POLARIZATION PHENOMENA IN THE COMPTON SCATTERING ON A MOVING ELECTRON AND THE POSSIBILITY OF OBTAINING BEAMS OF POLARIZED PHOTONS

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The photon polarization resulting from Compton scattering of soft photons on relativistic electrons is considered. It is shown that the degree of polarization of such photons can approach 100% both in the case of photons scattered under a given azimuthal angle and in the case where the polarization state is averaged over this angle. Consequently Compton scattering on relativistic electrons can serve as an efficient method for obtaining polarized γ quanta.

As has been shown in [1], soft photons Compton-scattered on relativistic electrons can possess considerable energies, comparable to the energy of the electrons. This makes it possible to obtain high-energy photons by scattering intense photon fluxes by beams from an electron accelerator. It has been shown that the number of γ quanta with energies close to the maximal value in the energy spectrum of secondary photons is relatively large, in distinction from the case of bremsstrahlung, which is usually utilized for the production of beams of high energy γ quanta. It has also been noted that the γ quanta obtained in this manner are polarized.

We consider below the peculiarities of the polarization properties of such γ quanta. This presents a certain interest, since the polarization of the primary photons can be chosen in any predetermined manner, for instance by using as primary photons the radiation produced by optical quantum generators (lasers).

The Stokes parameters of the primary photons ($\xi_i^{(1)}$) and of the secondary photons ($\xi_i^{(2)}$) for Compton-scattering on a stationary electron can be expressed in terms of the unit vectors:

$$\chi_1^{(1)} = [\mathbf{k}_1\mathbf{k}_2]/|[\mathbf{k}_1\mathbf{k}_2]|, \qquad \chi_2^{(1)} = [\mathbf{k}_1\chi_1^{(1)}]/|\mathbf{k}_1|; \qquad (1)^*$$

$$\chi_1^{(2)} = [\mathbf{k}_1\mathbf{k}_2]/|[\mathbf{k}_1\mathbf{k}_2]|, \qquad \chi_2^{(2)} = [\mathbf{k}_2\chi_1^{(2)}]/|\mathbf{k}_2|, \qquad (2)$$

in the following manner [2]:

$$\xi_1^{(2)} = F^{-1}[\sin^2\theta' + (1+\cos^2\theta')\,\xi_1^{(1)}],$$

$$\xi_2^{(2)} = 2F^{-1}\xi_2^{(1)}\cos\theta',$$

$$\xi_3^{(2)} = F^{-1}[(\omega_1'/\omega_2' + \omega_2'/\omega_1' - 2)\,\xi_2^{(1)} + 2\xi_3^{(1)}]\cos\theta'. \qquad (3)$$

*$[\mathbf{k}_1\mathbf{k}_2] = \mathbf{k}_1 \times \mathbf{k}_2$.

In Eqs. (1)—(3) $\mathbf{k}_1$ and $\mathbf{k}_2$ are the wave-vectors of the incident and scattered photon, respectively, ω_1' ω_2' are the corresponding frequencies, and θ' is the angle between the directions of these photons. The function F has the form:

$$F = \omega_1'/\omega_2' + \omega_2'/\omega_1' + (\xi_1^{(1)} - 1)\sin^2\theta'; \qquad (3a)$$

$\xi_1^{(1)}$ and $\xi_2^{(1)}$ determine the probabilities of two linear polarizations forming an angle of $\pi/4$, and $\xi_3^{(1)}$ determines the circular polarization. Let us consider the polarization properties of the photons in a fixed coordinate system x, y, z (the z-axis is taken along the momentum of the incident photon). The polarization of the incident photons is considered known in terms of these fixed unit vectors and we express the Stokes parameters $\xi_i^{(1)}$ in the form [3]

$$\xi_1^{(1)} = P\sin\beta\cos 2\alpha,$$

$$\xi_2^{(1)} = P\sin\beta\sin 2\alpha,$$

$$\xi_3^{(1)} = P\cos\beta, \qquad (4)$$

where P denotes the degree of polarization of the incident photons.

