ON INTERACTING FIELDS WITH DEFINITE SPIN

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The concept of the spin of an interacting field is examined. Requirements on the interaction of higher-spin particles are formulated, such that when they are satisfied each interacting field transfers only one angular momentum, i.e., has one definite spin. The conditions in question single out a certain restricted class of interactions (theories of class A), for which examples are given.

1. GENERAL CONSIDERATIONS

In the theory of interacting fields the spin of a field is defined in practice as the number of dynamically independent components.[1,2] As a rule the Heisenberg field operators have superfluous components. The supplementary conditions (s.c.) which restrict the number of degrees of freedom, and thus single out a particular spin, in general depend on the interaction and differ from the s.c. for the free field.

We note that for particles with spin 0 or $1/2$ the situation is the same in the free case and with any Lorentz-invariant interaction; in each of these cases there simply are no s.c. (the number of components is equal to the number of degrees of freedom).

The question arises, to what physical consequences do the differences in the s.c. lead? As an example let us consider the process which corresponds to the diagram of Fig. 1, where the initial and final states are connected by one virtual line of the field in question (in the general case with all radiative corrections). As is well known, if the intermediate particle has spin 0 (or $1/2$) it can transfer just the single amount of angular momentum, 0 (or $1/2$); the amplitude for such a process contains only the partial waves that correspond to total angular momentum 0 (or $1/2$).

In the simplest theory of a neutral vector field, where the s.c. is the same in form as for the free case,

$$\partial_\mu A_\mu(x) = 0, \tag{1}$$

the quanta of the vector field are also capable of transferring only one amount of angular momentum (unity). In general this is not true; in a number of theories the virtual quanta are capable of transferring several amounts of angular momentum. For example, in spite of the fact that in the simplest axial-vector theory the s.c.

$$\partial_\mu A_\mu = Cg\bar{\psi}\gamma_5\psi \tag{2}$$

(g is the coupling constant with the spinor field ψ, and C is a constant which can be expressed in terms of the masses of the two fields) reduces the number of dynamically independent components of the field A_μ to three, nevertheless this field transfers the angular momenta 0 and 1; the amplitude for the process represented by Fig. 1 contains partial waves corresponding to both of these values of the total angular momentum. This fact is connected with the difference between the s.c. (2) and the s.c. (1).

2. In the general case the field transfers several different angular momenta. Any field, however, (for example, a tensor field) with a finite number of components is capable of transferring only a limited number of angular momenta.

The answer to the question as to what set of angular momenta a certain field can transfer is given by the operator $\hat{s}^2$ for the square of the spin of the field in question[1)]:

$$\hat{s}^2 = \frac{1}{2} m_{\rho\sigma} m_{\rho\sigma} - p^{-2} m_{\lambda\rho} m_{\lambda\sigma} p_\rho p_\sigma \tag{3a}$$

$$= \frac{1}{2} s_{\rho\sigma} s_{\rho\sigma} - p^{-2} s_{\lambda\rho} s_{\lambda\sigma} p_\rho p_\sigma. \tag{3b}$$

It is constructed from generators of transformations of the field operators

$$p_\lambda = -i\partial_\lambda, \tag{4}$$

$$m_{\rho\sigma} = x_\rho p_\sigma - x_\sigma p_\rho + s_{\rho\sigma}, \tag{5}$$

[1)]The projections of the spin operator can be defined in other ways, for example in the form

$$\hat{s}_{\rho\sigma} = m_{\rho\sigma} + p^{-2}(p_\rho m_{\sigma\lambda} p_\lambda - p_\sigma m_{\rho\lambda} p_\lambda)$$
$$= s_{\rho\sigma} + p^{-2}(p_\rho s_{\sigma\lambda} p_\lambda - p_\sigma s_{\rho\lambda} p_\lambda), \quad \hat{s}^2 = \frac{1}{2}\hat{s}_{\rho\sigma}\hat{s}_{\rho\sigma}. \qquad (3a)$$

where the $s_{\rho\sigma}$ are the generators of Lorentz rotations of the field components.

