QUADRUPOLE RESONANCES OF ATOMIC NUCLEI

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The collective states of atomic nuclei excited during the absorption of quadrupole γ quanta are considered in the shell model approximation. It is shown that in heavy nuclei the residual interaction causes the appearance of two groups of states of distinctive energy and absorption probabilities for quadrupole γ quanta. The group of states displaced upwards from the position of the zero-order approximation levels form the so-called quadrupole resonance of the nuclear photodisintegration. The states displaced downwards play an important part in forming the effective quadrupole charge of a nucleon. The intensities of transitions into the specified states exhaust almost all the quadrupole nuclear transitions. In light nuclei the two states are less clearly displayed. The mean energies of the specified groups of quadrupole transitions are estimated for all the nuclei considered.

RECENTLY, considerable interest has arisen in the quadrupole resonance of the γ quanta absorption cross-section of atomic nuclei [1-4]. This question was first discussed by Migdal [5] and by Danos and Steinwedel [6].

In the present paper quadrupole resonance is studied from the viewpoint of shell model of the nucleus taking into account the effective pair interaction between nucleons. The magic nuclei O^{16}, Ca^{40}, Pb^{208} and the "almost magic" Zr^{90} will be discussed as examples.

In shell-model terms, quadrupole resonance is caused by the transfer of nucleons from the shell with principal quantum number N to the shell with principal quantum number N + 2. In the zero-order approximation—i.e., when the effective particle-hole pair interaction is neglected—the majority of states due to such nucleon transfer will be in the energy range $\sim 2\hbar\omega$ i.e., for example, for Pb^{208}, at ~ 18 MeV [7].

When the effective pair interaction is taken into account, it can lead [8] to the formation of two states Ψ_1 and Ψ_2 with distinct energies and radiative properties, one of which (Ψ_1) is displaced downwards from the position of the levels in the zero-order approximation, while the other (Ψ_2) is displaced upwards. The example of the dipole resonance shows that the energy difference between these states can be very significant. [9] For a coherent effect to be present (separation of the states Ψ_1 and Ψ_2) in the considered case of states 2+, it is necessary that a quadrupole contribution be present in the effective pair interaction [10,11].

Qualitative consideration [8], under the assumption of degeneracy in energy of the zero-order states and of a quadrupole-quadrupole pair interaction, shows that the separated states Ψ_1 and Ψ_2 exhaust all the quadrupole transitions, and that they can be obtained, apart from normalization, by acting on the ground state of the nucleus $|0\rangle$ with the operators $\hat{Q}_1$ and $\hat{Q}_2$ 1):

$$\hat{Q}_1 = e_p \sum r_p^2 Y_{20}(\Omega_p) + e_n \sum r_n^2 Y_{20}(\Omega_n),$$
$$\hat{Q}_2 = e_p \sum r_p^2 Y_{20}(\Omega_p) - e_n \sum r_n^2 Y_{20}(\Omega_n). \qquad (1)$$

Here e_p and e_n are certain constants which are automatically obtained when solving the qualitative problem specified above. It is simplest, however, to determine them from the orthogonality condition for the states $\Psi_1 = \hat{Q}_1|0\rangle$ and $\Psi_2 = \hat{Q}_2|0\rangle$. In a real case, when the interaction is not quadrupole-quadrupole and the zero-order levels are not degenerate, the states generated by the operators $\hat{Q}_1$ and $\hat{Q}_2$ will not be eigenfunctions of the complete Hamiltonian. The part of the interaction which is not quadrupole-quadrupole will partially redistribute the intensity of the quadrupole transitions over other states of the system.

