LEPTON DECAYS OF HYPERNUCLEI

V. A. LYUL'KA

Submitted to JETP editor December 13, 1962

J. Exptl. Theoret. Phys. (U.S.S.R.) **45**, 164-172 (August, 1963)

Lepton decays of hypernuclei are examined on the basis of the Lee-Yang four-fermion interaction. The proton and electron energy spectra and the electron angular correlations are obtained for the $He^4_\Lambda \to He^3 + p + e^- + \bar{\nu}$ decays. The muon decays $He^4_\Lambda \to He^4 + \mu^- + \bar{\nu}$ and $He^4_\Lambda \to He^3 + p + \mu^- + \bar{\nu}$ are also considered. A method of determining the decay interaction constants is discussed for three-particle muon decays. The probabilities of lepton decays of hypernuclei are estimated under the assumption of a universal V-A interaction.

1. INTRODUCTION

As is well known, a study of the lepton decays of strange particles has shown that the Feynman–Gell-Mann universal V-A interaction scheme is in good agreement wit the experimental data on β decay and on pion and muon decay, but does not describe lepton decays of hyperons. In this connection, the question arises of determining the variants of the decay interaction responsible for processes of this type. This question has been the subject of several papers[1-3], where it was shown that very extensive experimental information is necessary to establish the character of the interaction that leads to lepton decays of hyperons. It is also of interest to consider lepton decays of hypernuclei, which can give additional information on weak hyperon interactions; in this respect, three-particle decays of hypernuclei, the analysis of which is the simplest, are particularly useful. In addition, a study of the lepton decays of hypernuclei is of independent interest from the point of view, for example, of obtaining additional information on the structure of hypernuclei.

Filimonov[4] investigated the simplest lepton decays of the hypernuclei H^3_Λ and H^4_Λ:

$$H^3_\Lambda \to He^3 + e^- + \bar{\nu}, \tag{a}$$

$$H^4_\Lambda \to He^4 + e^- + \bar{\nu}. \tag{b}$$

However, the larger energy released in lepton decays of hypernuclei will lead to the possibility of disintegration of the nucleus, that is, to decays into four particles (and more), for example

$$H^3_\Lambda \to d + p + e^- + \bar{\nu}, \tag{c}$$

$$H^4_\Lambda \to H^3 + p + e^- + \bar{\nu}, \tag{d}$$

$$He^4_\Lambda \to He^3 + p + e^- + \bar{\nu}, \tag{e}$$

$$He^5_\Lambda \to He^4 + p + e^- + \bar{\nu}. \tag{f}$$

We note that such decays will in general be the simplest lepton decays for the hypernuclei He^4_Λ and He^5_Λ.

In addition, decays with muon emission are also possible:

$$H^4_\Lambda \to He^4 + \mu^- + \bar{\nu}, \tag{g}$$

$$He^4_\Lambda \to He^3 + p + \mu^- + \bar{\nu} \tag{h}$$

and others. The present paper is devoted to further study of lepton decays of hypernuclei[1)]. In particular, in Secs. 2 and 3 we consider the decay (c) and the decay (e) in detail, while Sec. 4 deals with decays (g) and (h).

2. MATRIX ELEMENT OF FOUR-PARTICLE DECAYS OF HYPERNUCLEI

We start the calculations with the Lee and Yang form of the four-particle interaction of the particles p, Λ, e^-, and $\bar{\nu}$:

$$H_{\text{int}} = \int H(x)\,dx,$$

$$\begin{aligned} H(x) = {} & (\bar{\psi}_p \psi_\Lambda)[\bar{\psi}_l (C_S + C'_S \gamma_5)\psi_\nu] \\ & + (\bar{\psi}_p \gamma_\mu \psi_\Lambda)[\bar{\psi}_l \gamma_\mu (C_V + C'_V \gamma_5)\psi_\nu] \\ & + \tfrac{1}{2}(\bar{\psi}_p \sigma_{\mu\nu} \psi_\Lambda)[\bar{\psi}_l \sigma_{\mu\nu}(C_T + C'_T \gamma_5)\psi_\nu] + (\bar{\psi}_p \gamma_\mu \gamma_5 \psi_\Lambda) \\ & \times [\bar{\psi}_l \gamma_\mu \gamma_5 (-C_A - C'_A \gamma_5)\psi_\nu] \\ & + (\bar{\psi}_p \gamma_5 \psi_\Lambda)[\bar{\psi}_l \gamma_S (C_P + C'_P \gamma_5)\psi_\nu], \\ & \sigma_{\mu\nu} = (\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu)/2i, \qquad \gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4. \end{aligned} \tag{1}$$

[1)]Problems connected with the two different neutrinos in decays (c), (f), (g), and (h) are not discussed in the present article.

