IONIZATION LOSSES IN AN INHOMOGENEOUS MEDIUM

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A general expression is obtained for the ionization losses of a fast particle in a medium with statistical inhomogeneities. The results are applied to the case of discrete energy losses, longitudinal losses in a plasma, and radiation from density fluctuations.

SEVERAL recently published papers are devoted to an analysis of the radiation from a fast particle in a medium with statistical inhomogeneities. In all these papers, however, only the radiation losses were calculated, and no full account was taken of the ionization losses. We develop below a general method for the analysis of ionization losses in a statistically inhomogeneous medium. Applications of the method to specific problems are limited to those cases in which the result can be obtained within the framework of the classical theory if the dielectric constant of the homogeneous medium is specified. Effects that call for a quantum approach will be considered separately.

1. DECELERATION FORCE AT A GIVEN POINT

In an inhomogeneous medium, the dielectric constant, together with the deceleration force, varies from point to point so that the rate of energy loss $d\epsilon/dt$ depends on the coordinate of the particle. For fast particles, as a rule, we can neglect deviations from linear motion of the particle, and we can write at the point $\mathbf{x} = \mathbf{v}t$

$$d\varepsilon/dt = e\mathbf{v}\mathbf{E}(\mathbf{v}t, t) = e\int d\omega \int d\mathbf{k}\mathbf{v}\mathbf{E}(\omega, \mathbf{k})\, e^{it(\mathbf{k}\mathbf{v}-\omega)}. \quad (1.1)$$

In order to find the deceleration of the particle it is necessary to express in (1.1) the electric field in terms of the current density of the charged particle and the dielectric constant of the medium. To take into account media with intrinsic dispersion, it is convenient to include all terms that result from the averaging of the microcurrents in the determination of the electrical induction, writing down Maxwell's equations in the form used by Landau and Lifshitz [1], Sec. 83.

For inhomogeneities that are stationary or slowly varying (over the time of flight of the charge) the most general connection between the induction and the field $\mathbf{E}$ is

$$D_i(\mathbf{r}, t) = \int_0^\infty d\tau \int d\mathbf{r}'\tilde{\varepsilon}_{is}(\tau, \mathbf{r}', \mathbf{r})\, E_s(t-\tau, \mathbf{r}-\mathbf{r}'). \quad (1.2)$$

The dependence of $\tilde{\epsilon}_{is}$ on $\mathbf{r}'$ describes the spatial dispersion and occurs in both homogeneous and inhomogeneous media. The dependence of $\tilde{\epsilon}_{is}$ on $\mathbf{r}$ is completely due to the presence of inhomogeneities and vanishes in a homogeneous medium.

Transformation to Fourier components yields

$$D_i(\mathbf{k}, \omega) = (2\pi)^{-3}\int d^3\mathbf{l}\,\varepsilon_{is}(\omega; \mathbf{k}, \mathbf{k}-\mathbf{l}, \mathbf{l})\, E_s(\omega, \mathbf{k}-\mathbf{l}). \quad (1.2')$$

In a homogeneous medium $\epsilon_{is}(\omega, \mathbf{k}, \mathbf{k}-\mathbf{l}, \mathbf{l}) = \epsilon_{is}^{(0)}(\mathbf{k}, \omega)\,\delta(\mathbf{l})\,(2\pi)^3$. The dependence of ϵ_{is} on $(\mathbf{k}-\mathbf{l})$ is essentially connected with the presence of inhomogeneities, and corresponds to the possible change in the momentum of an electromagnetic wave scattered by inhomogeneities. To change over to a medium in which the spatial dispersion is negligible, it is sufficient to leave out the dependence of ϵ_{is} on $\mathbf{k}$.

From Maxwell's equations and (1.2′) follows an equation for the vector potential

$$(k^2\delta_{ss'} - k_s k_{s'})\, A_{s'}(\omega, \mathbf{k}) - \omega^2 \int d\mathbf{l}\,\varepsilon_{ss'}(\omega, \mathbf{k}, \mathbf{k}-\mathbf{l}, \mathbf{l})\, A_{s'}(\omega, \mathbf{k}-\mathbf{l}) = 4\pi j_s(\omega, \mathbf{k}), \quad (1.3)$$

where a Coulomb gauge is used for the potential ($\varphi = 0$) and the system of units is such that $\hbar = c = 1$.