The spectrum of the eigenvalues of the operator $\hat{s}^2$ for a given field (i.e., for fixed $s_{\rho\sigma}$) gives the spectrum of the angular momenta that it can carry (Section 3). Generally speaking, a field can have several spins (i.e., can transfer several amounts of angular momentum). By means of $\hat{s}^2$ we can write down the condition which a field must satisfy for it to have a single definite spin s: it must be an eigenfunction of $\hat{s}^2$ with a prescribed eigenvalue $s(s+1)$.[2)]

Thus when we speak of the quantum number "spin of a field" we must first assign to it, in accordance with the principles of quantum mechanics, a definite operator—the operator (3).

3. It seems natural to divide theories of particles with higher spins into two classes in the following way:

A. Theories in which each of the interacting fields has a single definite spin, just as the free fields do.[3)]

B. Theories in which a single definite spin cannot be assigned to each of the interacting fields.

It will be shown below that a theory belongs to class A when and only when the interaction is so chosen that it does not change the form of the s.c. as compared with those for the free fields (it is of course understood that in the free case the s.c. single out just one spin).[4)]

What is the explanation of the fact that in theories of class B an interacting field that has the same number of degrees of freedom as the free field still transfers a larger number of values of the angular momentum? Precisely this question (but in a somewhat different form) was posed for theories of the vector field by Byers and Peierls[6] and was cleared up by Kemmer.[7] The components of the field which carry the additional angular momenta can be expressed in terms of the independent components of the fields. For example, the s.c. (2) means that the part of the axial-vector field with the spin 0 is a combination of the ψ. Kemmer pointed out that evidently there always exists a canonical transformation which leads to new fields that satisfy the same s.c. as in the free case. It is true that after such a transformation the theory takes a very complicated and in general nonlocal form. Thus a theory of class B can be reduced to a theory of class A at the price of introducing nonlocality and other complications.

4. The operator for the square of the spin of a closed quantum-mechanical system or a system of fields is one of the invariants of the inhomogeneous Lorentz group. It has been discussed in various aspects in connection with the classification of the representations of this group.[8-11] The application of this operator to the free fields is extremely useful, since it combines within itself all of the s.c. and allows us to obtain them in a unified way (Section 2). A free field with definite mass and definite spin transforms according to an irreducible representation of the inhomogeneous Lorentz group—it is an eigenfunction of the invariants of this group.

5. Unlike the operators of a free field, the operators of an interacting field no longer transform according to a single irreducible representation of the inhomogeneous group. In fact, an interacting field has a whole spectrum of eigenvalues of one invariant of this group, the mass operator $-p^2 \equiv \Box$. Indeed, the operator of an interacting field in momentum space is different from zero both for timelike and for spacelike, and also for isotropic and null, four-momenta.[5)] Actually it is only owing to this that the interaction of fields, their mutual interconversion, is possible.

As for the second invariant, the spin, in existing theories its spectrum of values for an interacting field is in practice artificially restricted by the fact that fields are used that transform according to finite-dimensional irreducible representations of the homogeneous Lorentz group.[6)]

Even with this restriction, however, as a rule the spin of the interacting field takes several values.[7)] The only fields that have just one spin value are the scalar and spinor fields.

[2)]This approach to the singling out of the spin 1 of a vector field has already been discussed in our papers [3-5].

[3)]In complete analogy with the fields with spins 0 and ½.

[4)]We also note that if we do not impose s. c. that single out one spin, then difficulties arise either with an indefinite metric or with keeping the energy positive.

[5)]In particular, there are nonzero masses for the interacting electromagnetic field. This causes the appearance of the Coulomb interaction.

[6)]If we drop this postulate (which is a purely mathematical one), then, for example, theories are conceivable in which the free field has only the spin 0 (or the spin ½) and the interacting field has a whole spectrum of spins.

[7)]Nevertheless we can assume that the wave functions of physical one-particle states transform according to an irreducible representation of the inhomogeneous group. We emphasize that all of the other quantum numbers (for example charge, isospin, parity, and so on) are the same for the free and interacting fields and are the same as for the corresponding one-particle states. Only the mass is necessarily smeared out.

The spectrum of spins of each field is simplest in theories of class A: as in the free case, this spectrum consists of only one value. Then each field will always be an eigenfunction of its operator $\hat{s}^2$, and in this sense we can say that the spin of each field is conserved.