In the nonrelativistic approximation Eq. (1) assumes the form

$$H(x) = (\psi_p^*\psi_\Lambda) A_0 + (\psi_p^*\sigma\psi_\Lambda)\mathbf{A},$$
$$A_0 = \bar{\psi}_l (C_S + C_S'\gamma_5 + C_V\gamma_4 + C_V'\gamma_4\gamma_5)\psi_\nu,$$
$$A_i = \bar{\psi}_l [\tfrac{1}{2}\varepsilon_{i\mu\nu}\sigma_{\mu\nu}(C_T + C_T'\gamma_5) - i\gamma_i\gamma_5(C_A + C_A'\gamma_5)]\psi_\nu. \quad (2)$$

Here σ are the usual Pauli matrices.

In the present and in the following sections we consider four-particle decays of type (c) and (e).

A study of pion decays of hypernuclei, occurring in accordance with the scheme $X_\Lambda^A \to X^{A-1} + p + \pi^-$ has shown that the energy spectrum and the angular distributions of the final particles are essentially determined, first, by the interaction between the particles in the final state (particularly the interaction in the system p, X^{A-1}) and, second, by the Pauli principle[6-8]. An account of both of these effects greatly complicates the analysis of the lepton decays into four particles. It can be assumed, however, that since much more energy is released in lepton decays than in pion decays, the effect of the final-state interaction will in this case be weakened (see also [9]). We therefore take into account only the Pauli principle in these calculations.

As was noted by Filimonov[4], the interaction (2) coincides in form with the nonrelativistic interaction for pion decays of hypernuclei, so that to obtain the matrix element of interest to us we can proceed as for the corresponding pion decays[7,8]. Consequently, using (2), we obtain the probabilities of transitions (c) and (e) by summing over the corresponding spin variables and averaging over the spin states of the decaying hypernucleus.

For the decay (c)

$$dw = \frac{2\pi}{(2\pi)^9}\Big[\Big(g_0 + \frac{5}{9}g_1\Big)|\Phi_1(\mathbf{P}_\Lambda, \mathbf{p}) - \Phi_2(\mathbf{P}_\Lambda, \mathbf{p})|^2 + \frac{4}{9}g_1|\Phi_1(\mathbf{P}_\Lambda, \mathbf{p}) + \frac{1}{2}\Phi_2(\mathbf{P}_\Lambda, \mathbf{p})|^2\Big]\delta^4(P_\Lambda + p + p_e + p_\nu - M_h)\, d\mathbf{P}_\Lambda d\mathbf{p}\, d\mathbf{p}_e d\mathbf{p}_\nu \text{ for } j = \frac{3}{2}, \quad (3)$$

$$dw = \frac{2\pi}{(2\pi)^9}\Big[\Big(g_0 + \frac{1}{9}g_1\Big)\Big|\Phi_1(\mathbf{P}_\Lambda, \mathbf{p}) + \frac{1}{2}\Phi_2(\mathbf{P}_\Lambda, \mathbf{p})\Big|^2 + \frac{8}{9}g_1|\Phi_1(\mathbf{P}_\Lambda, \mathbf{p}) - \Phi_2(\mathbf{P}_\Lambda, \mathbf{p})|^2\Big]\delta^4(\mathbf{P}_\Lambda + p + p_e + p_\nu - M_h)\, d\mathbf{P}_\Lambda\, d\mathbf{p}\, d\mathbf{p}_e d\mathbf{p}_\nu \text{ for } j = \frac{1}{2}. \quad (4)$$