We introduce the retarded Green's function $D_{ss'}(\mathbf{k}, \mathbf{k}', \omega)$ of the electromagnetic field in an inhomogeneous medium with the aid of the relation

$$A_i(\mathbf{k}, \omega) = \int d\mathbf{k}'\, D_{is}(\mathbf{k}, \mathbf{k}', \omega)\, j_s(\mathbf{k}', \omega). \quad (1.4)$$

Substituting (1.4) in (1.3) we obtain an integral equation for $D_{ss'}$:

$$(k^2\delta_{is} - k_ik_s) D_{ss'}(\mathbf{k}, \mathbf{k}', \omega) - \omega^2 \int d\mathbf{l}\,\varepsilon_{is}(\omega, \mathbf{k}, \mathbf{k}-\mathbf{l}, \mathbf{l}) D_{ss'}(\mathbf{l}, \mathbf{k}', \omega) = 4\pi\delta\,\delta_{is}(\mathbf{k}-\mathbf{k}'). \quad (1.5)$$

From (1.1) and (1.4), recognizing that $\mathbf{E}(\mathbf{k}, \omega) = i\omega\,\mathbf{A}(\mathbf{k}, \omega)$, we can obtain

$$\frac{d\varepsilon}{dt} = -\operatorname{Im} e \int \omega d\omega \int d\mathbf{k} \int d\mathbf{k}' v_s D_{ss'}(\mathbf{k}, \mathbf{k}', \omega)\, j_{s'}(\mathbf{k}', \omega)\, e^{it(\mathbf{kv}-\omega)}.$$

For a charge that moves uniformly in a straight line

$$\mathbf{j}(\mathbf{k}, \omega) = e\mathbf{v}\,(2\pi)^{-3}\,\delta(\omega - \mathbf{kv}),$$

and the formula for $d\epsilon/dt$ becomes

$$\frac{d\varepsilon}{dt} = -\frac{e^2}{(2\pi)^3}\int \omega d\omega \int d\mathbf{k} \int d\mathbf{k}'\delta(\omega - \mathbf{kv})\, v_s v_{s'} \times \operatorname{Im} D_{ss'}(\mathbf{k}, \mathbf{k}', \omega)\, e^{it(\omega - \mathbf{kv})}. \quad (1.6)$$

Thus, calculation of the ionization losses reduces to a calculation of the Green's function of the electromagnetic field $D_{ss'}(\mathbf{k}, \mathbf{k}', \omega)$.

In a homogeneous medium, the Green's function is of the form $D^{(0)}_{ss'}(\mathbf{k}, \omega)\,\delta(\mathbf{k} - \mathbf{k}')$, the dependence on t disappears from (1.6), and the energy loss is proportional to the time of flight. Substituting in (1.6)

$$D^{(0)}_{is}(\mathbf{k}, \omega) = 4\pi(\delta_{is} - k^{-2}k_ik_s)(k^2 - \omega^2\varepsilon_0^t(k, \omega))^{-1} - 4\pi k^{-2}k_ik_s(\omega^2\varepsilon_0^l(k, \omega))^{-1},$$

we readily obtain the usual formulas for ionization losses [2]. Formula (6) can be used to calculate the ionization losses in a specified region of an inhomogeneous medium.

2. ENERGY LOSSES IN A STATISTICALLY INHOMOGENEOUS MEDIUM

In a medium with random inhomogeneities, interest attaches to the energy losses averaged over the distribution of the inhomogeneities. Since the dependence of (1.6) on the inhomogeneities is contained only in $D_{ss'}(\mathbf{k}, \mathbf{k}', \omega)$, the problem reduces to a determination of the Green's function $\langle D_{ss'}(\mathbf{k}, \mathbf{k}', \omega)\rangle$ averaged over the distribution of the inhomogeneities.

