MESON PRODUCTION IN NEUTRINO EXPERIMENTS AND THE ISOTOPIC STRUCTURE OF WEAK INTERACTIONS

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In the weak interaction schemes under discussion it is assumed that the strangeness-conserving current of strongly interacting particles is an isotopic vector. It is shown that this hypothesis leads to a number of relations between cross sections for the production of mesons without change in strangeness in high energy neutrino and antineutrino collisions with nucleons. It is assumed that neutral symmetric currents exist.

IN high-energy neutrino or antineutrino collisions with nucleons, in addition to elastic processes of the type

$$\nu(\bar{\nu}) + N \to l^-(l^+) + N \quad \text{or} \quad \nu(\bar{\nu}) + N \to \nu(\bar{\nu}) + N$$

(if neutral symmetric currents exist), certain inelastic processes are possible in which pions and kaons are produced. The cross sections for these processes have been estimated in a number of papers [1-4] and found to be comparable to the cross sections for the elastic processes.

The simplest inelastic processes are the production of a pion in the collision of a neutrino or antineutrino with a nucleon. The amplitudes for these latter processes were obtained [5-7] with the help of dispersion relations. A comparison of the results with experiment may provide a test of the universal weak interaction hypothesis.[8-9] The application of the method of extrapolation of the experimental data to these processes will also make possible the determination of the neutrino form factors of mesons and test proposed hypotheses.[4,10]

However a study of the structure of weak interactions on the basis of the results of the above mentioned papers[4-7,10] requires the measurement of not only cross sections but also of correlation effects. The simplest task is to study the isotopic structure of the weak interactions on the basis of cross section data, as was pointed out by Lee and Yang[11] and Shekhter.[12]

The weak interaction schemes considered nowadays [8,11,13] possess the following symmetry property: the full strangeness conserving current of the strongly interacting particles transforms like an isotopic vector. In the work of Lee and Yang this property is referred to as the $\Delta I = 1$ rule for weak interactions without change in strangeness.[1)] This rule leads to a number of relations between the cross sections of different inelastic processes. Certain of these relations were obtained by Lee and Yang [11] and Shekhter.[12] In this work we obtain several other relations.

Corresponding to the type of weak interaction schemes being considered we assume that the part of the Lagrangian responsible for the lepton-baryon interaction without change in strangeness has the form

$$L = \frac{G}{\sqrt{2}}\left\{\frac{1}{\sqrt{2}}\bar{l}_i\gamma_\mu(1+\gamma_5)\nu_i I_\mu^{(+)} + \text{э. с.} + \xi\bar{\nu}_i\gamma_\mu(1+\gamma_5)\nu_i I_\mu^{(0)}\right\}, \tag{1}$$

where $l_i = e, \mu$; $\nu_i = \nu_e, \nu_\mu$; G is the universal weak interactions coupling constant; ξ is a constant; $I_\mu^{(\alpha)}$ is an isotopic vector. The possibility of existence of neutral symmetric currents, entering into the second term of the Lagrangian (1), has been discussed in a number of papers (see, for example,[14] and the literature quoted therein[2)]). The constant ξ in the Lagrangian (1) may be determined from the data on elastic neutrino or antineutrino scattering on nucleons. In the Bludman[13] two-neutrino scheme $\xi = 1/2$, and if ν_e and ν_μ are identical then in that scheme $\xi = 1$.

In neutrino or antineutrino collisions with nucleons the following processes may occur:

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1)This nomenclature is not to be understood as meaning that the total isotopic spin of the state changes by unity, but rather that the isotopic spin of the final state is equal to the sum of the isotopic spin of the initial state and unity, the sum to be performed according to the rules for the addition of angular momenta.

