periphery in comparison with the function $|1s\rangle$, the side of this triangle will be longer than that of the C^{12} triangle (orbits $|1s\rangle$, $|x\rangle$, and $|y\rangle$), so that if we designate the bonds we obtain a three-pronged star.

It is easy to find the form of the alpha geometry for other nuclei, too. For Ne^{20} this will be a three-faced bipyramid, while for Mg^{24} it is a four-faced bipyramid, but these figures differ from the corresponding figures of the alpha model in that in the latter the four-faced bipyramid for Mg^{24} has all edges of equal length, that is, it is an octahedron, a spherical top, while in the unified model, inasmuch as $\omega_z < \omega_\perp$, the z axis is singled out and the pyramid is no longer a spherical top but a "prolate body." We thus encounter here for the first time a phenomenon wherein quadrupole deformation distorts the alpha geometry constructed by the rules of the alpha model. The situation for Ne^{20} will be essentially the same.

We can formulate three general rules for the construction of the alpha geometry.

1. For oblate nuclei, in which $N = n_\perp$ for all the Nilsson orbits, the figure will be such that all alpha clusters lie in a single plane. Two examples of this type were considered above.

2. For prolate nuclei, in which $N = n_z$ for all the Nilsson orbits, a linear chain of alpha clusters is obtained. One example of this kind is Be^8, and another is the excited state of C^{12} with the orbits (No. 1)^4(No. 3)^4 (No. 6)^4 filled. According to very simple estimates[13], the excitation energy in this case, amounts to 10—15 MeV (it is possible that this is the 7.65-MeV level).

3. For nuclei with shells closed in the LS coupling, the alpha geometry should be described by a cubic group (the system comprises a spherical top). This includes Ca^{40} in addition to the O^{16} nucleus considered above. The set of orbitals $1s1p^3 1d^5 2s$ is broken up into two parts, $1s1p^3 1d^2 + 1d^3 2s$, and we find, using the same method as in the analysis of O^{16}, that the alpha geometry of Ca^{40} has the form of an "octahedron within an octahedron" with 24 bonds (there is one cluster over each center of the four pairwise opposite faces of the octahedron, and each cluster has three bonds to the vertices of the corresponding face). We see that even for the Ca^{40} case, which at first glance is sufficiently intricate, the alpha geometry is determined in a very simple fashion.

It is interesting to note that whereas the determined four-particle excitations correspond, as we see, to a change in the geometry of the arrangement of the alpha clusters, single-particle excitations which retain the orbital Young tableau correspond to rotation-vibration excitations for relative motion of the alpha clusters, with conservation of the alpha geometry (see the discussion of the alpha model below). Inasmuch as the transition from the shell orbitals to the hybrid orbitals does not change the total antisymmetrized wave function of the system, therefore, if the alpha geometry of the system is known, the separation of the coordinates of motion of the mass centers of the clusters can be carried out by the usual methods[2].

Let us note one interesting consequence of the foregoing. In our preceding investigations[14] we calculated the alpha-widths for the decays of the lower levels of nuclei of the Be^8 type ($J = 0^+, 2^+, 4^+$; $T = 0$) $\rightarrow \alpha + \alpha$ or $C^{12*} \rightarrow Be^{8*} + \alpha$ in the shell model, that is, we considered the transitions $1s^4 1p^n \rightarrow 1s^4 1p^{n-4}$. These transitions cannot be regarded as the vanishing of one alpha cluster,

corresponding to some hybrid orbital from the nucleus, since the function $1p^4$ cannot be reduced to any hybrid orbit populated with four nucleons. The emitted alpha particle is made up of nucleons belonging to different clusters.

The emission of a complete fully populated hybrid orbit is hindered by energy factors: the higher shell excitations of the final nucleus are energetically inaccessible. In the case of high-energy quasi-alpha-particle photodisintegration (γ, pt), which apparently takes place when $E_\gamma \approx 60$—80 MeV[15], and particularly in the case of quasi-elastic scattering (p, pα), these energy limitations become less rigid, and the picture becomes closer to one in which a whole hybrid orbit, populated with four nucleons, is knocked out of the nucleus. If we make the last assumption, that is, neglect the nucleon exchange between the different hybrid orbits, then, knowing the wave function of the hybrid orbit, we can readily calculate the spectrum of the residual nucleus in the shell basis. This spectrum will differ from the spectrum calculated by the shell theory[14]. A comparison of the experimental spectra with the two just-mentioned theoretical spectra will help clarify the role of the exchange effect. The same factor (formation of an alpha particle from different localized orbits) causes the reduced alpha width for the l^4 configuration to decrease rapidly with increasing l, since the alpha particle is "assembled" of nucleons at localized orbits that are farther and farther from one another.