We consider a substance which is in the mean homogeneous and in which no preferred direction remains after averaging over the distribution of the inhomogeneities, and we assume that the inhomogeneities are small. It follows therefore that the average electron density $\langle n(\mathbf{r})\rangle = n_0$ does not depend on the coordinates, and the deviations $\delta n(\mathbf{r}) = n(\mathbf{r}) - n_0$ of the electron density from the mean are small.

Equation (1.2′) takes into account in an inhomogeneous medium the scattering of the electromagnetic wave by the inhomogeneities, with the momentum $\mathbf{l}$ transferred to the inhomogeneity upon scattering. Therefore, when the density deviations from the mean are small, we can represent $\epsilon_{ij}(\mathbf{k}, \mathbf{k}-\mathbf{l}, \mathbf{l}; \omega)$ in the form

$$\varepsilon_{ij}(\mathbf{k}, \mathbf{k}-\mathbf{l}, \mathbf{l}; \omega) = \varepsilon^{(0)}_{ij}(\mathbf{k}, \omega)(2\pi)^3\delta(\mathbf{l}) + \delta n(\mathbf{l})\,\alpha_{ij}(\mathbf{k}, \mathbf{k}-\mathbf{l}; \omega), \quad (2.1)$$

where $a_{ij}(\mathbf{k}, \mathbf{k}-\mathbf{l}; \omega)$ is some function determined by the properties of the medium.

Using the foregoing and (1.7), we can rewrite (1.5) in a more convenient form

$$D_{is}(\mathbf{k}, \mathbf{k}', \omega) = D^{(0)}_{is}(\mathbf{k}, \omega)\,\delta(\mathbf{k}-\mathbf{k}') - D^{(0)}_{ij}(\mathbf{k}, \omega)\int d\mathbf{l}\,\xi_{jj'}(\mathbf{k}, \mathbf{k}-\mathbf{l}, \omega)\,\delta n(\mathbf{l})\, D_{j's}(\mathbf{k}-\mathbf{l}, \mathbf{k}', \omega),$$

$$\delta n(\mathbf{l}) = \int d\mathbf{r}\,\delta n(\mathbf{r})\, e^{-i\mathbf{lr}},$$

$$\xi_{jj'}(\mathbf{k}, \mathbf{k}-\mathbf{l}, \omega) = \frac{\omega^2}{4\pi}\alpha_{jj'}(\mathbf{k}, \mathbf{k}-\mathbf{l}, \omega). \quad (2.2)$$

From (2.2) we readily obtain an equation for the averaged Green's function $\langle D_{is}(\mathbf{k}, \mathbf{k}', \omega)\rangle$ for which we integrate this equation and average the result over the inhomogeneity distribution. This gives rise to the average of the product $\langle\delta n(\mathbf{l})\,\delta n(\mathbf{l}')\,D_{s's}(\mathbf{k}-\mathbf{l}-\mathbf{l}', \mathbf{k}', \omega)\rangle$, which differs from $\langle\delta n(\mathbf{l})\,\delta n(\mathbf{l}')\rangle\langle D_{s's}(\mathbf{k}-\mathbf{l}-\mathbf{l}', \mathbf{k}', \omega)\rangle$ by terms of the third and higher order in δn. In view of the smallness of δn, this difference can be neglected, so that we get an equation for $\langle D_{is}(\mathbf{k}, \mathbf{k}', \omega)\rangle$.

If no preferred direction remains after averaging over the inhomogeneities in the medium, then $\langle\delta n(\mathbf{l})\,\delta n(\mathbf{l}')\rangle$ should be proportional to $\delta(\mathbf{l}+\mathbf{l}')$, and the averaged Green's function should be of the form

$$\langle D_{is}(\mathbf{k}, \mathbf{k}', \omega)\rangle = D_{is}(\mathbf{k}, \omega)\,\delta(\mathbf{k}-\mathbf{k}') = \{(\delta_{is} - k^{-2}k_ik_s)D^t(k, \omega) + k^{-2}k_ik_sD^l(k, \omega)\}\,\delta(\mathbf{k}-\mathbf{k}'). \quad (2.3)$$

In the averaging, the term $\langle\delta n(\mathbf{r})\,\delta n(\mathbf{r}')\rangle$ is equal to

$$\langle\delta n(\mathbf{r})\,\delta n(\mathbf{r}')\rangle = n_0\delta(\mathbf{r}-\mathbf{r}') + n_0\nu(\mathbf{r}-\mathbf{r}').$$

The effects arising when $\mathbf{r} = \mathbf{r}'$ have no bearing on the inhomogeneities of the medium and consequently should be taken into account in the calculation of the dielectric constant of the homogeneous medium [3].

