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EMISSION OF A PHOTON BY A FAST PARTICLE INTERACTING WITH ELEMENTARY EXCITATIONS IN MATTER

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It is shown that when an elementary excitation is absorbed by a fast particle, a hard quantum may be emitted, with a probability which exceeds in some cases the bremsstrahlung probability.

1. To satisfy the energy and conservation laws when a photon is emitted by a free particle, participation of a third body, which takes up the excess momentum, is necessary. The maximum momentum is transferred to the third body in the case when the photon is radiated in the direction of motion of the particle. The momentum transfer depends in this case on the particle energy and on the photon frequency ω, and for relativistic particles its value is ($\hbar = c = 1$)

$$\Delta p_{min} = E_1 - E_2 - \omega + (E_1 - E_2) M^2/2E_1E_2. \qquad (1.1)$$

At high energies this quantity can become so small that it is possible to consider as the third body a long-wave elementary excitation, such as an acoustic or optical phonon, a plasmon, or some other quasiparticle with integer spin.

The simplest process of this type is the emission of a photon upon absorption (or emission) of a quasiparticle. It is seen from (1.1) that greater interest attaches to the case of absorption, for then (ω_q is the quasiparticle energy)

$$\Delta p_{min} = (\omega - \omega_q) M^2/2E_1E_2 - \omega_q \qquad (1.2)$$

and when

$$\omega < E_1 [1 + M^2/2\omega_q E_1]^{-1} \qquad (1.3)$$

emission is possible for an arbitrarily small momentum transfer (in this case the value of the momentum transfer is determined by the dependence of ω_q on q). When $E_1 > M^2/2\omega_q$ this is valid for the entire spectrum of radiated frequencies.

The probability of photon emission with absorption of an elementary excitation depends on the presence of such excitation in the substance. Recognizing that in the equilibrium state of the substance the number of Bose excitations is determined by equilibrium conditions and that it can become large at sufficiently high temperatures, we can conclude that the radiation mechanism considered here can become significant at high temperatures.

The total radiation intensity in a frequency interval $d\omega$ from a unit path of the fast particle can be written in the form

$$I(\omega)\, d\omega = \int \omega n(\omega_q)\, d\sigma(q, k, p)\, d^3q\, (2\pi)^{-3}, \qquad (1.4)$$

where $d\sigma(q, k, p)$ is the cross section for the emission of a photon k upon absorption of a quasiparticle q, and $n(\omega_q)$ is the average number of quasiparticles in the state with momentum q and energy ω_q. In the equilibrium state we have

$$n(\omega_q) = [\exp(\omega_q/T) - 1]^{-1}, \quad n(\omega_q) \approx (T/\omega_q) \text{ for } T \gg \omega_q. \qquad (1.5)$$

The indicated radiation mechanism is possible also for nonrelativistic particles if the radiated frequencies ω are small compared with the particle energy. In this case

$$\Delta p_{min} \approx (\omega - \omega_q) M/p_1 \qquad (1.6)$$

and from the condition that the quasiparticle momentum is bounded from above it follows that the radiated frequencies are bounded.

2. We denote by g(q) the vertex part corresponding to the absorption of a Bose excitation by a charged fermion. The smallness of q makes it possible to retain in the calculation of the matrix element the first nonvanishing approximation in q/p. It must also be noted that when condition (1.3) is satisfied the effective length of the considered process can exceed the interatomic distances, so that the decrease in the number of particles due to Coulomb scattering by the atoms of the substance will play a role. To take this into account, it is necessary to replace in the matrix element the vacuum Green's function of the virtual fermion by the Green's function of the particle in the matter:

$$G(p) = (\hat{p} - M + i\gamma_4\alpha/2)^{-1}, \qquad (2.1)$$

which takes into account the Coulomb scattering. In the approximation of exponential screening $\alpha = (n_0\lambda^{-3})(Z^2e^4/2\pi)$, where n_0 is the density of the atoms, and λ the inverse screening radius (the Thomas-Fermi radius of the atom in a dense medium or the Debye radius in a plasma).