6. All arguments relating to an interacting field can be made in the most practically convenient and mathematically correct way by dealing not with the field itself [say A(x)] but with the matrix elements

$$\langle 0 | A(x) | \Phi_j \rangle, \tag{6}$$

where Φ_j are physical states which are eigenfunctions of the operator of the square of the spin of the system of interacting particles (the total angular momentum in the center-of-mass system) (Section 3).

The diagram for this matrix element is shown in Fig. 2. By definition the field A(x) transfers those angular momenta j for which the matrix element (6) is different from zero. In theories of class A there is only one such angular momentum (one spin per field). We note that if a real particle of the field A(x) with definite spin were to decay into a system of particles in the state Φ_j the matrix element would be different from zero only for values of j equal to the spin of the free field.

It is obvious that the analyses of the diagrams of Figs. 1 and 2 lead to identical conclusions about the spin of the field, since a diagram of the type of Fig. 2 is a constituent part of the diagram of Fig. 1.

7. It will be shown in Section 4 that class A is not empty, and examples will be given of theories of particles with spin 1 which belong to this class. In the same place we shall indicate the close connection between the concept of spin in ordinary space and isotopic invariance.

8. As for zero masses, we shall here confine ourselves to the following remark. The restrictions on the matrix elements which we have found cannot be expressed in the operator form (14) or (17) if there are physical states with zero mass. In this case the interacting field will have a definite spin if the s.c. for it are satisfied in bracket expressions taken between physical states. This is the situation, for example, in the Fermi electrodynamics. In all gauge invariant theories of vector fields one can impose any restrictions on $\partial_\mu A_\mu$, since this quantity is quite arbitrary and is not determined by the equations of motion. The restriction $\langle \Psi_{\rm phys} | \partial_\mu A_\mu | \Psi_{\rm phys} \rangle = 0$ allows us to carry out the quantization by the Fermi method. Therefore gauge invariant theories of vector fields are theories which describe only quanta with spin 1. These questions are discussed in our previous papers,[3-5] of which the present paper is a further development.

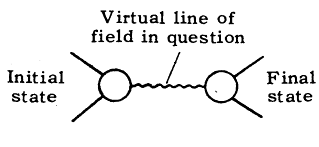

FIG. 1

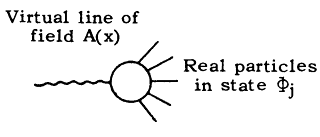

FIG. 2

2. HIGHER-SPIN FORMALISM BASED ON THE OPERATOR $\hat{s}^2$. FREE FIELDS

1. The use of the operator for the square of the spin is helpful even in the case of free fields, since it provides a single point of view for all the s.c. and unites them.

A field A transforms according to an irreducible representation of the inhomogeneous Lorentz group if

$$\Box A = m^2 A, \tag{7}$$

$$\hat{s}^2 A = s(s+1) A, \tag{8}$$

where m (the mass) and s (the spin) are fixed numbers.

2. Let us begin with integer spins. We shall work in the tensor formalism,[12] and to describe the spin s we shall use a tensor of the s-th rank, $\varphi_{\mu_1\mu_2\ldots\mu_s}$. For it the matrices $s_{\rho\sigma}$ in Eq. (5) are given by

$$\begin{aligned}(s_{\rho\sigma})_{\mu_1\ldots\mu_s;\,\nu_1\ldots\nu_s} = &-i(\delta_{\rho\mu_1}\delta_{\sigma\nu_1} - \delta_{\rho\nu_1}\delta_{\sigma\mu_1})\,\delta_{\mu_2\nu_2}\delta_{\mu_3\nu_3}\ldots\delta_{\mu_s\nu_s}\\ &- i\delta_{\mu_1\nu_1}(\delta_{\rho\mu_2}\delta_{\sigma\nu_2} - \delta_{\rho\nu_2}\delta_{\sigma\mu_2})\,\delta_{\mu_3\nu_3}\ldots\delta_{\mu_s\nu_s} - \ldots\\ &- i\delta_{\mu_1\nu_1}\delta_{\mu_2\nu_2}\ldots\delta_{\mu_{s-1}\nu_{s-1}}(\delta_{\rho\mu_s}\delta_{\sigma\nu_s} - \delta_{\rho\nu_s}\delta_{\sigma\mu_s}).\end{aligned} \tag{9}$$

