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AZIMUTHAL DISTRIBUTION IN JETS PRODUCED BY COSMIC-RAY PARTICLES IN PHOTOGRAPHIC EMULSIONS

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A new method of detecting various correlations in the azimuthal angular distribution of jet particles is proposed. The method is more efficient than the one described previously.[2,3] The effect of the momentum conservation law on the distribution of various random quantities is investigated. Azimuthal isotropy and independence of the emission angles of the secondary particles have been verified for 85 jets produced in photographic emulsions by singly-charged cosmic-ray particles with a mean energy of ~ 25 BeV ($n_h + n_g \leq 5$, $n_s \geq 8$). It is shown that the main cause of the violation of these conditions is the momentum conservation law. Apparently, the mesons have no tendency to be complanar. Finally, 117 interactions between singly-charged cosmic-ray particles and heavy emulsion nuclei have also been investigated ($n_h + n_g \geq 8$, $n_s \geq 10$). An azimuthal asymmetry of jet particles has been observed for $n_h + n_g > 15$.

THE azimuth angle distribution of jet particles is relevant for the study of the interaction of fast particles with nucleons and nuclei. Kraushaar and Marks,[1] assuming a two-center model of nucleon-nucleon high-energy collisions, predicted a correlation of type a) (see Fig. 1a) in the azimuth-angle distribution. The tendency of the mesons to be complanar may be due both to a large intrinsic momentum of the centers, and to a deflection from the direction of the primary particles. A correlation of type b) (see Fig. 1b) may occur when a single center, which does not propagate in the direction of the primary particle, is produced. There exist, of course, other reasons for correlations in the multiple-particle production in nuclear interactions.

We have investigated[2,3] 16 jets produced in the emulsion by singly-charged cosmic-ray particles, measured in the laboratory of J. Pernegr

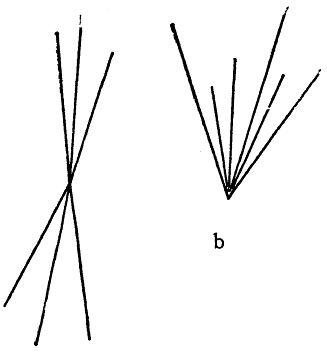

FIG. 1. Possible correlations in the azimuth-angle distribution of secondary particles (track of the primary particle is perpendicular to the plane of drawing).

(Czechoslovakia). The median energy of the primary nucleons, determined according to the Castagnoli formula,[4] was 110 BeV, the number of secondary shower particles n_s varied from 14 to 42, and the number of strongly ionizing particles was $n_h + n_g \leq 5$. It was shown by an original method that, at least for several of the jets investigated, there exists (with a probability close to unity) an azimuthal correlation which cannot be explained by the momentum conservation law.

In the present article we investigate new experimental material using a more powerful method than the one proposed earlier.

1. METHOD OF ANALYSIS

In addition to the azimuth angles φ_i ($i = 1,2\ldots, n_s$; $0 \leq \varphi_i \leq 2\pi$) of the secondary shower particles, counted from a certain axis, let us consider the azimuth angles ϵ_{ij} ($0 \leq \epsilon_{ij} \leq \pi$) between the i-th and j-th particles in the plane perpendicular to the direction of the primary particle. We construct the following random quantities

$$\beta_k = \sum_{i\neq j} \cos(k\varepsilon_{ij}) / \sqrt{n_s(n_s-1)} = \left\{\left[\sum_{i=1}^{n_s} \cos(k\varphi_i)\right]^2 + \left[\sum_{i=1}^{n_s} \sin(k\varphi_i)\right]^2 - n_s\right\} \Big/ \sqrt{n_s(n_s-1)}, \qquad k = 1, 2 \quad (1)$$

(where the factor $1/\sqrt{n_s(n_s-1)}$ is introduced for normalization). From Eq. (1) it follows that: 1) the β_k are independent of the axis chosen as the reference for the angles φ_i; 2) a correlation of type a) leads to an increase in β_2 but does not affect considerably β_1; 3) a correlation of type b) leads to an increase in both β_1 and β_2; 4) the β_k are bounded and can assume values in the interval

$$-\sqrt{\frac{n_s}{n_s-1}} \leqslant \beta_k \leqslant \sqrt{n_s(n_s-1)}. \qquad (2)$$

Assuming azimuthal isotropy, i.e., a uniform distribution of the angles φ_i within the interval $(0, 2\pi)$, and assuming statistical independence of the n_s angles φ_i, we can show that the mathematical expectation and the standard deviation of the β_k equal, respectively,

$$\nu(\beta_k) = 0, \quad \sigma(\beta_k) = 1. \qquad (3)$$

For the random quantities

$$\bar{\beta}_k = \frac{1}{n}\sum_{i=1}^{n} \beta_{ki}, \qquad (4)$$

averaged over n showers with arbitrary n_s (not necessarily equal), we have

$$\nu(\bar{\beta}_k) = 0, \quad \sigma(\bar{\beta}_k) = 1/\sqrt{n}. \qquad (5)$$

The quantities β_1 and β_2 are uncorrelated and, for large n_s, independent; they can be written in the form

$$\beta_k \approx \tfrac{1}{2}\chi_2^2 - 1, \qquad (6)$$

where the quantity χ_2^2 has a χ^2 distribution with two degrees of freedom. From Eqs. (2) and (6) it follows that the conditions of Lyapunov's theorem[5] are satisfied for any sequence of jets. According to this theorem, for a large number of jets n the quantities $\bar{\beta}_k$ can be assumed to be normally distributed. It is therefore easy to indicate a confidence interval[1)] which contains $\bar{\beta}_k$ with probability close to unity. If even one of the obtained values of $\bar{\beta}_k$ falls outside this interval, we can assert that the azimuthal isotropy and the independence of the angles φ_i are violated at least in a part of the investigated jets.