It is, however, important to note that this part of the interaction does not lead to a coherent effect in the intensity distribution of the quadrupole transitions [11], and cannot consequently change the range of positions of the levels into which quadrupole transitions proceed with considerable proba-

1)When the isotopic spin is a good quantum number $e_p = e_n = 1/2$, $\hat{Q}_1$ and $\hat{Q}_2$ give the transitions without and with change of isotopic spin, respectively.

bility. In a real case, therefore, instead of two levels exhausting all quadrupole transitions, we shall have two relatively sharp groups of levels, over which the quadrupole transitions will be distributed. The energy of these groups of levels can be approximately found by averaging the complete Hamiltonian of the system with respect to the states Ψ_1 and Ψ_2[10]. We have

$$E_1 = \langle \Psi_1 | H | \Psi_1 \rangle / \langle \Psi_1 | \Psi_1 \rangle,$$
$$E_2 = \langle \Psi_2 | H | \Psi_2 \rangle / \langle \Psi_2 | \Psi_2 \rangle. \quad (2)$$

Here $H = H_0 + V_{ph}$, where H_0 is the Hamiltonian of the zero-order approximation, and V_{ph} is the particle-hole interaction operator; $\langle \Psi_i | \Psi_i \rangle$ is the norm of the wave functions Ψ_1 and Ψ_2.

So far we have tacitly implied that the complete Hamiltonian is approximately diagonalized in the basis of the particle-hole functions. However, when a residual interaction between the nucleons is present, the eigenfunctions of the Hamiltonian H will in general be superpositions of particle-hole states and two-particle two-hole states (two nucleons go over into a neighboring shell). States of the latter type do not contribute directly to the intensity of quadrupole transitions, but can significantly redistribute them between the various levels of the nucleus. However, taking these states into account does produce coherent effects, and consequently does not significantly change the mean energy of either group. These states are not, therefore, taken into consideration.

We note yet another point. In evaluating (2) we did not include the entire choice of particle-hole states. The fact is that when a nucleon in the last-filled shell is excited, the nucleon can proceed directly into the continuous spectrum. For our approach it is, however, essential that the nucleon should be within the limits of the nucleus for a sufficient length of time (so that the particle experiences a sufficiently large number of interactions with holes). Therefore, all states which do not have a clearly displayed single-particle resonance were excluded. It was considered that they give a continuous background in the behavior of the absorption cross section.

As in the majority of nuclear spectra calculations, we took oscillator functions as the single-particle wave functions. Such a choice of single-particle functions is reasonable in the study of resonance processes, because inside the nucleus an oscillator function is close to the function of a smeared-out potential well. The effective interaction was taken as $V_{12} = g(1 - \alpha - \alpha \sigma_1 \cdot \sigma_2)$. The values of g and α were taken to be the same as in the calculations of the dipole levels.[9] The values resulting from calculations made on O^{16}, Ca^{40}, Zr^{90}, and Pb^{208} for the changes of energy ΔV_1 and ΔV_2 of the states Ψ_1 and Ψ_2 due to the residual interaction are given in Table I.

Table I

Nucleus	O^{16}	Ca^{40}	Zr^{90}	Pb^{208}
ΔV_1, MeV	−7.0	−9.2	−11.7	−14.0
ΔV_2, MeV	4.1	5.3	5.6	7.1

Thus, the energy splitting of the states Ψ_1 and Ψ_2 increases with increase of A, reaching for Pb^{208} the value of ~ 21 MeV. An important condition for the formation of the groups of levels specified above is, of course, that the energy interval over which the zero order levels are disposed should be small compared with the splitting of the states Ψ_1 and Ψ_2. Estimates show that in the heavy nuclei the majority of zero-order levels are contained in an interval of order ~ 3 MeV. In the heavy nuclei, therefore, the separate groups of levels should definitely be formed. In light nuclei of the type O^{16} and Ca^{40}, firstly, there is a very large splitting of the zero order levels, and, secondly, there is a small number of states displaced by the effective interaction. Therefore, for these nuclei the concept of the states Ψ_1 and Ψ_2 is very poor.

As already remarked, the groups of nuclear levels corresponding to Ψ_1 and Ψ_2 exhaust almost all quadrupole transitions. The group of states corresponding to Ψ_2 form a quadrupole resonance in the photon absorption cross section and correspond to quadrupole polarization oscillations of the neutrons relative to the protons.