Here j —spin of the hypernucleus H_Λ^3; P_Λ, p, p_e, p_ν —four vector momenta of the recoil nucleus, proton, electron, and neutrino, respectively; M_h is the mass of the hypernucleus

$$\Phi_1(\mathbf{P}_\Lambda, \mathbf{p}) = \int \psi_{H_\Lambda^3}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)\, e^{i\mathbf{q}_e\mathbf{r}_1} \exp\Big[\frac{i\mathbf{P}_\Lambda(\mathbf{r}_2 + \mathbf{r}_3)}{2}\Big] \times e^{i\mathbf{p}\mathbf{r}_1}\psi_d(\mathbf{r}_2, \mathbf{r}_3)\, d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{r}_3,$$

$$\Phi_2(\mathbf{P}_\Lambda, \mathbf{p}) = \int \psi_{H_\Lambda^3}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)\, e^{i\mathbf{q}_e\mathbf{r}_1} P_{12} \exp\Big[\frac{i\mathbf{P}_\Lambda(\mathbf{r}_2 + \mathbf{r}_3)}{2}\Big] \times e^{i\mathbf{p}\mathbf{r}_1}\psi_d(\mathbf{r}_2, \mathbf{r}_3)\, d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{r}_3. \quad (5)$$

In (7) $\mathbf{r}_1$, $\mathbf{r}_2$, and $\mathbf{r}_3$ are the radius vectors of the Λ particle, proton, and neutron, respectively; $q_e = p_e + p_\nu$; P_{12} is the operator for the permutation of the particle coordinates, and is the result of the Pauli principle.

For the decay (e) we have

$$dw = \frac{2\pi}{(2\pi)^9}(g_0 + g_1)F^2(\mathbf{P}_\Lambda, \mathbf{p})\,\delta^4(P_\Lambda + p + p_e + p_\nu - M_h) \times d\mathbf{P}_\Lambda\, d\mathbf{p}\, d\mathbf{p}_e d\mathbf{p}_\nu. \quad (6)$$

Here

$$F(\mathbf{P}_\Lambda, \mathbf{p}) = \int \psi_{He_\Lambda^4}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4)\, e^{i\mathbf{q}_e\mathbf{r}_1} \times (1 - P_{12}) \exp\Big[\frac{i\mathbf{P}_\Lambda(\mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_4)}{3}\Big] e^{i\mathbf{p}\mathbf{r}_1}\psi_{He^3} \times (\mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4)\, d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{r}_3 d\mathbf{r}_4. \quad (7)$$

The quantities g_0 and g_1 in (3), (4), and (5) are expressed in terms of the constants contained in the Hamiltonian (2) as follows:

$$g_0 = 2(a_{SS} + a_{VV}) + \frac{2(\mathbf{p}_l\mathbf{p}_\nu)}{E_l E_\nu}(a_{VV} - a_{SS}) + \frac{2m_l}{E_l}(a_{SV} + a_{VS}),$$
$$g_1 = 6(a_{TT} + a_{AA}) + \frac{2(\mathbf{p}_l\mathbf{p}_\nu)}{E_l E_\nu}(a_{TT} - a_{AA}) + \frac{6m_l}{E_l}(a_{TA} + a_{AT}),$$
$$a_{ij} = \frac{1}{2}(C_i C_j^* + C_i' C_j'^*); \quad (8)$$

m_l —lepton mass (in this case, electron), $E_l = \sqrt{p_l^2 + m_l^2}$ —total lepton energy 2).

From (3), (4), and (6) we see that if the Pauli principle is taken into account the probability of a type (c) lepton decay of H_Λ^3 depends on the j-spin of the hypernucleus, whereas for the He_Λ^3 decay there is no such dependence. To obtain the spectra of the particles produced in the decays of H_Λ^3 and He_Λ^4, it is necessary to know the wave functions contained in the integrals of (5) and (7). The wave function of H_Λ^3, which takes into account the particle correlations, is very complicated and this makes the computations difficult. However, by choosing for He_Λ^4 a wave function in the form

$$\psi_{He_\Lambda^4} = K\exp[-\tfrac{1}{2}\alpha_3(r_{23}^2 + r_{24}^2 + r_{34})]\,U_\Lambda(\rho), \quad (9)$$

where $U_\Lambda(\rho)$ describes the motion of the Λ particle relative to the core nucleus and $\rho = |\mathbf{r}_1 - \frac{1}{3}(\mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_4)|$, Dalitz and Liu[11] obtained