Then the operator $\hat{s}^2$ can be written in the form

$$\begin{aligned}(\hat{s}^2)_{\mu_1\ldots\mu_s;\,\nu_1\ldots\nu_s} = &\tfrac{1}{2}(s_{\rho\sigma})_{\mu_1\ldots\mu_s;\,\lambda_1\ldots\lambda_s}(s_{\rho\sigma})_{\lambda_1\ldots\lambda_s;\,\nu_1,\ldots\nu_s}\\ &- p^{-2}(s_{\rho\tau})_{\mu_1\ldots\mu_s;\,\lambda_1\ldots\lambda_s}(s_{\rho\sigma})_{\lambda_1\ldots\lambda_s;\,\nu_1\ldots\nu_s}p_\tau p_\sigma.\end{aligned} \tag{10}$$

Since this representation is reducible, $\hat{s}^2$ is not a single-valued operator, and its spectrum of eigenvalues consists of the numbers n(n+1), where n = 0, 1, 2, . . . , s. We are interested in the eigenfunctions for the maximum eigenvalue:

$$(\hat{s}^2)_{\mu_1\ldots\mu_s;\,\nu_1\ldots\nu_s}\varphi_{\nu_1\ldots\nu_s} = s(s+1)\,\varphi_{\mu_1\ldots\mu_s}. \tag{11}$$

It is convenient to work in the p representation, in which, by Eq. (7), $p^2 = -m^2 \neq 0$, and go over to the rest system. In this system the operator (3b) for the square of the spin takes the form

$$\hat{s}^2 = \tfrac{1}{2} s_{rs}s_{rs} \qquad (r,\, s = 1, 2, 3) \tag{12}$$

and for the tensor field it can be written in the form

$$(\hat{s}^2)_{\mu_1\mu_2\ldots\mu_s;\,\nu_1\nu_2\ldots\nu_s} = 2(s-v)\,\delta_{\mu_1\nu_1}\delta_{\mu_2\nu_2}\ldots\delta_{\mu_s\nu_s} + \left\{\begin{bmatrix} -2(\delta_{\mu_1\mu_2}\delta_{\nu_1\nu_2}-\delta_{\mu_1\nu_2}\delta_{\mu_2\nu_1})\,\delta_{\mu_3\nu_3}\ldots\delta_{\mu_s\nu_s} & \text{for } \mu_1,\mu_2,\nu_1,\nu_2\neq 4 \\ 0, & \text{if any of the indices } \mu_1,\mu_2,\nu_1,\nu_2 \text{ is equal to } 4 \end{bmatrix} + \text{all terms obtained by different choices of the pairs } \mu_i\nu_i \text{ and } \mu_k\nu_k\right\}. \quad (13)$$

Here v is the number of pairs $(\mu_i\nu_i)$ in which at least one of the numbers is a four.

An analysis of Eq. (11) leads to the well known s.c.[13,2]:

$$\begin{aligned} &\text{a) } \varphi_{\mu_1\ldots\mu_s} - \text{ is a completely symmetric tensor,}\\ &\text{b) } \varphi_{\mu\mu\mu_3\ldots\mu_s} = 0,\\ &\text{c) } \partial_\mu\varphi_{\mu\mu_2\ldots\mu_s} = 0. \end{aligned} \quad (14)$$

3. Let us now consider half-integral spins in the γ formalism proposed by Rarita and Schwinger[13] and by Tamm (cf. [14,15]). In this formalism the spin $s = k + 1/2$ is described by a tensor of the k-th rank which has one spinor Dirac index α, $\psi_{\alpha\mu_1\ldots\mu_k}$, or, briefly, $\psi_{\mu_1\ldots\mu_k}$. For $\psi_{\mu_1\ldots\mu_k}$ the matrices $s_{\rho\sigma}$ are written in the form

$$(s_{\rho\sigma})_{\mu_1\ldots\mu_k;\,\nu_1\ldots\nu_k} = \tfrac{1}{2}\sigma_{\rho\sigma}\delta_{\mu_1\nu_1}\ldots\delta_{\mu_k\nu_k} - i(\delta_{\rho\mu_1}\delta_{\sigma\nu_1}-\delta_{\rho\nu_1}\delta_{\sigma\mu_1})\,\delta_{\mu_2\nu_2}\ldots\delta_{\mu_k\nu_k} - \ldots - i\delta_{\mu_1\nu_1}\delta_{\mu_2\nu_2}\ldots\delta_{\mu_{k-1}\nu_{k-1}}(\delta_{\rho\mu_k}\delta_{\sigma\nu_k}-\delta_{\rho\nu_k}\delta_{\sigma\mu_k}) \quad (15)$$