Thus

$$\langle\delta n(\mathbf{l})\,\delta n(\mathbf{l}')\rangle = \varphi(\mathbf{l})\,\delta(\mathbf{l}+\mathbf{l}'),$$

$$\varphi(\mathbf{l}) = n_0(2\pi)^3\int d\mathbf{r}\,\nu(\mathbf{r})\, e^{-i\mathbf{l}r}, \quad (2.4)$$

and $\nu(\mathbf{r})$ is the fluctuation correlation function.

Taking the foregoing into account, the integral equation for the averaged Green's function reduces to an algebraic equation

$$D_{is}(\mathbf{k},\omega)=D_{is}^{(0)}(\mathbf{k},\omega)+D_{ij}^{(0)}(\mathbf{k},\omega)\int d\mathbf{l}\,\varphi(\mathbf{l})\times\xi_{jj'}(\mathbf{k},\mathbf{k}-\mathbf{l},\omega)D_{j's'}^{(0)}(\mathbf{k}-\mathbf{l},\omega)\,\xi_{s's''}(\mathbf{k}-\mathbf{l},\mathbf{k},\omega)D_{s''s}(\mathbf{k},\omega),$$

from which we readily obtain

$$D_{is}(\mathbf{k},\omega)=4\pi\left\{\frac{\delta_{is}-k_ik_sk^{-2}}{k^2-\omega^2(\varepsilon_0^t(k,\omega)+\Delta^t(k\omega))}-\frac{k_ik_sk^{-2}}{\omega^2(\varepsilon_0^l(k,\omega)+\Delta^l(k,\omega))}\right\},\tag{2.5}$$

where $\Delta^l(k,\omega)$ and $\Delta^t(k,\omega)$ do not depend on the direction of $\mathbf{k}$ and are determined by the formulas

$$\Delta^l(k,\omega)=(2\pi)^{-3}\int d\mathbf{q}\,\varphi(q)\frac{k_sk_\beta}{k^2}\omega^2\alpha_{\beta j}(\mathbf{k},\mathbf{k}-\mathbf{q},\omega)\times\left[\frac{\delta_{j\delta}-(\mathbf{k}-\mathbf{q})_j(\mathbf{k}-\mathbf{q})_\delta/(\mathbf{k}-\mathbf{q})^2}{(\mathbf{k}-\mathbf{q})^2-\omega^2\varepsilon_0^t(\mathbf{k}-\mathbf{q},\omega)}-\frac{(\mathbf{k}-\mathbf{q})_j(\mathbf{k}-\mathbf{q})_\delta/(\mathbf{k}-\mathbf{q})^2}{\omega^2\varepsilon_0^l(\mathbf{k}-\mathbf{q},\omega)}\right]\alpha_{\delta s}(\mathbf{k}-\mathbf{q},k,\omega),$$

$$\Delta^t(k,\omega)=(2\pi)^{-3}\frac{1}{2}\int d\mathbf{q}\,\varphi(q)\left(\delta_{s\beta}-\frac{k_sk_\beta}{k^2}\right)\omega^2\alpha_{\beta j}(\mathbf{k},\mathbf{k}-\mathbf{q},\omega)\times\left[\frac{\delta_{j\delta}-(\mathbf{k}-\mathbf{q})_j(\mathbf{k}-\mathbf{q})_\delta/(\mathbf{k}-\mathbf{q})^2}{(\mathbf{k}-\mathbf{q})^2-\omega^2\varepsilon_0^t(\mathbf{k}-\mathbf{q},\omega)}-\frac{(\mathbf{k}-\mathbf{q})_j(\mathbf{k}-\mathbf{q})_\delta/(\mathbf{k}-\mathbf{q})^2}{\omega^2\varepsilon_0^l(\mathbf{k}-\mathbf{q},\omega)}\right]\alpha_{\delta s}(\mathbf{k}-\mathbf{q},\mathbf{k},\omega).\tag{2.6}$$