2)We denote by E the total energy and by W the kinetic energy of the corresponding particle.

for $F(P_\Lambda, p)$ an explicit expression which assumes for $U_\Lambda(\rho) = D[\exp(-a\rho^2) + y\exp(-b\rho^2)]$ the form

$$F(\mathbf{P}_\Lambda, \mathbf{p}) = D\Big\{\left(\frac{\pi}{a}\right)^{3/2}\exp\left(-\frac{P_\Lambda^2}{4a}\right) + y\left(\frac{\pi}{b}\right)^{3/2}\exp\left(-\frac{P_\Lambda^2}{4b}\right) - \left(\frac{9}{5}\right)^{3/2}\exp\left[-\frac{1}{10\alpha_3}\left(\frac{\mathbf{P}_\Lambda}{3}-\mathbf{p}\right)^2\right]\Big[\left(\frac{\pi}{9/10\alpha_3 + a}\right)^{3/2} \times \exp\left[-\frac{9(\mathbf{P}_\Lambda + 2\mathbf{p})^2}{10(9\alpha_3+10a)}\right] + y\left(\frac{\pi}{9/10\alpha_3 + b}\right)^{3/2} \times \exp\left[\frac{9(\mathbf{P}_\Lambda + 2\mathbf{p})^2}{10(9\alpha_3+10b)}\right]\Big]\Big\}. \quad (10)$$

The values of the parameters contained in (9) and (10) are given in the papers of Dalitz et al [11,12].

In the next section, using (10), we calculate the energy and angular correlations for the decay (e).

3. PROTON AND ELECTRON ENERGY SPECTRA AND ELECTRON CORRELATION

To obtain the proton spectrum we integrate (6) over the momenta p_e and p_ν. To this end it is convenient to use the invariant integration method employed by Okun' and Shabalin [13] for the analysis of K-meson decays. Carrying out the indicated integration, we obtain [3)]

$$dW = (2\pi)^{-5}F^2(\mathbf{P}_\Lambda, \mathbf{p})\{\tfrac{1}{6}\, l_0[3(\Delta - W_{\rm nuc} - W_p)^2 - (\mathbf{P}_\Lambda + \mathbf{p})^2] + l_1[(\Delta - W_{\rm nuc} - W_p)^2 - (\mathbf{P}_\Lambda + \mathbf{p})^2\}\, P_\Lambda^2 p^2\, dP_\Lambda\, dp\, d\cos\chi. \quad (11)$$

Here χ is the angle between the vectors $\mathbf{P}_\Lambda$ and $\mathbf{p}$, $\Delta = M_h - M_{nuc} - M_p$, while M_{nuc} and M_p are the masses of the recoil nucleus and the proton,

$$l_0 = 4a_{VV} + 8a_{TT} + 4a_{AA}, \qquad l_1 = 2(a_{SS} + a_{AA} - a_{VV} - a_{TT}). \quad (12)$$

To obtain the energy distribution of the protons it is necessary to integrate in (11) with respect to $d\cos\chi$ and dP_Λ over the region admissible by the conservation law. This integration has been carried out in analogy with the procedure used by Mathur [14] for the K_{e4} decays. The result of such an integration, carried out numerically with an electronic computer, is shown in Fig. 1. Curves 1 and 2 pertain to the first and second terms of (11), respectively. Both curves have maxima at p ~ 140 MeV/c and decrease rapidly with increasing p. Thus, the lepton decays $He^4_\Lambda \to He^3 + p + e^- + \bar\nu$ occur with emission of protons that have momenta in the narrow interval near ~ 140 MeV/c.

[3)]In the analysis of electronic decays, terms proportional to m_l have been neglected in Eq. (8).