2)The question of neutral currents has also been investigated by King.[15] We have become familiar with this work only after the publication of our work.[14]

a) $\nu_i + p \to l_i^- + p + \pi^+$, a') $\bar{\nu}_i + n \to l_i^+ + n + \pi^-$;
b) $\nu_i + n \to l_i^- + n + \pi^+$, b') $\bar{\nu}_i + p \to l_i^+ + p + \pi^-$;
c) $\nu_i + n \to l_i^- + p + \pi^0$, c') $\bar{\nu}_i + p \to l_i^+ + n + \pi^0$;
d) $\nu_i + p \to \nu_i + p + \pi^0$, d') $\bar{\nu}_i + n \to \bar{\nu}_i + n + \pi^0$;
e) $\nu_i + p \to \nu_i + n + \pi^+$, e') $\bar{\nu}_i + n \to \bar{\nu}_i + p + \pi^-$;
f) $\nu_i + n \to \nu_i + n + \pi^0$, f') $\bar{\nu}_i + p \to \bar{\nu}_i + p + \pi^0$;
g) $\nu_i + n \to \nu_i + p + \pi^-$, g') $\bar{\nu}_i + p \to \bar{\nu}_i + n + \pi^+$.

The processes a)—c) and a')—c') are not subject to doubt since the existence of charged currents, responsible for these processes, has been demonstrated experimentally. The remaining processes d)—g) and d')—g') occur if neutral symmetric currents exist.

It follows from the Lagrangian (1) that the matrix elements for processes a)—g) and a')—g') have the form

$$M_n = \frac{G}{2}\bar{u}_{l_i}\gamma_\mu(1+\gamma_5)u_{\nu_i}T_\mu^{(n)}, \qquad n = \mathrm{a, b, c},$$

$$M_n = \frac{G}{2}\bar{v}_{\nu_i}\gamma_\mu(1+\gamma_5)v_{l_i}T_\mu^{(n)}, \qquad n = \mathrm{a', b', c'},$$

$$M_n = \frac{G}{\sqrt{2}}\xi\bar{u}_{\nu_i}\gamma_\mu(1+\gamma_5)u_{\nu_i}T_\mu^{(n)}, \qquad n = \mathrm{d, e, f, g},$$

$$M_n = \frac{G}{\sqrt{2}}\xi\bar{v}_{\nu_i}\gamma_\mu(1+\gamma_5)v_{\nu_i}T_\mu^{(n)}, \quad n = \mathrm{d', e', f', g'}, \tag{2}$$

where $T_\mu^{(n)}$ are strong interaction matrix elements:

$$\begin{aligned}
&T_\mu^{(a)} = \langle p\pi^+|I_\mu^{(+)}|p\rangle, && T_\mu^{(a')} = \langle n\pi^-|I_\mu^{(-)}|n\rangle,\\
&T_\mu^{(b)} = \langle n\pi^+|I_\mu^{(+)}|n\rangle, && T_\mu^{(b')} = \langle p\pi^-|I_\mu^{(-)}p\rangle,\\
&T_\mu^{(c)} = \langle p\pi^0|I_\mu^{(+)}|n\rangle, && T_\mu^{(c')} = \langle n\pi^0|I_\mu^{(-)}|p\rangle,\\
&T_\mu^{(d)} = \langle p\pi^0|I_\mu^{(0)}|p\rangle, && T_\mu^{(d')} = \langle n\pi^0|I_\mu^{(0)}|n\rangle,\\
&T_\mu^{(e)} = \langle n\pi^+|I_\mu^{(0)}|p\rangle, && T_\mu^{(e')} = \langle p\pi^-|I_\mu^{(0)}|n\rangle,\\
&T_\mu^{(f)} = T_\mu^{(d')}, && T_\mu^{(f')} = T_\mu^{(d)},\\
&T_\mu^{(g)} = T_\mu^{(e')}, && T_\mu^{(g')} = T_\mu^{(e)}.
\end{aligned} \tag{3}$$

Charge symmetry requires that

$$T_\mu^{(a)} = T_\mu^{(a')},\quad T_\mu^{(b)} = T_\mu^{(b')},\quad \ldots\quad, T_\mu^{(g)} = T_\mu^{(g')}. \tag{4}$$

However it does not yet follow from these relations that the cross sections for processes (a)—(g) are equal to the cross sections for the corresponding processes (a')—(g').