Substituting (2.6) in (1.6), we easily obtain the final formula for the energy losses averaged over the inhomogeneity distribution

$$\left\langle\frac{d\varepsilon}{dt}\right\rangle=-\,\mathrm{Im}\,\frac{e^2}{2\pi^2}\int\omega d\omega\int\frac{d\mathbf{k}}{k^2}\delta(\omega-\mathbf{kv})\times\left\{\frac{k^2v^2-\omega^2}{k^2-\omega^2(\varepsilon_0^t(k,\omega)+\Delta^t(k,\omega))}-\frac{1}{\varepsilon_0^l(k,\omega)+\Delta^l(k,\omega)}\right\}.\tag{2.7}$$

Formulas (2.5) and (2.7) were obtained by expanding the kernel of the integral equation (2.2) in powers of the small deviation of the density from the equilibrium value. In other words, terms of the third degree in δn were discarded from the kernel of the integral equation. Therefore (2.5) and (2.7) contain in the denominators a correction Δ which is brought about by the account of the density fluctuations. It is obvious that the influence of the fluctuations can be appreciable only in the region of frequencies where $\epsilon^l(\omega,\mathrm{k})$ or $\mathrm{k}^2-\omega^2\epsilon^t(\omega,\mathrm{k})$ is small. Outside this region we can confine ourselves to the first term of the expansion in Δ or carry out the expansion directly without resorting to the integral equation[4,5]. The frequency region of importance to ionization losses is precisely the one where $\epsilon(\omega)\to 0$ and the expansion in Δ is impossible. For radiation of transverse waves below the threshold of Cerenkov radiation, expansion in Δ becomes possible and leads to the previously obtained results[4,5] (Sec. 5).

As is well known, the imaginary and real parts of $\epsilon_0^t(\mathrm{k},\omega)$ and $\epsilon_0^l(\mathrm{k},\omega)$ are respectively odd and even functions of the frequency. Using (2.6), we can show that analogous properties are possessed by the imaginary and real parts of the functions $\Delta^l(\mathrm{k},\omega)$ and $\Delta^t(\mathrm{k},\omega)$. This makes it possible to confine the integration in (2.7) only to the region of positive frequencies by integrating over the parallel v-component of the wave vector $\mathbf{k}$ and introducing the vector $\mathbf{q}=\mathbf{k}-\mathbf{v}(\omega/v^2)$, which is the component of the vector $\mathbf{k}$ perpendicular to the velocity.

It is also convenient to divide (2.7) by the particle velocity v, and thus change over to the average energy loss per unit path, which with account of the statements made above can be obtained in the form

$$\left\langle\frac{d\varepsilon}{dx}\right\rangle=\int_0^\infty I(\omega)\,d\omega,$$

where $I(\omega)$ is the spectral density of the average energy loss per unit path

$$I(\omega)=\frac{2e^2\omega}{\pi v^2}\int_0^\infty\frac{q\,dq}{q^2+(\omega/v)^2}\times\mathrm{Im}\left\{\frac{q^2v^2}{q^2+(\omega/v)^2(1-v^2(\varepsilon_0^t+\Delta^t))}-\frac{1}{\varepsilon_0^l+\Delta^l}\right\},\tag{2.8}$$

where ϵ_0^t, ϵ_0^l, Δ^t, and Δ^l are functions of the variables ω and $[q^2+(\omega/v)^2]^{1/2}$. When $\Delta^t=\Delta^l=0$ formula (2.8) goes over into the well known formula for the spectral density of the energy loss in a homogeneous medium[1,2].