The foregoing enables us to obtain the matrix element of the radiation of a photon upon absorption of an elementary excitation, in the form

$$(2\pi)^4 \frac{eg(q)}{\sqrt{2\omega}} (\bar{u}_2\gamma_\mu u_1)\left[\frac{2E_2}{-2p_{1\nu}k_\nu - i\alpha E_2} + \frac{2E_1}{2p_{2\nu}k_\nu - i\alpha E_1}\right]$$

$$\times\, \delta(p_{1\nu} + q_\nu - k_\nu - p_{2\nu}). \qquad (2.2)$$

It follows therefore that the energy radiated by a relativistic unpolarized particle per unit path in the frequency interval $d\omega$ is equal to

$$I(\omega)\,d\omega = \frac{e^2}{8\pi^2}\omega^2 d\omega \int \frac{d^3q}{(2\pi)^3} n(\omega_q)|g(q)|^2 \int d\Omega_k \frac{M^2}{E_1(E_1+\omega_q-\omega)}$$

$$\times \left|\frac{2(E_1-\omega)}{2p_{1\nu}k_\nu + i\alpha E_2} - \frac{2E_1}{2p_{2\nu}k_\nu - i\alpha E_1}\right|^2$$

$$\times\delta(E_1 + \omega_q - \omega - \sqrt{M^2 + (\mathbf{p}_1 - \mathbf{k} - \mathbf{q})^2}). \qquad (2.3)$$

For frequencies satisfying the condition $\omega_q \ll \omega \ll E$, condition (1.3) is satisfied at not too high energies, and we therefore consider this limiting case first. We assume also that g(q) does not depend on the direction of q. In this case we can readily integrate over the directions of the vectors **q** and k. It is necessary to recognize here that the limits of integration with respect to the angle ϑ between p and q depend on the value of $|q|$: when $|q| > q_1 \equiv \omega_q - \omega M^2/2E^2$, the conservation laws limit $\cos\vartheta$ from above to the value $q_1/|q|$. Taking the foregoing into account we readily obtain

$$I(\omega) = \frac{e^2\omega}{(2\pi)^3}\frac{M^2}{E^2}\Big\{\int_{q_1} q\,dq\,n(\omega_q)|g(q)|^2[f(q) - f(-q_1)]$$

$$+ \int_0^{q_1} q\,dq\,n(\omega_q)|g(q)|^2[f(q) - f(-q)]\Big\},$$

$$f(q) = \frac{\omega_q + \beta q}{(\omega_q + \beta q)^2 + (\alpha/2)^2} + \frac{2}{\alpha}\operatorname{arctg}\frac{2(\omega_q + \beta q)}{\alpha}. \qquad (2.4)^*$$

3. We consider by way of an example the emission of a hard transverse quantum by a relativistic particle, brought about by the absorption of a longitudinal wave. The frequency ω_q and the wave vector q of the longitudinal wave are connected, as is well known, by the relation

$$\varepsilon(\omega_q, q) = 0, \qquad (3.1)$$

where ϵ is the dielectric constant of the substance (the absorption is assumed small). Relation (3.1) determines the dependence of ω_q on q.

To solve the problem we must determine the explicit form of the vertex part g(q), which can be done with the aid of the following auxiliary considerations. It is known that the longitudinal part of the ionization losses of a relativistic particle on a unit path has the form

$$\frac{dE}{dx} = \frac{e^2}{4\pi}\frac{1}{\pi^2 v}\int\frac{d^3q}{q^2}\int q_4\,dq_4\,\delta(q_4 - \mathbf{qv})\,\pi\delta(\varepsilon(q_4, q)). \qquad (3.2)$$

It is obvious that the same expression should also be obtained if we calculate the quantum-mechanical energy lost to radiation of longitudinal quanta in the first perturbation theory approximation for $q \ll p$, i.e., without account of the recoil upon radiation. The matrix element of the radiation of a longitudinal quantum with $q \ll p$ is equal to

$$(2\pi)^4 g(q)\,\delta(E_1 - E_2 - \omega_q)\,\delta(\mathbf{p}_1 - \mathbf{p}_2 - \mathbf{q}).$$

Hence

$$\frac{dE}{dx} = \frac{1}{4\pi^2 v}\int d^3q\,|g(q)|^2\,\delta(E_1 - \omega_q - \sqrt{M^2 + (\mathbf{p}_1 - \mathbf{q})^2}). \qquad (3.3)$$

Recognizing that when $q \ll p$ we also have $E_1 - \sqrt{M^2 + (p_1 - q_1)^2} \approx q\cdot v$, we can readily obtain from a comparison of (3.3) and (3.2)

$$|g(q)|^2 = \frac{e^2}{q^2}\left[\left|\frac{\partial\varepsilon}{\partial q_4}\right|_{q_4=\omega_q}\right]^{-1}. \qquad (3.4)$$

Neglecting the spatial dispersion, we can assume that for frequencies larger than the natural frequencies of the electrons

$$\varepsilon(\omega) = 1 - (\omega_L/\omega)^2, \quad \omega_L^2 = n_0e^2Z/m_e.$$

In this case

$$|g(q)|^2 = e^2\omega_L/2q^2 \qquad (3.5)$$

and $\omega_q = \omega_L$ does not depend on q.

