INTERFERENCE PHENOMENA IN RADIATIVE $K^0_{2\pi}$ DECAYS

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The interference of the decays of K_1^0 and K_2^0 mesons in a K^0 beam into $\pi^+ + \pi^- + \gamma$ is considered. It is shown that the circular polarization of the photons, which appears in this case, depends on the magnitude and sign of the mass difference between the K_1^0 and K_2^0 meson and on the phase difference of the final-state pion-pion interaction. The angular and energy distributions for photons and pions are obtained. The angular correlation between the production planes of the positron-electron pair and the pion pair in the decay mode $K^0 \to \pi^+ + \pi^- + e^+ + e^-$ is also analyzed and an estimate of the internal conversion coefficient is given.

1. It is well-known that with respect to the weak decay interaction the neutral K meson behaves like a mixture of the particles K_1^0 and K_2^0, which have respective lifetimes τ_1 and τ_2 ($\tau_1 \ll \tau_2$), masses m_1 and m_2, and the CP-parities +1 and −1. Therefore the matrix element $\mathcal{M}$ of K^0 decay will contain two terms corresponding to the decay of the K_1^0 and K_2^0 components of the K^0 beam, respectively:

$$\mathcal{M} = \frac{1}{\sqrt{2}} e^{-im_1t}\left[M_1 e^{-t/2\tau_1} + e^{i(m_1-m_2)t} M_2 e^{-t/2\tau_2}\right]. \quad (1)$$

This fact leads to interesting interference phenomena[1-6] which depend on the mass difference $\Delta m = m_1 - m_2$ and on the time t elapsed since the generation of the K^0 meson. One such effect is the polarization of photons emitted in the $K^0 \to \pi^+ + \pi^- + \gamma$ decay. The dependence of the angle between the vector of transverse polarization of the photon and the normal to the decay plane has been investigated in the paper of Barshay and Iso[6]. We consider the corresponding effects for the circular polarization of the photons produced in the same process, as well as the correlation of the emission planes of the electron positron pair and the pion pair in the K^0 decay with internal conversion of the photon.

In order to describe the polarization we utilize the Stokes parameters ξ_j (j = 1, 2, 3), which can be obtained from the polarization density matrix ρ[7]:

$$\xi_j = \mathrm{Sp}\,(\rho\tau_j),$$

$$\rho_{\lambda\mu} = \mathcal{M}^{(\lambda)}\mathcal{M}^{(\mu)*} \Big/ \sum_\nu |\mathcal{M}^{(\nu)}|^2, \quad \lambda, \mu, \nu = 1, 2. \quad (2)$$

Here the decay matrix element $\mathcal{M}$ for the $K^0 \to \pi^+ + \pi^- + \gamma$ decay is written in the form $\mathcal{M}^{(\lambda)} = \mathcal{M}_\alpha \epsilon_\alpha^{(\lambda)}$; the index λ corresponds to the two orthogonal unit vectors $\boldsymbol{\epsilon}^{(1)}$ and $\boldsymbol{\epsilon}^{(2)}$ of the photon polarization, and τ_j are the Pauli matrices. Separating in the matrix elements $M_1^{(\lambda)}$ and $M_2^{(\lambda)}$ the phase factors due to the final-state interaction of the two pions, $M_1^{(\lambda)} = N_1^{(\lambda)} e^{i\delta_1}$, $M_2^{(\lambda)} = N_2^{(\lambda)} e^{i\delta_2}$, and introducing the notation $1/\tau = 1/\tau_1 + 1/\tau_2$, $\delta = \delta_2 - \delta_1$, $T = t\Delta m + \delta$, we obtain

$$\mathcal{M}^{(\lambda)}\mathcal{M}^{(\mu)*} = \tfrac{1}{2}\{e^{-t/\tau_1} N_1^{(\lambda)} N_1^{(\mu)} + e^{-t/\tau_2} N_2^{(\lambda)} N_2^{(\mu)} + e^{-t/2\tau}[\cos T\,(N_1^{(\lambda)} N_2^{(\mu)} + N_2^{(\lambda)} N_1^{(\mu)}) + i \sin T\,(N_2^{(\lambda)} N_1^{(\mu)} - N_1^{(\lambda)} N_2^{(\mu)})]\}. \quad (3)$$