$[\sigma_{\rho\sigma} = -i(\gamma_\rho\gamma_\sigma - \delta_{\rho\sigma})]$. Again it is simpler to make the calculations in the rest system, in which

$$(\hat{s}^2)_{\mu_1\ldots\mu_k;\,\nu_1\ldots\nu_k} = [\tfrac{3}{4} + 2(k-v)]\,\delta_{\mu_1\nu_1}\ldots\delta_{\mu_k\nu_k} + \left\{\begin{bmatrix} -i\sigma_{\mu_1\nu_1} & \text{for } \mu_1,\nu_1\neq 4 \\ 0 & \text{for } \mu_1 \text{ or } \nu_1 \text{, equal to } 4\end{bmatrix}\delta_{\mu_2\nu_2}\ldots\delta_{\mu_k\nu_k} + \text{terms in which } \mu_1\nu_1 \text{ is replaced by } \mu_2\nu_2,\ \mu_3\nu_3\ldots,\mu_k\nu_k\right\} - \left\{\begin{bmatrix} -2(\delta_{\mu_1\mu_2}\delta_{\nu_1\nu_2}-\delta_{\mu_1\nu_2}\delta_{\mu_2\nu_1})\,\delta_{\mu_3\nu_3}\ldots\delta_{\mu_k\nu_k} & \text{for } \mu_1,\mu_2,\nu_1,\nu_2\neq 4\\ 0, & \text{if any of the indices } \mu_1,\mu_2,\nu_1,\nu_2 \text{ is equal to } 4\end{bmatrix} + \text{all terms obtained by different choices of the pairs } \mu_i\nu_i \text{ and } \mu_j\nu_j\right\}. \quad (16)$$

Here again v is the number of pairs $(\mu_i\nu_i)$ in which at least one of the numbers is a four. In the present case $\hat{s}^2$ has the eigenvalues $r(r+1)$, where $r = 1/2, 3/2, \ldots, k+1/2$. We again look for the eigenfunctions of s^2 with the maximum eigenvalue. An analysis of Eq. (11) in this case gives the well known s.c.[13,15,2]:

$$\begin{aligned} &\text{a) } \psi_{\mu_1\ldots\mu_k} - \text{is completely symmetric in } \mu_1\ldots\mu_k\\ &\text{b) } \gamma_{\mu_1}\psi_{\mu_1\mu_2\ldots\mu_k} = 0,\\ &\text{c) } \partial_{\mu_1}\varphi_{\mu_1\mu_2\ldots\mu_k} = 0. \end{aligned} \quad (17)$$

In the case of half-integral spins one ordinarily uses the Dirac equation instead of the Klein-Gordon equation (7).

Naturally a precisely similar treatment of the s.c. on the basis of the operator for the square of the spin is also possible for the field operators in the formalism of Gel'fand and Yaglom.[16]

3. INTERACTING FIELDS WITH HIGHER SPINS

1. We now go on to the analysis of interacting Heisenberg fields $\varphi_{\mu_1\ldots}$ and $\psi_{\mu_1\ldots}$ with arbitrary spins. Let Φ_{jP} be the state vector of a system of particles for which the total four-momentum is P_μ ($P^2 < 0$, $P_0 > 0$) and the spin (total angular momentum in the center-of-mass system) is j:

$$\hat{S}^2\Phi_{jP} = j(j+1)\,\Phi_{jP} \quad (18)$$

(the spin projection does not concern us). The operator $\hat{S}^2$ is of the form

$$\hat{S}^2 = \tfrac{1}{2}\hat{M}_{\rho\sigma}\hat{M}_{\rho\sigma} - \hat{P}^{-2}\hat{M}_{\rho\sigma}\hat{M}_{\rho\tau}\hat{P}_\sigma\hat{P}_\tau, \quad (19)$$