The formation of the state Ψ_2 has an important consequence for (γ, n) reactions. During the quadrupole absorption of a γ quantum, protons and neutrons can be emitted (and because of the Coulomb barrier for protons, neutrons will be emitted preferentially). For sufficiently energetic photo-neutrons it is, therefore, natural to expect angular asymmetry similar to the asymmetry for protons (due to the interference of the dipole and quadrupole photo-neutrons). Such an asymmetry has, in fact, been observed experimentally.[12]

The state Ψ_1 is also of great interest; it is very important for detecting the effective quadrupole charge of a nucleon.[13] This state corresponds approximately to "in-phase" oscillations of neutrons and protons (for example, surface oscillations of the nucleus).

To find the absolute energies E_1 and E_2, it is necessary to know the position of the levels in the zero order approximation. At present, however, there are poor experimental and theoretical data on this question. Rough estimates using results of [7,14] give for Pb^{208} and Zr^{90}, respectively, $E_0 \approx 18$ and 21 MeV, so that $E_1 \approx 4$ MeV, $E_2 \approx 25$ MeV for Pb, and $E_1 \approx 9$ MeV, $E_2 \approx 27$ MeV for Zr. The value of E_2 for the heavy nuclei agrees reasonably with the quadrupole resonance energies observed experimentally in the cross sections for the (γ, p) reactions in Pb [15] and W [16].

The state Ψ_1 has not yet, however, been found in any of the nuclei considered. The distinctive feature of the state Ψ_1 by which it can be observed in experiment is the large value of the reduced matrix element of the quadrupole transition from this state to the ground state (it is easy to see that this reduced matrix element is equal to the corresponding matrix element of the state Ψ_2). In recent experiments on inelastic scattering of electrons [17] by Pb^{208}, this state was not observed in the range of excitation energies up to ~ 6 MeV. Our estimate of E_1 is, however, a poor approximation, because, first, the zero order level is not known with good accuracy; second, the state Ψ_1 contains a perceptible contribution of the wrong excitation with respect to the center of gravity (0.1 in the square of amplitude), and taking this fact into account should lead to some increase of E_1; third, the energy of the state Ψ_1 is rather more critical to the type of force and to the amplitude of the pair interaction than is the energy of the state Ψ_2. Great interest therefore attaches to further searches for this state at somewhat larger energies ($E^* \gtrsim 6$ MeV).

In the nuclei O^{16} and Ca^{40} the zero order levels are split over quite a wide energy range. Using Tyrren's data, [18] on the binding energy of nucleons in the deep shells, and assuming an energy separation between unfilled shells of $\sim 7-8$ MeV, we obtained the estimates given in Table II for the zero order level positions for O^{16} and Ca^{40}.

It is apparent that, owing to the large spread of energies, states including all quadrupole transitions will not be formed. It is important to note, however, that a large contribution to the quadrupole sum will be provided by states grouped in O^{16} nuclei in the 25 MeV region, and in Ca^{40} nuclei in the 19—20 MeV region. Correspondingly strong quadrupole absorption of quanta should be expected in the ~ 21 MeV region in O^{16} nuclei and in the ~ 15 MeV region in Ca^{40} nuclei (states with $T = 0$). Other levels with significant probability for quadrupole transitions will be spread in the ranges 27—35 MeV and 44—50 MeV (O^{16}), and 24—30 MeV and 33—39 MeV (Ca^{40}). For more accurate specification of the position of the states of interest to us and the transition intensities, additional insight into the single particle levels of nuclei is required.

Table II

Nucleus	Type of transition	Transition energy, MeV
O^{16}	$1s \to 1d$	44—50
	$1p_{1/2} \to 1f_{5/2}$	~25
	$1p_{3/2} \to 1f_{7/2}$	~25
	$1p_{3/2} \to 1f_{5/2}$	~31
Ca^{40}	$1p \to 1f,\ 2p$	33—39
	$1d_{3/2} \to 1g_{7/2}$	19—20
	$1d_{5/2} \to 1g_{9/2}$	
	$2s \to 2d$	23—27
	$1d_{5/2} \to 1g_{7/2}$	~28

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