2. It follows from CP-invariance that the photons emitted in the K_1^0 and K_2^0 decays are essentially of type E1 and M1, respectively, the E1 radiation being produced mainly by internal bremsstrahlung (we neglect direct E1 emission) and the M1 radiation being produced mainly by direct emission. The corresponding gauge-invariant matrix elements for the $K^0_{2\pi\gamma}$-decay have the form

$$N_1^{(\lambda)} = (eG/\sqrt{2})\, a_1 [(p_-k)(p_+\varepsilon^{(\lambda)}) - (p_+k)(p_-\varepsilon^{(\lambda)})],$$

$$a_1 = L_1^{-3}/(p_+k)(p_-k),$$

$$N_2^{(\lambda)} = (eG/\sqrt{2})\, a_2 [\varepsilon_{\alpha\beta\gamma\delta}\, p_{+\alpha} p_{-\beta} \varepsilon_\gamma^{(\lambda)} k_\delta], \quad a_2 = L_2^{-3}/M^4, \quad (4)$$

where $e^2 = \alpha = 1/137$, $G = 10^{-5}/M^2$, M is the nucleon mass, $p_\pm$ and k are the 4-momenta of the $\pi^\pm$ mesons and the photon, while L_1 and L_2 are quantities with the dimension of a length, determined by the virtual strong interactions as well as by the final state pion-pion interaction. Note that GL_1^{-3} equals the experimentally known constant of radiationless K_1^0-decay. For a rough estimate of L_2 we consider the simplest perturbation theory diagrams containing a baryon loop (cf. Appendix). Since both matrix elements in (4) are antisymmetric with respect to the interchange $p_+ \rightleftarrows p_-$, the final states of pion-pion scattering correspond to a spin J = 1 and isospin T = 1.

From an analysis of the S-matrix on the basis of unitarity and T-invariance and from the vanishing (as a consequence of CP-invariance) of the matrix element for the decay $K_2^0 \rightarrow \pi^+ + \pi^-$ it follows that the phase δ_2 of the matrix element N is equal to the phase shift δ_{11} in the J = T = 1 state. In general, the phase δ_1 does not coincide with δ_{11} and remains unknown. The influence of the pion-pion interaction on the amplitude of radiative K decay has previously been taken into account by means of the method of dispersion relations[9] and on the basis of a resonance model with an intermediate strongly interacting B meson[8]. The correction to the $K^+_{2\pi\gamma}$-decay amplitude due to pion-pion interaction in the J = T = 1 state, corresponding to internal bremsstrahlung, is small and cannot be larger than 10—20% (according to an estimate by I. G. Ivanter on the basis of [9]). From the work of Chew[8] it also follows that the resonance with J = T = 1 only weakly influences the internal bremsstrahlung and for the direct emission it can make itself felt, probably, only in the K_2^0 mode. We will approximately consider that the contribution of the pion-pion interaction does not change the amplitude of the K_1^0 decay and that it can increase the K_2^0 amplitude by a factor of two. Accordingly, and on the basis of an estimate of L_1 in terms of the experimental value of the probability for the K_2^0 decay, $w(K_1^0 \rightarrow \pi^+ + \pi^-) = 0.78 \times 10^{10}$ sec^{-1}, and in estimating L_2 on the basis of the results obtained in the Appendix, we will in what follows use the following values:

$$L_1 \approx 1/0.6\,M, \quad \varkappa = L_2^{-3}/L_1^{-3} = 4n, \quad n = 1, 2, 3, 6$$

(values of n equal to 2 and 6 correspond to a possible doubling of the K_2^0-decay amplitude because of pion-pion interaction).