3. EXPERIMENTAL POSSIBILITIES OF DETERMINING THE ALPHA GEOMETRY AND THE DEGREE OF SEPARATION OF THE ALPHA CLUSTERS

The degree of separation of the alpha clusters can be determined experimentally from the matrix elements of the electric transitions. Let us take, for example, Be^9. The spatial separation of the alpha clusters will cause the quadrupole moment Q to exceed the shell value[2], so that it is possible to determine from the experimental value of Q the parameter x which determines the degree of separation of the alpha clusters. Calculation[16] yields $x = 0.6$, which corresponds to a noticeable overlap of the alpha clusters. A value of x close to 0.6 was obtained by Wildermuth and co-workers[17] by minimization of the energy of Be^9 with specified phenomenological pair interaction. The separation of the clusters is due principally to the Majorana forces, which, unlike the Wigner forces, cause the alpha clusters to be repelled not as a result of the exchange interaction integrals, but the direct integrals, that is, the repulsion is appreciably intensified. (Estimates of the direct and exchange integrals in shell theory are given in [5].)

It was noted above that the (p, 2p) reaction in Be^9 at $E_p = 200$—400 MeV also indicates an appreciable overlap of the alpha clusters. More accurately, the large value of the interval (10—12 MeV) between the two broad cross section maxima (knock-out of 1p and 1s proton, respectively), which is comparable with the binding energy of the proton in Be^8 (17 MeV), indicates that the frequencies of the internal motion of the nucleons in the alpha clusters and of the relative motion of the alpha clusters are comparable. The first maximum corresponds to the ground state with respect to the relative motion of the clusters in the final nucleus, while the second corresponds to the first excited state. The alpha model situation would correspond to a larger ratio of the frequencies of internal and relative motion. The excitation of different vibrational states would then determine the fine structure of a single broad maximum.

The intensification of Q and of the probabilities of the E2 transitions in Be^9 can be explained also from the point of view of the unified model. However, in the heavier nuclei, a difference is observed between the unified model and the CM. Thus, for example, in C^{12} the levels 0^+, 2^+, 3^-, ... form according to the alpha model a rotational band. Consequently, on going over from the shell model ($x = 1$) to the separated clusters ($x < 1$), there will be an intensification not only of the E2 transitions but simultaneously also of the E3 transitions, something which does not take place in the unified model. Here, as in the case of Be^9, we can determine the degree of overlap of the alpha clusters from the intensification of the gamma transitions. The CM will be directly confirmed if the one parameter x can account for the observed intensification of both the E2 and E3 transitions. An analysis of the intensification of the E2 and E3 transitions in C^{12} will show whether the Q–Q forces play some independent role in C^{12}, as for example in Mg^{24} (see above).

Another effective experimental method of investigating the alpha clustering of nucleons is apparently the scattering of high-energy electrons. It is known[18] that the total cross section of elastic and inelastic scattering, as a function of the angle, yields information concerning the form factor of the pair density of the protons (Debye-

gram). Inasmuch as alpha clustering is characterized by relatively large radii, it is sufficient to confine oneself to a range of several times 10 MeV, in the excitation energies of the final nuclei, and if the experimental accuracy is high, it is possible to obtain information both with respect to the degree of separation of the alpha clusters (that is, the extent to which the wave function of the CM differs from that of the shell model) and on their relative geometry. The excitation cross sections of the individual levels can yield information on the electric transitions of high multipolarity. The alpha clustering causes an intensification of an entire group of $E\lambda$ transitions with $\lambda \leq \lambda_0$. The values of λ_0 and of the intensity of the individual transitions are determined by the number of alpha clusters and by the alpha geometry[16]. Unfortunately, there are still very few experimental data.