4. Substituting (3.5) in (3.4) we can obtain after a simple integration the spectral density of the radiation produced by the absorption of a longitudinal quantum, in the form

$$I(\omega) = \frac{n}{2}(\omega_L)\frac{\omega_L}{(2\pi)^3}e^4\left[\frac{\omega}{\alpha}\left(\frac{M}{E}\right)^2\left(\frac{\pi}{2} - \operatorname{arc\,tg}\frac{\omega M^2}{\alpha E^2}\right)\right.$$

$$\left. - \frac{2M^4\omega^2}{M^4\omega^2 + \alpha^2E^4}\right]\ln\left|\frac{2\alpha E q_{max}}{M^2\omega}\right|. \qquad (4.1)$$

The appearance in (4.1) of the maximum momentum that bounds the region of integration with respect to q from above is connected with the neglect of spatial dispersion in (3.5). The use of the more accurate formula (3.4) ensures automatic cutoff of the integral for large q.

In the limiting cases (4.1) can be simplified. In the frequency region $\omega \ll \alpha(E/M)^2$ we have

*arctg = $\tan^{-1}$.

$$I(\omega) = n(\omega_L)\left(\frac{e^2}{4\pi}\right)^2\left(\frac{\omega}{2\alpha}\right)\left(\frac{M}{E}\right)^2 \ln\left|\frac{2\alpha E q_{max}}{M^2\omega}\right|. \qquad (4.2)$$

In the frequency region $\alpha(E/M)^2 \ll \omega < \omega(E/M)^2$, $\omega \ll E$, which exists only when $\alpha E \ll M^2$, the following formula holds

$$I(\omega) = n(\omega_L)\left(\frac{e^2}{4\pi}\right)^2 \frac{\omega_L}{\pi} \ln\left|\frac{2\alpha E q_{max}}{M^2\omega}\right|, \qquad (4.3)$$

which is considerably larger than (4.2). In condensed media the region of existence of the solution (4.3) for $Z \lesssim 10$ and $n_0 \sim 10^{22}$ cm^{-3}, $\alpha \sim Z^2 10^{-10} m_e$ is possible for $E \ll (M/Zm_e)^2 \times 10^{15}$ eV and, for example, for an electron with energy 10^9 eV it lies between 10^4 and 10^7 eV. The ratio of the intensity (4.3) to the bremsstrahlung intensity is small under the most unfavorable condition (electron) at normal temperature, owing to the large value of the argument ω_L/T of the exponential in (1.5). However, when the temperature is increased to values $T \sim \omega_L$ the radiation mechanism considered here is more intense than bremsstrahlung.

The most favorable case for the observation of the effect under consideration is the emission of a heavy particle in a medium of light elements at maximally high temperatures. In a dense medium with $Z \sim 1$ we have $\omega_L \sim T$ at temperatures near 10,000°K, and in this case the radiation (4.3) is more intense than bremsstrahlung. When $\omega_L \sim 10T$ we have $n(\omega_L) \sim 10^{-5}$ and the radiation mechanism (4.3) is more intense in dense media than bremsstrahlung by a factor $(M/m_e)Z^{-3/2}$, so that even for an electron the radiation at $Z \sim 1$ is comparable with bremsstrahlung. It must be noted that owing to the exponential dependence of (1.5) on ω_L, at a specified temperature $T < \omega_L$ the radiation increases with decreasing density of the medium. In a rarefied gas the radiation upon absorption of a longitudinal quantum becomes more intense than bremsstrahlung at much lower temperatures.

In a high temperature plasma it is possible to have a case where there is no region in which (4.3) is applicable, owing to the fact that α exceeds ω_L, and only formula (4.2) remains applicable. In this case, however, the inequality $T \gg \omega_L$ is satisfied and $n(\omega_L) \approx T/\omega_L$. Taking into account the dependence of the Debye radius on the temperature and on the density, we can obtain $\alpha \approx e^2T/2\pi$, so that in this case the temperature dependence in (4.2) remains only under the logarithm sign and the radiation decreases abruptly with increasing particle velocity. At a density $n_0 \sim 10^{14}$ cm^{-3} and for $Z \sim 1$, the ratio of the intensities of the considered mechanism of radiation and bremsstrahlung amounts to $(T \gg \omega_L)$

$$(I(\omega)/I(\omega_T)) \sim 10^{15}(M/m_e)(\omega/m_e)(M/E)^2$$

and for $\omega \sim m_e$ we have in the case of an electron a ratio in excess of unity up to energies $\sim 10^{11}$ eV.

The example under consideration shows that radiation upon absorption of an elementary Bose excitation must be taken into account in the analysis of the radiation from a particle in matter.

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