3. For the calculation of the Stokes parameters ξ_j we choose $\epsilon^{(1)}$ and $\epsilon^{(2)}$ in the form

$$\varepsilon^{(1)} = [\mathbf{p}_+\mathbf{k}]/|[\mathbf{p}_+\mathbf{k}]|, \qquad \varepsilon^{(2)} = [\mathbf{k}\varepsilon^{(1)}]/|\mathbf{k}| \qquad (5)^*$$

and we go over to the rest system of the K^0 meson. Then Eqs. (2)—(4) yield:

$$N_1^{(1)} = N_2^{(2)} = 0, \quad N_1^{(2)}/a_1 = N_2^{(1)}/a_2 = (eG/\sqrt{2})\,m\,|[\mathbf{p}_+\mathbf{k}]| \equiv \sqrt{I_0},$$

$$\xi_1 = -2\xi_0^{-1}a_1a_2e^{-t/2\tau}\cos T, \quad \xi_2 = -2\xi_0^{-1}a_1a_2e^{-t/2\tau}\sin T,$$

$$\xi_3 = \xi_0^{-1}(-a_1^2e^{-t/\tau_1} + a_2^2e^{-t/\tau_2}),$$

$$\xi_0 = I_0^{-1}\sum_\nu |\mathcal{M}^{(\nu)}|^2 = a_1^2e^{-t/\tau_1} + a_2^2e^{-t/\tau_2}. \qquad (6)$$

The beam of photons turns out to be totally polarized ($\sum_j \xi_j^2 = 1$), its circular polarization being determined by the parameter

$$\xi_2 = \frac{-2(a_1/a_2)e^{-t/2\tau_0}\sin(t\Delta m + \delta)}{1 + (a_1/a_2)^2e^{-t/\tau_0}}, \qquad (7)$$

where $1/\tau_0 = 1/\tau_1 - 1/\tau_2$; ξ_2 depends on the magnitude and sign of Δm and on the relative phase δ, and reaches its maximum absolute value $|\xi_2| = 1$ for $(a_1/a_2)\exp(-t^*/2\tau_0) = 1$, which corresponds to $t^* \sim 2 \times 10^{-9}$ sec.

The circular polarization ξ_2 can be measured experimentally by scattering photons on the polarized electrons of magnetized iron. For the case when the photon incidence angle ϑ with respect to the electron spins is small (i.e., $\cos\vartheta \sim 1$) and their energy ω is large compared with the electron mass m_e, the energy of the scattered photons ω' satisfying the condition $1 - \cos\theta \gg m_e/\omega$ is independent of ω inside a large interval of scattering angles θ[10]. In this case ξ_2 is related to the angles ϑ and θ by the formula

$$\xi_2 \approx (1 - R)\cos\theta/(1 + R)\cos\vartheta, \qquad (8)$$

where R is the ratio of the measured scattering probabilities for the two opposite orientations of the spins of the iron electrons.

As can be seen from Eq. (7), owing to the strong time dependence of the circular polarization, the values of ξ_2 which are large enough to be measured are obtained in a relatively narrow interval around t*. By using the angular distribution and the estimate of the possible values of κ, obtained in Sec. 4 below, we can obtain the dependence of the quantity $\xi_2^0 = |\xi_2/\sin T|$ on t and ω. Figure 1 represents the curves $\xi_2^0(t)$ computed in the approximation $\omega \ll m/2$ for $\omega = 10$ MeV. The same curves determine the variation of ξ_2^0 with ω (in semilogarithmic scale) for $t = 2 \times 10^{-9}$ sec. For comparison the graph of sin T for $m = 1 \times 10^{-10}$ sec^{-1} is also included (since we do not know of the phase δ we have, for clarity, chosen the phase such that sin T = 0 for $t = 1 \times 10^{-9}$ sec).

*$[\mathbf{p}_+\mathbf{k}] = \mathbf{p}_+ \times \mathbf{k}$.