After integrating in (6) with respect to dP_Λ (thereby eliminating the momentum δ-function), and then using the energy δ-function to integrate with respect to $d\cos\varphi$ (φ is the angle between the vectors $\mathbf{p}$ and $\mathbf{q}_e = \mathbf{p}_e + \mathbf{p}_\nu$), we obtain the energy spectrum and angular correlation of the electrons (the limits of integration with respect to dp are p_{min} and p_{max}, which are determined from the conservation laws for specified E_e, E_ν, and $\cos\theta$, where θ is the angle between $\mathbf{p}_e$ and $\mathbf{p}_\nu$):

$$dw(E_e) = 2M_{\rm nuc}(2\pi)^{-5}(l_0 I_0 + l_1 I_1)\, E_e^2 dE_e \quad (13)$$

for the electron spectrum and

$$dw(\cos\psi) = 2M_{\rm nuc}(2\pi)^{-5}(l_0 J_0 + l_1 J_1)\, d\cos\psi \quad (14)$$

for the angular distribution. We have used here the notation

$$I_i = \int_{(\cos\psi)_{min}}^{1}\int_{q_{e,\,min}}^{q_{e,\,max}} \Phi(q_e, E_e, \cos\psi)\, f_i(\cos\theta)\, q_e\, dq_e \cos\psi,$$

$$J_i = \int_{0}^{E_{e,\,max}}\int_{q_{e,\,min}}^{q_{e,\,max}} \Phi(q_e, E_e, \cos\psi)\, f_i(\cos\theta)\, q_e\, dq_e E_e^2 dE_e, \quad (15)$$

$$f_0(\cos\theta) = 1, \quad f_1(\cos\theta) = 1 - \cos\theta,$$

ψ —angle between the vectors $\mathbf{p}_e$ and $\mathbf{q}_e$, and

$$\Phi(q_e, p_e, \cos\psi) = \int_{p_{min}}^{p_{max}} F^2(\mathbf{p}, \mathbf{q}_e)\, p\, dp. \quad (16)$$

In the derivation of (13) and (14) we have changed from the integration variables p_e and p_ν to the variables p_e and q_e. The quantities $q_{e,min}$ and $q_{e,max}$, contained in (15), are the solutions of the equation

$$(M_{\rm nuc} + M_p)^2 = M_h^2 + 2E_e^2 - 2M_h E_e - 2E_e q_e\cos\psi - 2(M_h - E_e)(q_e^2 + E_e^2 - 2q_e E_e\cos\psi)^{1/2} \quad (17)$$

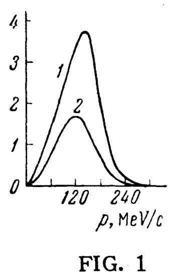

FIG. 1

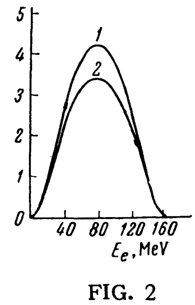

FIG. 2

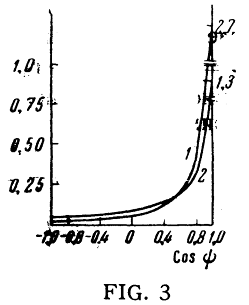

FIG. 3

FIG. 1. Proton energy spectrum (in arbitrary units). The ordinate of curve 1 has been increased by a factor 2.

FIG. 2. The functions $I_0E_e^2$ (curve 1) and $I_1E_e^2$ (curve 2) of formula (20) for the energy spectrum of the electrons in decay (e) (in arbitrary units).

FIG. 3. The functions J_0 (curve 1) and J_1 (curve 2), of formula (21) for the angular correlation of electrons in decay (e) (arbitrary units).

for given fixed E_e and $\cos\psi$ (the lower limit of integration with respect to dq_e in (15) is zero if the corresponding root is negative). The quantities $(\cos\psi)_{min}$ and $E_{e,max}$ are determined from the condition of the existence of real positive solutions of (17).

The results of the calculations for the energy spectrum and for the angular distribution of the electrons are shown in Figs. 2 and 3, respectively. Since we have neglected the electron mass, the same curves pertain to the energy spectrum and to the neutrino correlation. The electron spectra have a maximum at ~ 80 MeV and are symmetrical, which is also the consequence of the smallness of the electron mass compared with its energy. The angular-distribution curves increase rapidly in the region of small angles, thus evidencing that the lepton decays of the type under consideration occur with emission of the electron and neutrino in approximately the same direction.