Along with the scattering of high-energy electrons, very valuable information concerning the relative separation of the clusters in the nucleus can also be obtained from other reactions with fast particles. Particularly useful in this respect may be reactions of quasi-elastic scattering of fast particles with knocking out of nucleon clusters, that is, reactions of the type (p, pd), (p, pα), (α, 2α) etc. An investigation of the angular correlations of the products of these reactions makes it possible to find the momentum distribution of the clusters inside the nucleus. Comparing these results with the momentum distribution calculated by the shell model[19], we can explain the difference between the wave function of the nucleus and that of the shell model, and determine the degree of separation of the clusters. Experiments of this type are being carried out at present[20], but so far they cover only too small a group of nuclei to be able to present some sort of systematic analysis of the problem. For a clarification of the degree of deviation of the wave functions of the nuclei from the shell model and for the construction of a more realistic model of the lightest nuclei it would be very useful to investigate the momentum distribution of the protons in all the p-shell nuclei (this investigation is almost complete), and also of the deuterons, tritons, and alpha particles.

4. LIGHT NUCLEI AND THE ALPHA MODEL

In conclusion, in connection with the O^{16} experimental data discussed above, let us make a few remarks concerning the description of light nuclei by the alpha model. The alpha-model interpretation of O^{16}[21] was based on the lowest excited levels 0^+, 1^-, 2^+, 3^-, and 4^+. Recently a 0^- level was discovered at ~ 10 MeV, this apparently agreeing very well with the Dennison-Kameny variant "b"[21]. However, the results presented above obtained by Gorodetzky and co-workers[9], and also the fact that the lifetimes of the lower excited levels 0^+ and 2^+[22] are 15—20 times larger than those obtained with the alpha model[21], make it necessary to reject the Dennison-Kameny scheme. In addition, an analysis of the negative-parity levels of O^{16} by the shell theory (Elliott and Flowers[23]) has shown that the lower levels 3^-, 2^-, and 1^-, are well described as single-particle excitations $p^{-1}d$ or $p^{-1}(2s + 1d)$ with a Young tableau [4444], so that the level 0^- which is close to them is also a single-particle excitation $p^-(1d + 2s)$, except that the spin is S = 1 (such levels lie somewhat lower than the levels with T = 1). Therefore the 0^- level is apparently simply not an "alpha-model level," and it is incorrect to regard it as a type-E double excitation.

It is apparently possible to retain for the levels 1^-, 3^-, and 2^- the alpha-model interpretation. Furthermore, as is evidenced by the quadrupole moment O^{17}, the type-E 2^+ level, which "soaks in"[21] almost the entire quadrupole sum for the transitions between the levels with T = 0[24], should lie at 15—20 MeV[25]. This region apparently contains also the "breathing" level 0^+[25]. According to the shell theory, both levels are single-particle excitations. Thus, if the level 2^- (8.88 MeV) is interpreted as a vibrational level of the type E, there is a tremendous tunnel splitting.

There is another circumstance which complicates the alpha-model interpretation of the spectra of light nuclei. The distance between clusters is usually determined from data on the rotational spectra, which yield the moment of inertia, and consequently, in accordance with the classical formulas, also the distances between the clusters. In fact, however, the formulas for the rotational energy of, say, Be^8 and C^{12}, contain in the case of the shell theory only exchange integrals, so that their classical interpretation is impossible[3]. In addition, if the wave function of the nucleus is close to that of the shell model, then owing to the large distance between the levels of the different shells it becomes impossible to combine the levels of different parity in a single rotational series, as is the case in the alpha model, according to which in C^{12} a single rotational series is made up of the levels 0^+, 2^+, 3^-, 4^+... and in O^{16} the levels 0^+, 3^-, 4^+.... The magnitude of the tunnel splitting

and the close agreement between the arrangement of these levels (which are rotational in accordance with the alpha model) and the rotational interval rule are apparently sufficiently sensitive "indicators" of the cluster overlap.

The example of O^{16} demonstrates that, owing to the overlap of the alpha clusters, the alpha model is on the whole not fruitful for the analysis of the level schemes of light nuclei. Nonetheless, a comparison of the experimental results with its predictions, where possible, is very useful since it helps clarify under what circumstances the deviations from the asymptotic alpha-model values are insignificant. For problems where the alpha model representations are appropriate, they have the advantage of the greater simplicity. By way of an example we can cite the description of the lower levels of O^{17}[26], where use is made principally of the symmetry properties of the wave function of the O^{16} core written in the CM, and the degree of overlap of the clusters actually does not play any role.