3. DISCRETE ENERGY LOSSES IN THIN FILMS

As is well known, on passing through thin metallic plates a charged particle loses energy in discrete batches, corresponding to the excitation of the collective oscillations of the electrons in the metal. For a non-absorbing medium the energy losses are determined by the poles of the integrand of (2.7). In the case of a homogeneous medium, the longitudinal losses are determined by the expression

$$I(\omega) = -\frac{2e^2\omega}{\pi v^2}\int\limits_{\omega/v}^{\infty}\frac{dk}{k}\,\mathrm{Im}\,\frac{1}{\varepsilon_0^l(k,\omega)}, \qquad (3.1)$$

so that in the non-absorbing medium the energy loss ω_i can have a value satisfying the condition

$$\varepsilon_0^l(k, \omega_i) = 0. \qquad (3.2)$$

In a homogeneous medium with low absorption, $I(\omega)$ differs from zero only near the frequencies ω_i satisfying the condition (3.2). The presence of density fluctuations leads to a shift in the characteristic frequencies ω_i. Actually, in this case we have in place of (3.1)

$$I(\omega) = -\frac{2e^2\omega}{\pi v^2}\int\limits_{\omega/v}^{\infty}\frac{dk}{k}\,\mathrm{Im}\,\frac{1}{\varepsilon_0^l(k, \omega) + \Delta^l(k, \omega)} \qquad (3.1')$$

and the characteristic frequencies are determined from the condition (we can neglect the imaginary part of Δ^l)

$$\varepsilon_0^l(k, \omega_i) + \Delta^l(k, \omega_i) = 0. \qquad (3.2')$$

To obtain the characteristic frequencies ω_i it is necessary to know the explicit form of $\Delta^l(k, \omega)$ and consequently also the form of $\alpha_{ij}(k, k-1, \omega)$.

The quantity $\alpha_{ij}(k, k-1, \omega)$ should, generally speaking, be calculated quantum mechanically, but in the particular case when spatial dispersion can be neglected (which is valid in small-angle electron scattering), it is sufficient to know the form of $\alpha_{ij}(0, 0, \omega)$. This quantity can be expressed in terms of the dielectric constant of the medium. Indeed, from (1.2) we have, neglecting spatial dispersion,

$$D_i(\mathbf{r}, t) = \int\limits_{-\infty}^{t} dt' \int d\mathbf{r}' \tilde{\varepsilon}_{ij}(t-t'; \mathbf{r}, \mathbf{r}') E_j(\mathbf{r}', t').$$

The dependence of $\tilde{\epsilon}_{ij}(t-t'; \mathbf{r}, \mathbf{r}')$ on $\mathbf{r}'$ is due to the presence of the inhomogeneities. In a homogeneous medium we have $\tilde{\epsilon}_{ij}(t-t'; \mathbf{r}, \mathbf{r}') = \epsilon_{ij}^{(0)}(t-t')\,\delta(\mathbf{r}-\mathbf{r}')$. Therefore, for small deviations of the electron density from the mean, we can represent $\tilde{\epsilon}_{ij}(t-t'; \mathbf{r}, \mathbf{r}')$ in the form

$$\tilde{\varepsilon}_{ij}(t-t'; \mathbf{r}, \mathbf{r}') = \tilde{\varepsilon}_{ij}^{(0)}(t-t')\,\delta(\mathbf{r}-\mathbf{r}')$$
$$+ \delta(\mathbf{r}-\mathbf{r}')\,\delta n(\mathbf{r}')\,\partial\varepsilon_{ij}^{(0)}(t-t')/\partial n_0. \qquad (3.3)$$

Going to Fourier transforms in (3.3) and comparing the relation obtained with (2.1), we get directly

$$\alpha_{ij}(0, 0, \omega) = \partial\varepsilon_{ij}^{(0)}(\omega)/\partial n_0. \qquad (3.4)$$

In this case therefore $\Delta^l(0, \omega)$ can be calculated without resorting to quantum mechanics.