4. MUON DECAYS OF HYPERNUCLEI

In the present section we investigate the decays (g) in detail and calculate the meson energy spectrum for the decay (h). A distinguishing feature of muon decays is the need for taking into account in (8) terms that are proportional to the muon mass. Taking this circumstance into account, we obtain for the probability of the decay $H^4_\Lambda \to He^4 + \mu^- + \bar{\nu}$, retaining the notation of Filimonov[4],

$$dw = \tfrac{1}{6}\pi^{-3}F^2(p_N)\{E_\mu(\Delta_m - E_\mu)L_2 - L_1[\tfrac{1}{2}(\Delta_m^2 - p_N^2 - m_\mu^2) - E_\mu(\Delta_m - E_\mu)] + m_\mu(\Delta_m - E_\mu)L_3\}\, p_N dp_N p_\mu E_\mu^{-1} dp_\mu. \quad (18)$$

Here p_N is the momentum of the recoil nucleus, $\Delta_m = M_h - M_{nuc}$, and m_μ is the muon mass. In the derivation of (18) we have neglected, as in [4], the kinetic energy W_{nuc} of the recoil nucleus ($W_{nuc}^{max} \sim 3$ MeV) compared with Δ_m. Further

$$L_1 = 6S_1(a_{TT} - a_{AA}) + 6S_0(a_{VV} - a_{SS}),$$
$$L_2 = 18S_1(a_{TT} + a_{AA}) + 6S_0(a_{SS} + a_{VV}),$$
$$L_3 = 18S_1(a_{AT} + a_{TA}) + 6S_0(a_{SV} + a_{VS}). \quad (19)$$

The coefficients S_0 and S_1 for different hypernuclei are given in the paper of Filimonov[4]. The function $F(p_N)$ has the form

$$F(p_N) = \left[\frac{48\alpha_3\alpha_4}{(3\alpha_3 + 4\alpha_4)^2}\right]^{3/4}(3\pi\alpha_4)^{3/4} D\left\{\left(\frac{3}{2}\alpha_4 + a\right)^{-3/2} \times \exp\left[-\frac{9p_N^2}{96\alpha_4 + 64a}\right] + y\left(\frac{3}{2}\alpha_4 + b\right)^{-3/2}\exp\left[-\frac{9p_N^2}{96\alpha_4 + 64b}\right]\right\}. \quad (20)$$

Using (18)–(20) we can calculate by standard methods the energy spectra of the muons and of the recoil nuclei. The results of the calculations are shown in Figs. 4 and 5 for the muon spectrum and recoil-nucleus spectrum, respectively. Both the muon spectrum and the recoil-nucleus spectrum depend, generally speaking, on the values of the interaction constants, but in the case of the former no such dependence is observed in the absence of a contribution from the Fierz terms (curves 2 and 3), as in the case of electron decays[4].

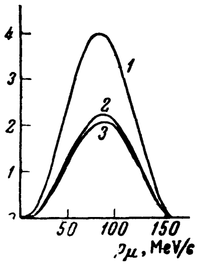

FIG. 4. Muon spectrum in decay (h) (arbitrary units). Curve 1 pertains to the case $a_{SS} = a_{VV} = a_{SV} = a_{VS} = G^2/2$.
Curve 2 – $a_{VV} = a_{SV} = a_{VS} = 0$; $a_{SS} = G^2/2$.
Curve 3 – $a_{SS} = a_{VS} = a_{SV} = 0$; $a_{VV} = G^2/2$.
The ordinates of curves 2 and 3 have been increased by a factor 2.

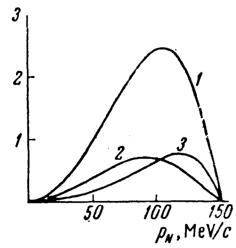

FIG. 5. Spectrum of recoil nuclei for decay (g). The designations of the curves are the same as in Fig. 4.