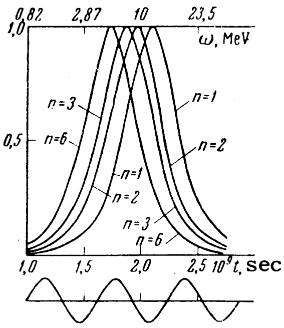

FIG. 1. The dependence of the reduced circular polarization $\xi_2^0 = |\xi_2/\sin T|$ in the $K_{2\pi\gamma}^0$ decay on t (for ω = 10 MeV) and on ω (for t = 2×10^{-9} sec); the curve $\sin T \equiv \sin(t\Delta m + \delta)$ is shown in the lower part.

4. From the formula for the probability of the $K_{2\pi\gamma}^0$ decay

$$dw = (2\pi)^{-5}\int\frac{|\mathcal{M}|^2 d^3p_+ d^3p_- d^3k}{2m2\omega_+2\omega_-2\omega}\,\delta^4(P - p_+ - p_- - k) \quad (9)$$

($\omega_\pm$ are the energies of the $\pi^\pm$ mesons) it is easy to obtain the expression for the angular distribution of the pions (in the rest system of the K^0 meson). Using Eqs. (3), (4), and (6) and denoting the cosine of the angle between the directions of emission of one of the pions and the photon by z, we obtain

$$\frac{dw}{dz} = \frac{\alpha G^2(1-z^2)}{256\pi^3\mu}\left\{L_2^{-6}e^{-t/\tau_2}\frac{m^4\mu^2}{M^8}\int\frac{\sqrt{(x^2-1)^3}(x_0-x)^3\,dx}{(2x_0+z\sqrt{x^2-1}-x)^4} + L_1^{-6}e^{-t/\tau_1}\int\frac{\sqrt{(x^2-1)^3}\,dx}{(2x_0+z\sqrt{x^2-1}-x)^2(x_0-x)(x-z\sqrt{x^2-1})^2}\right\}, \quad (10)$$

where μ is the pion mass, $x = \omega_\pm/\mu$, and $x_0 = m/2\mu$ is the maximum energy of the pion. Since the integration in (10) leads to very involved expressions we represent the angular distributions graphically in Fig. 2 (separately for the K_1^0 and K_2^0 components). The curves have been obtained by means of approximate graphical integration, and in order to insure convergence for $x \to x_0 = 1.78$ a cut-off has been imposed on x for $\bar{x} = 1.75$ (corresponding to ω_{min} = 10 MeV).

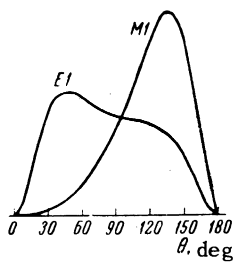

FIG. 2. The angular distributions of the photons (relative to the direction of emission of one of the pions) in the decays $K_1^0 \to \pi^+ + \pi^- + \gamma$ (E1) and $K_2^0 \to \pi^+ + \pi^- + \gamma$ (M1)

In analogy with the angular distribution, we obtain for the energy spectra of the pions and photons[1)] produced in the $K_{2\pi\gamma}^0$-decay, in the rest system of the K^0 meson:

$$\frac{dw}{dx} = \frac{\alpha G^2\mu}{128\pi^3 m^2}\left\{L_2^{-6}e^{-t/\tau_2}\frac{\mu^4 m^4\sqrt{(x^2-1)^3}(x_0-x)^2}{6M^8(x_0-x+1/4x_0)} + L_1^{-6}e^{-t/\tau_1}\left[\frac{x-1/2x_0}{x_0-x}\ln\frac{x-1/2x_0+\sqrt{x^2-1}}{x-1/2x_0-\sqrt{x^2-1}} - \frac{\sqrt{x^2-1}}{x_0-x}\right]\right\}, \quad (11)$$