Under a rotation by an angle φ around the z axis, β remains unchanged and α is increased by φ. Therefore in terms of the unit vectors $\chi_i^{(1)}$ the $\xi_i'^{(1)}$ will have the following expressions as functions of the $\xi_i^{(1)}$:

$$\xi_1'^{(1)} = \xi_1^{(1)}\cos 2\varphi - \xi_2^{(1)}\sin 2\varphi,$$

$$\xi_2'^{(1)} = \xi_1^{(1)}\sin 2\varphi + \xi_2^{(1)}\cos 2\varphi, \qquad \xi_3'^{(1)} = \xi_3^{(1)}. \qquad (5)$$

Using the expressions for $\xi_i'^{(1)}$ and taking into account the equations (3) we obtain the following expressions of the Stokes parameters $\xi_i'^{(2)}$ in terms of the unit vectors $\chi_i^{(2)}$:

$$\xi_1'^{(2)} = (F')^{-1}[\sin^2\theta' + \xi_1^{(1)}\cos 2\varphi\,(1+\cos^2\theta') - \xi_2^{(1)}\sin 2\varphi\,(1+\cos^2\theta')],$$

$$\xi_2'^{(2)} = 2(F')^{-1}\cos\theta'\,(\xi_1^{(1)}\sin 2\varphi + \xi_2^{(1)}\cos 2\varphi),$$

$$\xi_3'^{(2)} = \left[\left(\frac{\omega_1'}{\omega_2'}+\frac{\omega_2'}{\omega_1'}-2\right)\xi_1^{(1)}\sin 2\varphi + \left(\frac{\omega_1'}{\omega_2'}+\frac{\omega_2'}{\omega_1'}-2\right)\xi_2^{(1)}\cos 2\varphi + 2\xi_3^{(1)}\right]\frac{\cos\theta'}{F'}, \tag{6}$$

where, by Eqs. (3a) and (5), F′ is of the form

$$F' = \frac{\omega_1'}{\omega_2'}+\frac{\omega_2'}{\omega_1'} + (\xi_1^{(1)}\cos 2\varphi - \xi_2^{(1)}\sin 2\varphi - 1)\sin^2\theta'. \tag{6a}$$

If the primary photon and the electron move at angles $\theta = 0,\pi$ with respect to each other before colliding, the polarization of the scattered photons, determined by the parameters $\xi_i'^{(2)}$ in terms of the unit vectors (2) in the rest system of the incident electron, will remain unchanged in terms of the same unit vectors in the laboratory system (l.s.), provided the frequencies ω_1' and ω_2' and the angle θ' are expressed in the rest system of the electron. This can be verified in the following manner. It is easy to observe that the direction of the electrical vector in the state $\chi_1^{(2)}$ does not change under the transformation from the rest system of the electron to the l.s. If the polarization state of the scattered photon is such that its electric vector is directed along the vector $\chi_2^{(2)}$, it is easy to show by performing a Lorentz transformation, that in the l.s. this vector will be directed along the new unit vector $\chi_2^{(2)}$ (which is orthogonal to $\chi_1^{(1)}$ and $k_{2\,\mathrm{lab}}$). Since any polarization state can be represented as a superposition of these two orthogonal states with invariant amplitudes, it is obvious that the parameters $\xi_i'^{(2)}$ remain unchanged when going over to the l.s. Similarly, the parameters for the initial photon remain unchanged. Thus we arrive at the conclusion, that Eqs. (6) give the polarization parameters of the scattered photon in the laboratory system (in terms of unit vectors fixed in the scattering plane).

In the rest system of the electron the angle θ' is connected with the l.s. energy ω_2 of the scattered photons through the relation ($\hbar = c = 1$)

$$\omega_2 = \omega_1 \times \frac{1 - v_1\cos\theta_1 + v_1(\cos\theta_1 - v_1)\cos\theta' + v_1\sqrt{1-v_1^2}\sin\theta_1\sin\theta'}{1 - v_1^2 + (\omega_1/\varepsilon_1)(1-\cos\theta')(1-v_1\cos\theta_1)}, \tag{7}$$

where v_1 and ϵ_1 are the velocity and energy of the electron, θ_1 is the angle between the directions of the primary photon and electron, ω_1 is the energy of the incident photon in the l.s. For $\theta_1 = \pi$ this relation becomes

$$\cos\theta' = \left\{\omega_1\left(1-\frac{\omega_2}{\varepsilon_1}\right) - \frac{1}{1+v_1}\left(\frac{m}{\varepsilon_1}\right)^2\omega_2\right\}\Big/\,\omega_1\left(v_1 - \frac{\omega_2}{\varepsilon_1}\right). \tag{8}$$

Using this relation between θ_2' and ω_2 one can obtain the dependence of the degree of polarization of the scattered γ quanta on their energy. One must take it into account that

$$\omega_1' = \omega_1(1 - v_1\cos\theta_1)/\sqrt{1-v_1^2}, \tag{9}$$

$$\omega_2' = \omega_1'/[1 + (\omega_1'/m)(1-\cos\theta')]. \tag{10}$$

It follows from (6) that by varying the polarization state of the incident photons one can obtain for certain angles φ a sufficiently large degree of polarization of the desired type.