If we investigate the distribution of the decays in the region of the two independent variables p_μ and p_N admissible by the conservation laws, then we can determine some combinations of constants contained in the Hamiltonian (3). We consider here a possible method of determining these constants. It is analogous to the method used by Belov, Mingalev, and Shekhter[1] for the decay of free hyperons, and also by Filimonov[4] for the study of electron decays of hypernuclei. If we break up the region of variation of p_μ and p_N into parts, then the integral probability that the decay occurs in the i-th region can be written in the form

$$w_i = \frac{\Delta_m^5}{36\pi^3} p_i, \quad (21)$$

where p_i is some linear combination of L_1, L_2, and L_3, with coefficients that depend on the type of the breakdown:

$$p_i = a_1^i L_1 + a_2^i L_2 + a_3^i L_3. \quad (22)$$

Thus, if we subdivide the region of variation of p_N and p_μ (bounded by the continuous lines in Fig. 6) by the curves (shown dashed in Fig. 6) that bound the region

$$|\Delta_m - \tfrac{1}{2}p_\mu - E_\mu| \leqslant p_N \leqslant \Delta_m + \tfrac{1}{2}p_\mu - E_\mu, \quad (23)$$

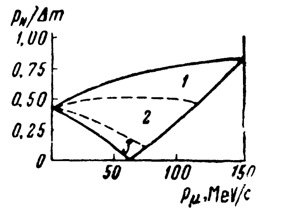

FIG. 6

then the values of the coefficients a_j^i are obtained by integrating (18) inside of each of the three regions. We present the values of the coefficient $a_j^i = 10^{-2} b_j^i$ for the indicated breakdown:

$$b_1^1 = 0.085; \quad b_2^1 = 0.514; \quad b_3^1 = 0.390; \quad b_1^2 = -0.102;$$
$$b_2^2 = 0.381; \quad b_3^2 = 0.317; \quad b_1^3 = -0.016; \quad b_2^3 = 0.046;$$
$$b_3^3 = 0.042. \tag{24}$$

If we find the probability w_i experimentally, then the determination of L_1, L_2, and L_3 reduces to a solution of a system of three linear equations with constant coefficients. Since it follows from the analysis of the pion decays of the hypernuclei that the spins of H_Λ^3 and H_Λ^4 are apparently equal to ½ and zero, respectively, we can, in the analysis of the muon decays of hypernuclei, determine on the basis of (19) the following combination of constants:

For H_Λ^3:

$$a_{TT} - a_{AA} - 9a_{SS} + 9a_{VV},$$
$$a_{TT} + a_{AA} + 3a_{SS} + 3a_{VV},$$
$$a_{AT} + a_{TA} + 3a_{SV} + 3a_{VS}, \tag{25}$$

for H_Λ^4

$$a_{VV} - a_{SS}, \quad a_{VV} + a_{SS},$$
$$a_{SV} + a_{VS}.$$

For such hypernuclei as He_Λ^4 and He_Λ^5, as was already indicated, the simplest lepton decays will be into four particles. We calculate here the muon spectrum in the decay (h) $He_\Lambda^4 \rightarrow He^3 + p + \mu^- + \bar{\nu}$:

For the probability of this decay we have

$$dw = \frac{2\pi}{(2\pi)^9} [l_0 E_\mu E_\nu + l_1 (p_\mu p_\nu)$$
$$+ l_2 m_\mu E_\nu] F^2 (P_\Lambda p) \, dP_\Lambda \, dp \, d\mathbf{p}_\mu \, d\mathbf{p}_\nu / E_\mu E_\nu. \tag{26}$$

The values of l_0 and l_1 are given by (11), while for l_2 we have

$$l_2 = 2a_{SV} + 2a_{VS} + 6a_{TA} + 6a_{AT}. \tag{27}$$

An account of terms proportional to the muon mass m_μ in (26) entails great computational difficulties, so that in order to gain an idea of the character of the muon spectrum we confine ourselves to an analysis of only the first two terms in (26). In this case the calculations are analogous to the case of electronic decays, the only difference being that Eq. (17) is replaced by

$$(M_{\text{nuc}} + M_p)^2 = M_h^2 + 2p_\mu^2 + m_\mu^2 - 2M_h E_\mu - 2q_\mu p_\mu \cos\psi$$
$$- 2(M_h - E_\mu)(q_\mu^2 + p_\mu^2 - 2q_\mu p_\mu \cos\psi)^{1/2}. \tag{28}$$

Figure 7 shows the muon spectra. Curves 1 and 2 are connected with the first and second terms in (26), respectively. The spectra have maxima at muon momentum ~ 80 MeV/c.