$$\frac{dw}{d\omega} = \frac{\alpha G^2}{64\pi^3 m^2}\left\{L_2^{-6}e^{-t/\tau_2}\frac{m^4\omega^3\sqrt{(\omega_0-\omega)^3}}{12M^8\sqrt{m/2-\omega}} + L_1^{-6}e^{-t/\tau_1}\left[\frac{m/2-\omega-\mu^2/m}{\omega}\ln\frac{\sqrt{m/2-\omega}+\sqrt{\omega_0-\omega}}{\sqrt{m/2-\omega}-\sqrt{\omega_0-\omega}} - \frac{\sqrt{(m/2-\omega)(\omega_0-\omega)}}{\omega}\right]\right\}, \quad (12)$$

where $\omega_0 = (m^2 - 4\mu^2)/2m$ is the maximal photon energy. For $t \gtrsim 1.25 \times 10^{-9}$ sec, the spectra are dominated by the second term in the curly brackets of Eqs. (11) and (12) (internal bremsstrahlung), and for $t \gtrsim 2.5 \times 10^{-9}$ sec —by the first term (direct emission), whereas the direct emission produced by the K_2^0 component corresponds to spectra with sufficiently large maxima at $\omega_\pm = x\mu \sim$ 145—150 MeV and $\omega \sim$ 115—120 MeV, respectively.

We note that the spectrum of the pions emitted in the K_1^0 decay, as described by Eq. (11), differs somewhat from the $\pi^\pm$ spectrum produced by internal bremsstrahlung in the radiative K^+-decay, as obtained by Good[11] (Good's curve yields values by 20—30% lower in its principal part). On the other hand the formula obtained by Chew[8] for the spectrum of internal bremsstrahlung photons for the radiative K^+ decay differs from the second term in Eq. (12) by having the factor $\frac{1}{2}(m/2 - \omega)/\omega$ in front of the logarithm in place of $(m/2 - \omega - \mu^2/m)/\omega$.

The integration of the second term in Eq. (12) in the calculation of the total probability for the $K_{2\pi\gamma}^0$ decay cannot be carried out to the end in terms of elementary functions. Therefore we find the result approximately, and in addition we assume a lower limit $\omega_{min} = \epsilon \ll \omega_0$ for the integrals that diverge as $\omega \to 0$. The integration yields

$$w = \frac{\alpha G^2}{128\pi^3 m}\left\{5.1\cdot 10^{-7}L_2^{-6}e^{-t/\tau_2} + 1.16\left[\ln\frac{m}{\varepsilon} - 1.65\right]L_1^{-6}e^{-t/\tau_1}\right\}. \quad (13)$$

Hence, for t = 0, $L_1^{-1} = 0.6\,M$, and $L_2^{-1} = 1.37\,M$ (or 0.95 M) (cf. the Appendix) we obtain for the total probabilities of the $K_1^0 \to \pi^+ + \pi^- + \gamma$ and $K_2^0 \to \pi^+ + \pi^- + \gamma$ modes:

$$w_1 \approx 0.5\,[\ln(m/\varepsilon) - 1.65]\cdot 10^8\ \text{sec}^{-1},$$
$$w_2 \approx 3.6\cdot 10^2\beta^2\ \text{sec}^{-1}\ \text{or}\ w_2 \approx 4\cdot 10\beta^2\ \text{sec}^{-1}, \quad (14)$$

where β, which is equal to 1 or 2, is a coefficient determined by the final-state pion-pion interaction.

1)The energy spectrum of the photons produced in $K_{2\pi\gamma}^0$ decay [the first term in Eq. (12)] has been obtained by Chew.[8]

It follows that the ratio of probabilities for the radiative and ordinary decays of the K_1^0 meson for $\epsilon \approx 65$ MeV is $w_1/w_0 \approx 0.3\,\alpha \approx 2\times 10^{-3}$.