Let us see how the characteristic frequency is shifted if the electrons in the metal are regarded as a degenerate Fermi gas. In this case ($T = 0$)

$$\varphi(q) = \begin{cases} -(2\pi)^{-3}\,[4\pi p_0^3/3 - 2\pi p_0^2 q/2 + 2\pi (q/2)^3/3] & q \leqslant 2p_0, \\ 0 & q > 2p_0, \end{cases} \qquad (3.5)$$

where p_0 is the momentum on the Fermi surface ($p_0 = (3\pi^2)^{1/3} n_0^{1/3}$). Substituting (3.4) and (3.5) in (2.6′) we get after integration

$$\Delta^l(\omega) = -\omega_0^4/3\omega^4. \qquad (3.6)$$

Now, solving (3.2′), we determine the energy lost by a heavy charged particle in a thin metallic plate

$$\hbar\omega_1 = \hbar\omega_0\left(1 + \tfrac{1}{2}\Delta^l(\omega_0)\right), \qquad \omega_0^2 = 4\pi e^2 n_0/m. \qquad (3.7)$$

For example, the values of ω_0 and ω_1, calculated from (3.7), and ω_{exp}, determined experimentally by considering the discrete losses [6], amount to respectively 24, 20, and 19 for B; 25, 21, and 22 for C; 28, 23, and 22 for Mn; 11, 9, and 10 for Mg; 32, 27, 23 for Zn; and 24, 25, and 30 for Au (all quantities in eV). It follows therefore that an account of the density fluctuation correlation greatly improves the agreement with experiments.

4. LONGITUDINAL LOSSES IN A MEDIUM WITH DENSITY FLUCTUATIONS

If the particle traveling through the medium is non-relativistic, then the energy losses are determined by the second term in (2.8), since the first term is of order v^2/c^2 with respect to the second. Such a particle loses energy to the formation of longitudinal waves. However, the macroscopic theory of energy loss by fast charged particles is valid only in the case when the impact parameters, which are characteristic of collisions between fast particles and the electrons of the medium, are much larger than the interatomic distances. However, as shown in Sec. 99 of [1], the concept of dielectric constant can be used also at wavelengths $\lambda \sim a$. Here, of course, it must be remembered that in the equations that relate **E** with **D** the quantities pertain to a field which is not averaged over physically infinitesimal volumes. Then the expression for the total loss of a nonrelativistic heavy particle is written in the form

$$\left\langle \frac{d\varepsilon}{dx} \right\rangle = -\frac{2e^2}{\pi v^2}\int\limits_0^{\infty}\omega\, d\omega \int\limits_{\omega/v}^{\infty}\frac{dk}{k}\,\mathrm{Im}\,\frac{1}{\varepsilon_0^l(k, \omega) + \Delta^l(k, \omega)}. \qquad (4.1)$$

In this expression $\omega_{eff} \sim \omega_0$ and $k \gtrsim \omega_0/v$, while in the calculation of $\Delta^l(k, \omega)$ we have $q_{eff} \lesssim 1/r_f$, where r_f is the characteristic dimension of the fluctuations.

In the case of a completely ionized gas, $r_f \sim r_D = \sqrt{\kappa T/4\pi e^2 n_0}$ where κT is the electron temperature. In this case therefore $q \ll k$ (if the parameter of gas approximation $\kappa T \gg e^2 n_0^{1/3}$ is satisfied for the medium, which is always the case in a real plasma). If $q \ll k$, then $\alpha_{ij}(\mathbf{k}, \mathbf{k}-\mathbf{q}, \omega)$ can again be expressed in terms of $\epsilon_{ij}^{(0)}(\mathbf{k}, \omega)$

$$\alpha_{ij}(\mathbf{k}, \mathbf{k}, \omega) = \partial \varepsilon_{ij}^{(0)}(\mathbf{k}, \omega)/\partial n_0. \tag{4.2}$$

Taking the foregoing into account, we can calculate the explicit form of $\Delta^l(k, \omega)$. We then obtain the following expression for the energy losses:

$$\left\langle \frac{d\varepsilon}{dx} \right\rangle = -\frac{2e^2}{\pi v^2} \int_0^\infty \omega\, d\omega \int_{\omega/v}^\infty \frac{dk}{k} \operatorname{Im} \left[\varepsilon_0^l(k, \omega) - \frac{n_0^2}{3} \left(\frac{\partial \varepsilon_0^l(k, \omega)}{\partial n_0} \right)^2 (\varepsilon_0^l(k, \omega))^{-1} \right]^{-1}. \tag{4.3}$$

In the particular case when spatial dispersion is neglected, (4.3) goes over into (3.1′). The use of this formula makes it possible to consider discrete losses in thin films also in the case of large angle deflections.