[1] J. A. Wheeler, Phys. Rev. **52**, 1083, 1107 (1937).

[2] J. K. Perring and T. Skyrme, Proc. Phys. Soc. **A69**, 600 (1956). B. F. Bayman and A. Bohr, Nucl. Phys. **5**, 536 (1958). Neudachin, Smirnov, and Yudin, JETP **37**, 1781 (1959), Soviet Phys. JETP **10**, 1256 (1960).

[3] K. Wildermuth and Th. Kanellopoulos, Nucl. Phys. **9**, 449 (1958).

[4] Pearlstein, Tang, and Wildermuth, Nucl. Phys. **18**, 23 (1960); Phys. Rev. **120**, 224 (1960) and **123**, 548 (1961). Kopaleĭshvili, Vashakidze, Mamasakhlisov, and Chilashvili, JETP **38**, 1758 (1960). Soviet Phys. JETP **11**, 1268 (1960); Nucl. Phys. **23**, 430 (1961).

[5] A. M. Lane and J. P. Elliot, Handbuch der Physik **39**, (1957).

[6] Tyren, Hillman, and Maris, Nucl. Phys. **7**, 10 (1958).

[7] B. Roth and K. Wildermuth, Nucl. Phys. **20**, 10 (1960). R. K. Sheline and K. Wilermuth, Nucl. Phys. **21**, 196 (1960).

[8] Eyring, Walter, and Kiball, Quantum Chemistry, Wiley, N.Y., 1944.

[9] Gorodetzky, Mennrath, Chevalier, and Scheibling, Phys. Lett. **1**, 14 (1962).

[10] H. Morinaga, Phys. Rev. **101**, 254 (1956).

[11] N. Vinh-Man and G. E. Brown, Phys. Lett. **1**, 36 (1962).

[12] V. G. Neudachin and V. N. Orlin, JETP **41**, 874 (1961), Soviet Phys. JETP **14**, 625 (1962); Nucl. Phys. **31**, 338 (1962).

[13] S. G. Nilsson, Dan. Mat.-Fys. Medd. **29**, No. 16 (1955).

[14] Balashov, Neudachin, Smirnov, and Yudin, JETP **37**, 1385 (1959), Soviet Phys. JETP **10**, 983 (1960); Yu. F. Smirnov and D. Chlebowska, Nucl. Phys. **26**, 306 (1961).

[15] V. N. Maikov, Yadernye reaktsiĭ pri malykh i srednikh energiyakh (Nuclear Reactions at Medium and Low Energies), AN SSSR, 1958. V. V. Balashov and V. N. Fetisov, Nucl. Phys. **27**, 337 (1961).

[16] Matthies, Neudachin, and Smirnov, Izv. AN SSSR, in press.

[17] Tang, Khanna, Herndon, and Wildermuth, Nucl. Phys. **35**, 353 (1962).

[18] J. H. Smith, Phys. Rev. **95**, 271 (1954). L. Shiff, Nuovo cimento **5**, 1223 (1957). K. W. McVoy, Proc. Rutherford Jubilee Conf. London (1961).

[19] V. V. Balashov and A. N. Boyarkina, Izv. AN SSSR, ser. fiz., in press.

[20] Ruhla, Riou, Garron, Jacmart, and Massonet, Phys. Lett. **2**, 44 (1962).

[21] D. M. Dennison, Phys. Rev. **96**, 378 (1954). S. D. Kameny, Phys. Rev. **103**, 358 (1956).

[22] F. Ajzenberg and T. Lauritsen, Nucl. Phys. **11**, 1 (1959). Bromley, Gove, Kuehner, Litherland, and Almquist, Phys. Rev. **114**, 758 and 785 (1959).

[23] J. P. Elliot and B. H. Flowers, Proc. Roy. Soc. **A242**, 57 (1957).

[24] D. H. Wilkinson, Proc. Intern. Conf. on Nucl. Structure, Kingston (1960).

[25] S. Fallieros, Dissertation, University of Maryland (1959).

[26] Matthies, Neudachina, and Smirnov, JETP **42**, 592 (1962), Soviet Phys. JETP **15**, 411 (1962); Izv. AN SSSR **26**, 1060 (1962), Columbia Tech. Transl. p. 1067; Nucl. Phys. **38**, 63 (1962).

Translated by J. G. Adashko
22