Let us denote by k_μ and k'_μ respectively the 4-momenta of the incident and outgoing lepton, by ω and ω' the energies of these particles in the barycentric frame and by E the total energy in that frame. A calculation of cross sections for the processes considered gives

$$d\sigma_n = \frac{2}{\omega\omega'}\frac{E-\omega}{E}(2\pi)^4\delta^4\left(\sum p\right)\prod\frac{d^3p}{(2\pi)^3}F_{\mu\nu}^{(n)}\overline{\sum T_\mu^{(n)}T_\nu^{(n)*}}, \tag{5}$$

where the symbol $\overline{\Sigma(\ldots)}$ denotes an average over the polarizations of the particles in the initial state and a sum over the polarizations in the final state. The quantities $F_{\mu\nu}^{(n)}$ are defined as follows:

$$F_{\mu\nu}^{(n)} = \left(\frac{G}{2}\right)^2[k_\mu k'_\nu + k_\nu k'_\mu - kk'\delta_{\mu\nu} + \varepsilon_{\mu\nu\alpha\beta}k_\alpha k'_\beta], \quad n = \mathrm{a, b, c},$$

$$F_{\mu\nu}^{(n)} = \left(\frac{G}{2}\right)^2[k_\mu k'_\nu + k_\nu k'_\mu - kk'\delta_{\mu\nu} - \varepsilon_{\mu\nu\alpha\beta}k_\alpha k'_\beta], \quad n = \mathrm{a', b', c'},$$

$$F_{\mu\nu}^{(n)} = \left(\frac{\xi G}{\sqrt{2}}\right)^2[k_\mu k'_\nu + k_\nu k'_\mu - kk'\delta_{\mu\nu} + \varepsilon_{\mu\nu\alpha\beta}k_\alpha k'_\beta],$$

$$n = \mathrm{d, e, f, g},$$

$$F_{\mu\nu}^{(n)} = \left(\frac{\xi G}{\sqrt{2}}\right)^2[k_\mu k'_\nu + k_\nu k'_\mu - kk'\delta_{\mu\nu} - \varepsilon_{\mu\nu\alpha\beta}k_\alpha k'_\beta],$$

$$n = \mathrm{d', e', f', g'}. \tag{6}$$

According to Eq. (4), the sums $\overline{\Sigma T_\mu^{(n)}T_\nu^{(n)*}}$ for each pair of processes a) and a'), b) and b'), etc., are equal to each other. However the cross sections for the processes in each pair are not equal because, according to Eq. (6), the last terms in $F_{\mu\nu}^{(n)}$ have different signs. Let us consider these processes at high energies when the lepton mass may be neglected.

Let θ denote the angle between the momenta of the leptons **k** and **k'**. It is clear that the indicated last terms in $F_{\mu\nu}^{(n)}$ vanish for $\theta = 0$. Thus for $\theta = 0$ one has the equalities

$$d\sigma_a/d\sigma_{a'} = d\sigma_b/d\sigma_{b'} = \ldots = d\sigma_g/d\sigma_{g'} = 1, \qquad \theta = 0. \tag{7}$$

Let us consider now processes for which $F_{\mu\nu}^{(n)}$ is the same (accurate to within a numerical coefficient), namely processes a)—g). For these processes there exist relations between differential cross sections at all angles. Consequently they are also valid for the total cross sections. Let us note first of all that the last of the relations (3) and the equalities (4) give

$$T_\mu^{(d)} = T_\mu^{(f)}, \qquad T_\mu^{(e)} = T_\mu^{(g)}. \tag{7}$$

Therefore

$$\sigma_d = \sigma_f, \qquad \sigma_e = \sigma_g. \tag{8}$$

Let us denote by T_μ^J matrix elements of the type (3) with total isotopic spin J ($J = 1/2, 3/2$). After standard calculations we obtain

$$T_\mu^{(e)} = T_\mu^{3/2}, \qquad T_\mu^{(b)} = \tfrac{1}{3}(T_\mu^{3/2} + 2T_\mu^{1/2}),$$

$$T_\mu^{(c)} = T_\mu^{(e)} = \tfrac{1}{3}\sqrt{2}(T_\mu^{1/2} - T_\mu^{1/2}), \qquad T_\mu^{(d)} = \tfrac{1}{3}(2T_\mu^{3/2} + T_\mu^{1/2}). \tag{9}$$