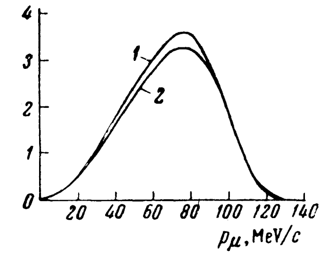

FIG. 7. Energy spectrum of muons in decay (h) (arbitrary units).

5. ESTIMATES OF THE PROBABILITIES OF LEPTON DECAYS OF HYPERNUCLEI

Since the values of the constants in the Hamiltonian of the decay interaction (1) are unknown at present, to estimate the probabilities of the lepton decays of the hypernuclei we propose that a V-A interaction obtains with a universal coupling constant $G = 1.01 \times 10^{-5} M^{-2}$. The calculations then yield:

for decay (e)

$$w_e = 1.64 \cdot 10^7 \text{ sec}^{-1} = 0.28/\tau_{e\Lambda};$$

for decay (g)

$$w_\mu = 0.40 \cdot 10^7 \text{ sec}^{-1} = 0.43/\tau_{\mu\Lambda};$$

for decay (h)

$$w_\mu = 0.12 \cdot 10^7 \text{ sec}^{-1} = 0.13/\tau_{\mu\Lambda}.$$

Here $\tau_{e\Lambda}$ and $\tau_{\mu\Lambda}$ are the lifetimes of the free Λ particles for the electron and muon decays, respectively.

Dalitz and Liu[7] have found that the probabilities of the muon decays for He_Λ^4 and H_Λ^4 are

$$w_\pi (He_\Lambda^4) = 0.60/\tau_\Lambda, \quad w_\pi (H_\Lambda^4) = 0.65/\tau_\Lambda,$$

τ_Λ —lifetime of the free Λ particle.

Recognizing that in the universal V-A theory $\tau_\Lambda/\tau_{e\Lambda} = 1.7\%$ and $\tau_\Lambda/\tau_{\mu\Lambda} = 0.26\%$, we obtain for the ratio of the probabilities of the lepton and pion decays $w_e/w_\pi = 0.80\%$ for decay (e), $w_\mu/w_\pi = 0.13\%$ for decay (g), and $w_\mu/w_\pi = 0.06\%$ for decay (h).

The author expresses his gratitude to V. A. Filimonov for a discussion of the results of the present work.

[1] Belov, Mingalev, and Shekhter, JETP **38**, 541 (1960), Soviet Phys. JETP **11**, 392 (1960).

[2] C. H. Albright, Phys. Rev. **115**, 750 (1959).

[3] D. R. Harrington, Phys. Rev. **120**, 1482 (1960).

[4] V. A. Filimonov, Paper at 3rd Inter-university Conference on Elementary-particle Physics, Uzhgorod, 1961.

[5] D. T. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956).

[6] N. Byers and W. N. Cottingham, Nucl. Phys. **11**, 554 (1959); I. Szymanski, Nuovo cimento **15**, 45 (1960); V. A. Lyul'ka, JETP **42**, 1629 (1962), Soviet Phys. JETP **15**, 1131 (1962).

[7] R. H. Dalitz and L. Liu, Phys. Rev. **116**, 1312 (1959).

[8] V. A. Lyul'ka, JETP **39**, 471 (1960), Soviet Phys. JETP **12**, 331 (1961).

[9] Y. Y. Chen, Nuovo cimento **19**, 36 (1961).

[10] M. Leon, Phys. Rev. **113**, 1604 (1959).

[11] R. H. Dalitz, Phys. Rev. **112**, 605 (1958).

[12] R. H. Dalitz and B. W. Downs, Phys. Rev. **111**, 967 (1958).

[13] L. B. Okun' and E. P. Shabalin, JETP **37**, 1775 (1959), Soviet Phys. JETP **10**, 1252 (1960). E. P. Shabalin, JETP **39**, 345 (1960), Soviet Phys. JETP **12**, 245 (1961). L. B. Okun', Lektsii po teorii slabykh vzaimodeistviĭ elementarnykh chastits (Lectures on the Theory of Weak Interactions of Elementary Particles), Dubna, 1961.

[14] V. S. Mathur, Nuovo cimento **14**, 1322 (1959).

Translated by J. G. Adashko
30