where $\hat{P}_\lambda$ and $\hat{M}_{\rho\sigma}$ are the generators of displacements and Lorentz rotations for the state vectors, and are the integrals of the motion for the system of interacting fields. It is well known that Lorentz invariance has as a consequence a connection between these operators and the operators p_λ and $m_{\rho\sigma}$ of Eqs. (4) and (5) for a field A(x):

$$p_\lambda A = [A, \hat{P}_\lambda], \quad (20)$$

$$m_{\rho\sigma}A = [A, \hat{M}_{\rho\sigma}]. \quad (21)$$

2. It is easy to see that the matrix elements

$$\langle 0|\varphi_{\mu_1\mu_2\ldots\mu_s}(x)|\Phi_{jP}\rangle, \quad \langle 0|\psi_{\mu_1\mu_2\ldots\mu_k}(x)|\Phi_{jP}\rangle, \quad (22)$$

are in general different from zero for $j = 0, 1, \ldots s$ and $j = 1/2, 3/2, \ldots k+1/2$, respectively.

In fact, when we use the translational invariance, namely the fact that

$$\langle 0|\varphi_{\mu_1\ldots\mu_s}(x)|\Phi_{jP}\rangle = e^{iPx}\langle 0|\varphi_{\mu_1\ldots\mu_s}(0)|\Phi_{jP}\rangle, \quad (23)$$

the relation (21), and the property of the vacuum $\hat{M}_{\rho\sigma}|0\rangle = 0$, we can convince ourselves that

$$(\hat{s}^2)_{\mu_1\ldots\mu_s;\,\nu_1\ldots\nu_s}\langle 0|\varphi_{\nu_1\ldots\nu_s}(x)|\Phi_{jP}\rangle = \langle 0|\varphi_{\mu_1\ldots\mu_s}(x)\,\hat{S}^2|\Phi_{jP}\rangle = j(j+1)\langle 0|\varphi_{\mu_1\ldots\mu_s}(x)|\Phi_{jP}\rangle. \quad (24)$$

By Eq. (23) the operators p_λ in $\hat{s}^2$ can be replaced by the c-numbers P_λ, and the problem of the eigenvalues of the operator $\hat{s}^2$ as applied to such matrix elements can be solved in exactly the same way as in the free case (see Section 2). In particular, the spectrum of the eigenvalues is of the form $n(n+1)$, where $n = 0, 1, \ldots, s$. It follows that $j = 0, 1, \ldots, s$. Similarly, for the case of half-integral spins we find $j = 1/2, 3/2, \ldots, k+1/2$.

The purpose of this paper is to obtain the conditions under which the field will transfer only one angular momentum, i.e., there will be only a single possible value of j. Exactly as in the free case we can choose for this single value the maximum value,[8)] namely s. For integer spins the conditions that single out the matrix element corresponding to this maximum value can be written

a) $\langle 0 | \varphi_{\mu_1\ldots\mu_s}(x) | \Phi_{jP}\rangle$ — is a completely symmetric tensor
b) $\langle 0 | \varphi_{\mu\mu\mu_3\ldots\mu_s}(x) | \Phi_{jP}\rangle = 0,$
c) $\langle 0 | \partial\mu_1 \varphi_{\mu_1\mu_2\ldots\mu_s}(x) | \Phi_{jP}\rangle = 0.$ (25)

Since the vectors Φ_{jP} for physical states form a complete system, on the basis of the hypotheses of locality, covariance, and positive definiteness and by the theorem of Federbush and Johnson[17,18] we can remove the brackets in the conditions (25). In other words, an interacting field which describes a single integer spin s must obey the same s.c. as in the free case, namely the s.c. (14).

Of course the converse is always true; the conditions (25) follow from the conditions (14).

Thus if and only if the interacting field $\varphi_{\mu_1\ldots\mu_s}(x)$ obeys the same s.c. as the free field is it possible (and indeed required) to say that it has a definite spin s.

By referring to the theorem of Federbush and Johnson we avoid having to consider the matrix elements[9)]

$$\langle \Phi_{j_1P_1} | \varphi_{\mu_1\ldots\mu_s}(x) | \Phi_{j_2P_2}\rangle = e^{i(P_2-P_1)x} \langle \Phi_{j_1P_1} | \varphi_{\mu_1\ldots\mu_s}(0) | \Phi_{j_2P_2}\rangle, \quad (26)$$

which would make the argument depend essentially on whether the vector $P_2 - P_1$ is timelike, isotropic, spacelike, or a null vector. These four possibilities correspond to four different classes of representations of the inhomogeneous Lorentz group, and each requires a special approach.