From these relations it follows first of all that

$$\sigma_e/\sigma_c = 2\xi^2. \tag{10}$$

Further, we have

$$T_\mu^{(a)} = T_\mu^{(b)} + \sqrt{2}T_\mu^{(c)}.$$

From here it follows that

$$\sqrt{\sigma_a} + \sqrt{\sigma_b} \geqslant \sqrt{2\sigma_c}, \qquad \sqrt{\sigma_a} + \sqrt{2\sigma_c} \geqslant \sqrt{\sigma_b},$$

$$\sqrt{\sigma_b} + \sqrt{2\sigma_c} \geqslant \sqrt{\sigma_a}. \tag{11}$$

These "triangle inequalities" were obtained by Lee and Yang[11] and by Shekhter.[12]

Analogously one obtains from Eqs. (9) and (10) the following relation

$$\sigma_a + \sigma_b = \sigma_c (1 + 2\sigma_d/\sigma_e). \tag{12}$$

For processes a′)—g′) there are analogous relations:

$$\sigma_{d'} = \sigma_{f'}, \qquad \sigma_{e'} = \sigma_{g'}. \tag{8′}$$

$$\sigma_{e'}/\sigma_{c'} = 2\xi^2, \tag{10′}$$

$$\sqrt{\sigma_{a'}} + \sqrt{\sigma_{b'}} \geqslant \sqrt{2\sigma_c} \quad \text{etc}, \tag{11′}$$

$$\sigma_{a'} + \sigma_{b'} = \sigma_{c'} (1 + 2\sigma_{d'}/\sigma_{e'}). \tag{12′}$$

These results are also applicable to processes of type a)—g) and a′)—g′) with the replacements $p \rightarrow K^+$, $n \rightarrow K^0$, $\pi^{\pm,0} \rightarrow \Sigma^{\pm,0}$ or $\pi^{\pm,0} + \Lambda$.

One may prove analogously that the cross sections for the processes

h) $\nu_i + n \rightarrow l_i^- + \Lambda + K^+$, h′) $\bar{\nu}_i + p \rightarrow l_i^+ + \Lambda + K^0$,

k) $\nu_i + n \rightarrow \nu_i + \Lambda + K^0$, k′) $\bar{\nu}_i + p \rightarrow \bar{\nu}_i + \Lambda + K^+$,

l) $\nu_i + p \rightarrow \nu_i + \Lambda + K^+$, l′) $\bar{\nu}_i + n \rightarrow \bar{\nu}_i + \Lambda + K^0$

satisfy the relations

$$d\sigma_h/d\sigma_{h'} = d\sigma_k/d\sigma_{k'} = d\sigma_l/d\sigma_{l'} = 1, \qquad \theta = 0, \tag{13}$$

$$\sigma_k = \sigma_l, \qquad \sigma_{k'} = \sigma_{l'}, \tag{14}$$

$$\sigma_k/\sigma_h = \sigma_{k'}/\sigma_{h'} = \xi^2. \tag{15}$$

In this fashion the proposed isotopic structure of the strangeness conserving current of strongly interacting particles leads to a number of relations between the cross sections for the processes a)—g), a′)—g′), h)—l) and h′)—l′). If one considers only total cross sections for processes involving charged currents then the results obtained by Lee and Yang [11] and Shekhter [12] exhaust all the possible relations. However if one considers in addition differential cross sections then one has the additional relations (7) and (13). If the processes due to the neutral currents exist one has additional relations between cross sections for these processes and cross sections for processes due to charged currents.

At the present time high energy neutrino and antineutrino experiments are being performed. The experimental data may allow a test of the above relations and, consequently, a test of the proposed isotopic structure of the weak interactions.

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