5. Let us now consider the radiative $K_{2\pi}^0$ decay with internal conversion of the photon into an electron-positron pair. The matrix element has the form [compare with Eq. (4)]

$$N_1 = \left(\frac{eG}{\sqrt{2}}\right) 4\pi e L_1^{-3}\left[\frac{(2p_+ + k)_\gamma}{2(p_+k)+k^2} - \frac{(2p_- + k)_\gamma}{2(p_-k)+k^2}\right] L_\gamma,$$

$$N_2 = (eG/\sqrt{2})\, 4\pi e L_2^{-3} M^{-4}\varepsilon_{\alpha\beta\gamma\delta} p_{+\alpha} p_{-\beta} L_\gamma k_\delta;$$

$$L_\gamma = k^{-2}\bar{u}(k_-)\gamma_\gamma v(k_+). \tag{15}$$

For the differential probability $\bar{w}$ of the $K^0 \to \pi^+ + \pi^- + e^+ + e^-$ process we obtain

$$\frac{d^4\bar{w}}{dQ^2 d\omega d\zeta d\varphi} = \frac{\alpha^2 G^2 Q^2 \gamma^3(Q^2)\,|\mathbf{k}|}{3\cdot 2^{11}\pi^5 m^3 k^2}\Big\{L_2^{-6}e^{-t/\tau_2}(1-\zeta^2)(1+2\cos^2\varphi)\frac{\mathbf{k}^2 m^4}{16M^8}$$

$$+ L_1^{-6}e^{-t/\tau_1}\frac{(1-\zeta^2)(3-2\cos^2\varphi)\,\omega^2 + 2\zeta^2(m-\omega)^2 k^2/Q^2}{[\omega^2-\zeta^2\mathbf{k}^2\gamma^2(Q^2)]^2}$$

$$+ L_1^{-3}L_2^{-3}e^{-t/2\tau}\cos T\,(1-\zeta^2)\sin 2\varphi\,\frac{\omega\,|\mathbf{k}|\,m^2}{M^4[\omega^2-\zeta^2\mathbf{k}^2\gamma^2(Q^2)]}\Big\}. \tag{16}$$

Here

$$Q^2 = (p_+ + p_-)^2,\quad \gamma^2(Q^2) = (Q^2 - 4\mu^2)/Q^2,$$

$$\zeta = \cos(\widehat{\mathbf{Rk}}),\quad \mathbf{R} = \mathbf{p}_+ - \mathbf{p}_-,$$

and φ is the angle between the planes of emission of the pion pair and the electron-positron pair.

For small ζ, when the second term in the denominator of the second term of (16) is negligible, the probability of the K_1^0 decay has an angular dependence of the form $f_1(\varphi) = 3 - 2\cos^2\varphi = 3\sin^2\varphi + \cos^2\varphi$, and the probability for K_2^0 decay has the angular dependence $f_2(\varphi) = 1 + 2\cos^2\varphi = 3\cos^2\varphi + \sin^2\varphi$. Thus, $f_1(\varphi)$ is obtained from $f_2(\varphi)$ by means of the substitution[2] $\sin^2\varphi \rightleftarrows \cos^2\varphi$. An approximate numerical estimate of the total decay probability leads to the result

$$\frac{d\bar{w}}{d\varphi} \approx \frac{\alpha^2G^2}{2^9\pi^5 m}\{L_2^{-6}e^{-t/\tau_2}\,1.5\cdot 10^{-6}(1+2\cos^2\varphi)$$

$$+ L_1^{-6}e^{-t/\tau_1}\,[(3-2\cos^2\varphi)+0.5]$$

$$+ L_1^{-3}L_2^{-3}e^{-t/2\tau}\cos T\cdot 5.5\cdot 10^{-4}\sin 2\varphi\} \tag{17}$$

(the coefficients of the terms in this formula are of the correct order of magnitude). Comparing Eqs. (13) and (17) one obtains for the conversion coefficient for the case of K_2^0 decay the estimate $\bar{w}/w \approx 0.6\%$.