In the case of fields $\psi_{\mu_1\mu_2\ldots\mu_k}$ which have a Dirac spinor index in addition to the vector indices, analogous arguments lead to the conclusion that in order for a field $\psi_{\mu_1\ldots\mu_k}$ to describe the single spin value $k + 1/2$ it must obey the s.c. (17). Theories of class A are theories in which each field satisfies the conditions (14) or (17) which define the fields with a single spin value.

[8)]The other possibilities lead to equivalent representations for fields with a given spin, in which the Heisenberg operator will have more vector indices than are needed for the description of this spin.

[9)]In the case of a field with spin s these matrix elements are different from zero when $\mathbf{j}_1 + \mathbf{j}_2 + \mathbf{S} = 0$ (according to the usual rule for the addition of angular momenta).

[10)]Such equations are equivalent to the conditions that define irreducible representations of the inhomogeneous Lorentz group [for example, the conditions (7) and (8)].

4. THEORIES OF CLASS A

There are well known equations for free fields which contain within themselves all of the s.c. that single out one spin value.[10)] These include all of the equations of Gel'fand and Yaglom[16] in the tensor formalism, the Proca equation for spin 1, and the Rarita-Schwinger equation for spin $3/2$. We can also state such an equation for spin 2:

$$\tfrac{1}{2}(\Box\varphi_{\mu\nu} - \partial_\mu\partial_\lambda\varphi_{\lambda\nu} - \partial_\nu\partial_\lambda\varphi_{\mu\lambda}) + \tfrac{1}{2}(\Box\varphi_{\nu\mu} - \partial_\nu\partial_\lambda\varphi_{\lambda\mu} - \partial_\mu\partial_\lambda\varphi_{\nu\lambda}) + \delta_{\mu\nu}\partial_\lambda\partial_\rho\varphi_{\lambda\rho} - \tfrac{1}{3}(\delta_{\mu\nu}\Box - \partial_\mu\partial_\nu)\varphi_{\rho\rho} - m^2\varphi_{\mu\nu} = 0. \quad (27)$$

We know of the following theories with interaction which belong to class A:

1. The theory of a neutral vector field interacting with a conserved current (cf. e.g., [3]):

$$\Box A_\mu - \partial_\mu\partial_\nu A_\nu - m^2 A_\mu = -j_\mu, \quad \partial_\mu j_\mu = 0. \quad (28)$$

2. The Yang-Mills theory, and all of its generalizations as indicated by Gell-Mann and Glashow[20]:

$$\partial_\mu G^i_{\mu\nu} - m^2 A^i_\nu = -j^i_\nu, \qquad \partial_\mu j^i_\mu = 0, \quad (29)$$

where

$$j^i_\nu = 2\gamma_0 [i\bar{\psi}\gamma_\nu M^i\psi + c_{ikj} G^j_{\nu\mu} A^k_\mu]$$

(the notation is the same as in [20]). When the conservation of the currents is taken into account Eqs. (28) and (29) each have as a consequence the supplementary condition (1), which singles out spin 1.[4,5] Thus class A is not an empty class.

It is also natural to state the converse question: how to include an interaction in such a way that the theory will belong to class A, i.e., so that in the theory not only the fields with spin 0 and $1/2$, but also the other fields will have definite spin. In the case of fields with spin 1 we have solved this converse problem[21] and have shown that the only theories of class A with dimensionless coupling constants are those in which the equations of motion are of the form (29). In other words, the only cases in which it is possible to include an interaction with a vector field in such a way that the spin of this field is 1 are the following:

a) when the vector field is a neutral field;

b) when three vector fields form an isotopic triplet and the theory as a whole is isotopically invariant;

c) in the case of symmetries of higher orders.

Possibility b) brings out a deep connection between isotopic invariance and the concept of spin in ordinary space.

We are carrying out a similar analysis for the higher spins.

We emphasize that there exist interactions which definitely do not belong to class A. First among such cases is the electromagnetic interaction of charged particles with higher spins. This interaction violates isotopic invariance, and therefore does not allow us to assign a definite spin to an interacting field with spin 1 or higher.

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