5. RADIATION IN A MEDIUM WITH STATISTICAL INHOMOGENEITIES

We investigate the spectral density of the energy loss in a transparent nonmagnetic medium with statistical inhomogeneities of the electron density, in a frequency region where $\epsilon_0(\omega)$ does not go through zero and the condition $v^2\epsilon_0 < 1$ is satisfied.

At such frequencies Cerenkov radiation is impossible in a homogeneous medium, and there are likewise no longitudinal losses, i.e., the spectral density of the losses vanishes in a homogeneous medium.

The presence of inhomogeneities leads to radiation connected with the polarization of the inhomogeneities by the uniformly moving charge. The spectral density of the energy losses has in this case the form

$$I(\omega) = \frac{2e^2\omega}{\pi v^2} \operatorname{Im} \int_0^\infty \frac{q\,dq}{q^2 + (\omega/v)^2} \times \left\{ \frac{q^2 v^2}{q^2 + (\omega/v)^2 (1 - v^2 [\varepsilon_0^t + \Delta^t])} - \frac{1}{\varepsilon_0^l + \Delta^l} \right\}. \tag{5.1}$$

Assuming that the influence of the inhomogeneities is small, we can expand in powers of Δ^t and Δ^l. In the first approximation, taking into account the absence of losses in the homogeneous medium, we can obtain

$$I(\omega) = \frac{2e^2\omega}{\pi v^2} \int_0^\infty \frac{q\,dq}{q^2 + (\omega/v)^2} \left\{ \frac{q^2 v^2 \omega^2}{[q^2 + (\omega/v)^2 (1 - v^2 \varepsilon_0^t)]^2} \times \operatorname{Im} \Delta^t \left(\omega, \sqrt{q^2 + \left(\frac{\omega}{v}\right)^2} \right) + (\varepsilon_0^l)^{-2} \operatorname{Im} \Delta^l \left(\omega, \sqrt{q^2 + (\omega/v)^2}\right) \right\}. \tag{5.2}$$

This expression contains only the imaginary parts of Δ^l and Δ^t.

When the radiated wavelengths greatly exceed the electron density correlation distance, $\varphi(l)$ can be replaced in (2.6) by $\varphi(0) = n_0(2\pi)^3$. If an ideal gas is considered, it must be taken into account that the correlation of the electron-density fluctuations is connected with the existence of stable atoms, so that the correlation length is the dimension of the atom. Spatial dispersion for wavelengths larger than the atomic dimensions is also insignificant, α_{ij} has the form (3.4), and it follows from (2.6) that ($\operatorname{Im} \epsilon_0 \to 0$):

$$\operatorname{Im} \Delta^l = \operatorname{Im} \Delta^t = (n_0/6\pi)(\partial \varepsilon_0/\partial n_0)^2 \omega^3 \sqrt{\varepsilon_0} \tag{5.3}$$

with the spectral density of the radiation assuming the form

$$I(\omega) = \frac{n_0 e^2 \omega^4}{3\pi^2 v^2} \left(\frac{\partial \varepsilon_0}{\partial n_0} \right)^2 \sqrt{\varepsilon_0} \int_0^{q_{max}} \frac{q\,dq}{q^2 + (\omega/v)^2} \times \left[\frac{q^2 v^2 \omega^2}{[q^2 + (\omega/v)^2 (1 - v^2 \varepsilon_0)]^2} + \frac{1}{\varepsilon_0^2} \right], \tag{5.4}$$

which coincides with the result of Kapitza[4] and Ter-Mikaelyan[5], obtained by other methods. In the case when $1 - v^2\epsilon_0$ is small, expansion is impossible and it is necessary to start with the more exact formula (5.2).

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