[2]The presence of such an angular correlation has been noted by Chew.[8]

APPENDIX

Since there is no theory of strong interactions, the matrix elements for the $K^0 \to \pi^+ + \pi^- + \gamma$ decay cannot be calculated. However, for a rough estimate of these matrix elements one can treat the decay by means of perturbation theory, considering diagrams with baryon loops[3]. We assume that the momenta of the particles that leave the loop are considerably smaller than the momenta and masses of the baryons inside the loop, and we retain only the first nonvanishing terms in the expansions of the corresponding integrals in powers of the momenta of the outgoing particles. The baryon weak interaction current will be written in the form

$$J_\alpha = f\,[\bar{p}\gamma_\alpha(1+A\gamma_5)\Lambda + \bar{p}\gamma_\alpha(1+B\gamma_5)n];$$

$$f^2 = G/\sqrt{2},\quad A\approx 1,\quad B\approx 1.25. \tag{A.1}$$

The decay mode[4] $K_2^0 \to \pi^+ + \pi^- + \gamma$ is described by Feynman diagrams of the kind represented in Fig. 3 (the first diagram contains two closed fermion loops, the second one contains one loop which intersects itself; the remaining diagrams are obtained from these two by permuting the external lines in all possible ways). The contribution to the element N_2 appears only in the third order of the expansion, N_2 being expressed in terms of divergent integrals of two kinds:

$$N_2 = 2eGg_K g_\pi^2\pi^{-2}M^{-1}R_2(h', h'')\,\varepsilon_{\alpha\beta\gamma\delta}p_{+\alpha}p_{-\beta}\varepsilon_\gamma k_\delta,$$

$$i\pi^2 h' = \frac{1}{M^2}\int\frac{d^4q}{q^2-M^2},\quad i\pi^2 h'' = \int\frac{d^4p}{(p^2-M^2)^2}. \tag{A.2}$$

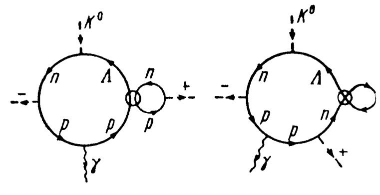

FIG. 3. Feynman diagrams describing the direct photon emission in the decay $K_2^0 \to \pi^+ + \pi^- + \gamma$; the plus and minus signs denote respectively positive and negative pions, and the small circle represents the weak interaction.

The function $R_2(h', h'')$ has the form of a sum $\Sigma_i \nu_i C_i h_i$ where the sum goes over all diagrams, the ν_i are numerical coefficients, $C = AB \pm 1$

[3]Similar estimates of the matrix elements for different modes of decay of K mesons have been given in the papers of Oneda, Pati, and Sakita (cf., e.g., [12]).

[4]We recall that it is not necessary to compute the matrix element N_1 of the $K_1^0 \to \pi^+ + \pi^- + \gamma$ decay, since the corresponding constant GL_1^{-3} can be obtained directly from the experimental probability of the $K_1^0 \to \pi^+ + \pi^-$ decay.

or AB, and h are integrals of the type h′ or h″. In order to estimate the absolute value and the sign of the integrals h in R_2, we compare the probabilities for the various decay modes ($K_{\pi 3}$, $K_{\pi 2}$, $K_{\mu 3}$, $K_{\mu 2}$ and also $\pi \rightarrow \mu + \nu$) calculated in terms of diagrams with appropriate baryon loops (we assume everywhere $g_\pi^2 \approx 14$ and $g_K^2 \approx 1$) with the experimental data. As a result one obtains for R_2 the rough estimate: $0.1 \lesssim R_2 \lesssim 1.0$. For our calculations we select two values: $R_2 = 0.6$ and $R_2 = 0.2$. Comparing Eqs. (A.2) and (4) and also computing the constant GL_1^{-3} (to which corresponds a value $R_1 = -0.05$) from the experimental rate of $K^0_{2\pi}$ decay, we obtain

$$L_1 \approx 1/0.6\,M, \qquad \varkappa = L_2^{-3}/L_1^{-3} \approx 4n, \qquad n = 1,\ 3 \quad \text{(A.3)}$$

(The second value of n corresponds to $R_2 = 0.2$).

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