The experimental conditions often do not permit to separate scattered photons emitted under certain azimuthal angles φ. For such practically important cases it is necessary to know the average over all possible φ of the polarization of the beam of scattered photons. For γ quanta emitted within small angles θ_2 of the z axis one may choose the polarization unit vectors along the fixed x and y axes. One must take into account the fact that for Compton scattering on relativistic electrons, the angles θ_2 are very small over practically the whole interval of frequencies, with the exception of the frequencies $\omega_2 \approx \omega_1$. In order to carry out the indicated averaging over φ, we express the Stokes parameters $\xi_i'^{(2)}$ of the scattered γ quanta in terms of the fixed set of unit vectors. For this it is necessary to rotate the unit vectors $\chi_i^{(2)}$ by an angle $\pm\varphi$. Thus

$$\xi_1^{(2)} = \xi_1'^{(2)}\cos 2\varphi \mp \xi_2'^{(2)}\sin 2\varphi,$$

$$\xi_2^{(2)} = \pm\xi_1'^{(2)}\sin 2\varphi + \xi_2'^{(2)}\cos 2\varphi, \qquad \xi_3^{(2)} = \xi_3'^{(2)}. \tag{11}$$

The $\xi_i^{(2)}$ will then have the following form

$$\xi_1^{(2)} = (F')^{-1}\left\{\xi_1^{(1)}[\cos^2 2\varphi\,(1\pm\cos\theta')^2 \mp 2\cos\theta'] - \tfrac{1}{2}\xi_2^{(1)}(1\pm\cos\theta')^2\sin 4\varphi + \cos 2\varphi\sin^2\theta'\right\},$$

$$\xi_2^{(2)} = (F')^{-1}\left\{-\tfrac{1}{2}\xi_1^{(1)}(1-\cos\theta')^2\sin 4\varphi + \xi_2^{(1)}[\mp 2\cos\theta' + \sin^2 2\varphi\,(1\pm\cos\theta')^2] - \sin 2\varphi\sin^2\theta'\right\},$$

$$\xi_3^{(2)} = (F')^{-1}\cos\theta'\,\{\xi_1^{(1)}\sin 2\varphi\,(\omega_1'/\omega_2' + \omega_2'/\omega_1' - 2) + \xi_2^{(1)}\cos 2\varphi\,(\omega_1'/\omega_2' + \omega_2'/\omega_1' - 2) + 2\xi_3^{(1)}\}. \tag{12}$$

After averaging the $\xi_i^{(2)}$ over all φ with a weight factor proportional to the scattering cross section, we obtain

$$\bar{\xi}_1^{(2)} = \frac{(1 \mp \cos\theta')^2}{2(\omega_1'/\omega_2' + \omega_2'/\omega_1' - \sin^2\theta')}\xi_1^{(1)},$$

$$\bar{\xi}_2^{(2)} = \frac{(1 \mp \cos\theta')^2}{2(\omega_1'/\omega_2' + \omega_2'/\omega_1' - \sin^2\theta')}\xi_2^{(1)},$$

$$\bar{\xi}_3^{(2)} = \frac{2\cos\theta'}{\omega_1'/\omega_2' + \omega_2'/\omega_1' - \sin^2\theta'}\xi_3^{(1)}. \quad (13)$$

The degree of polarization $\bar{P}$ of the scattered photons will be

$$\bar{P}^2 = \frac{0.25(1 \mp \cos\theta')^4(\xi_1^{2(1)} + \xi_2^{2(1)}) + 4\cos^2\theta'}{(\omega_1'/\omega_2' + \omega_2'/\omega_1' - \sin^2\theta')^2}. \quad (14)$$

It follows from Eq. (13) that one can obtain beams of hard γ quanta with a high degree of polarization (going up to 100%). The type of polarization of these γ quanta can be varied by varying the polarization state of the primary photons.

The availability of highly polarized γ-quanta allows one to obtain polarized electrons and positrons from their interactions with matter (for example by Compton scattering[3] or pair production[4]), and also to produce other polarized particles.

Undoubtedly the availability of beams of polarized γ quanta (which moreover possess an energy spectrum of very convenient form[1]) is of interest to the solution of a variety of physical problems, such as photoproduction, nuclear photodisintegration, and similar processes.

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