Assuming that the angles φ_i are statistically independent and arbitrarily distributed, the mathematical expectation of β_k is

$$\nu(\beta_k) = \sqrt{n_s(n_s-1)}\,[\nu^2(\cos k\varphi) + \nu^2(\sin k\varphi)] \geqslant 0. \qquad (7)$$

For any symmetrical distribution of the azimuth angles we have $\nu(\beta_1) = 0$. If we write the distribution density in the form

$$p(\varphi) = \frac{1}{2\pi}(1 + a\cos 2\varphi), \qquad 0 \leqslant a \leqslant 1, \qquad (8)$$

we obtain the following formulas for the mathematical expectation and the variance of β_2:

$$\nu(\beta_2) = \sqrt{n_s(n_s-1)}\,\frac{a^2}{4},$$

$$\sigma^2(\beta_2) = 1 + \frac{a^2}{2}(n_s-2) - \frac{a^4}{8}(2n_s-3). \qquad (9)$$

According to (8), the orientation of the plane of preferential particle emission does not vary from one experiment to another. If it is different for a series of trials but the relative positions of the secondary particle tracks in the jets are the same as before, then the distribution of β_k, which depend only on ϵ_{ij}, remains unchanged and Eqs. (9) remain valid.

In order to compare the efficiency of the above method and of that used earlier,[2,3] we calculated the mathematical expectation of the normalized quantity[2)]

$$\alpha'_m = (\alpha_m - 1) \Big/ \sqrt{\frac{2}{m-1}\,\frac{n_s-1}{n_s}}, \qquad m = 2, 3, 4, \ldots \qquad (10)$$

For the distribution (8) we obtain the following formula, averaging over the different orientations of the plane of preferential emission:

$$\overline{\nu(\alpha'_m)} = \sqrt{n_s(n_s-1)}\,\frac{a^2}{8\sqrt{2\pi^2}}\,\frac{m^2}{\sqrt{m-1}}\sin^2\frac{2\pi}{m}. \qquad (11)$$

The value of $\nu(\alpha'_m)$ attains a maximum for m = 7, but even then it is 2.3 times smaller than $\nu(\beta_2)$.

To find the efficiency of the method we have to calculate the detection probability of some effect as a function of n and n_s. In the calculations it was assumed that all n showers have the same multiplicity and the same distribution density (8). Assuming a normal distribution of the quantity $\bar{\beta}_2 > 2/\sqrt{n}$ according to the Lyapunov theorem, and using Eqs. (9), we can easily calculate the probability of finding a $\bar{\beta}_2 > 2/\sqrt{n}$ (in such a case the effect can be said to be detected). In Table I the probabilities are given for a = 0.2 and 0.4, and for different n and n_s.

The β_k criterion is useful for the dectection of not only azimuthal correlations of type a) or b), but, for example, also pair correlations which may occur as a result of the decay of short-lived particles into two charged particles. Let us assume that the angles φ_i ($i = 1, 2, \ldots, n_s$) are isotropically distributed, that the first $n_s - l$ angles φ_i are independent, and that the remaining angles can be expressed through the first ones according

1)Its limits can be taken as $-2/\sqrt{n}$ and $+2/\sqrt{n}$.

2)The quantities α_m are random variables used to detect azimuth-angle correlations in [2,3]. Assuming that the azimuth angles are isotropic and independent, we have $\nu(\alpha'_m) = 0$ and $\sigma(\alpha'_m) = 1$.

Table I

n_s	n = 25	50	100	500
5	—	—	—	0.17
	0.16	0.25	0.42	0.97
10	—	—	0.16	0.55
	0.47	0.71	0.92	1.00
15	—	0.19	0.31	0.87
	0.74	0.93	1.00	1.00
20	0.19	0.30	0.48	0.98
	0.89	0.99	1.00	1.00

Note: Only the probabilities greater than 0.15 are given. In each position, the upper value corresponds to a = 0.2, and the lower one to a = 0.4 [see Eq. (8)].

to the formula

$$\varphi_{n_s-l+i} = \varphi_i, \quad i = 1, 2, \ldots, l, \quad l \leqslant n_s/2. \qquad (12)$$

In that case, the mathematical expectation of β_k is

$$\nu(\beta_k) = \frac{2l}{\sqrt{n_s(n_s-1)}}. \qquad (13)$$

2. EFFECT OF THE MOMENTUM CONSERVATION LAW

The momentum conservation law affects the statistical independence of the angle φ_i. It is therefore important to find out how it changes the distribution of β_k if nothing else affects the azimuthal isotropy and the independence of the angles φ_i. For this purpose, we analyzed the tables of "random stars"[6] that imitate pp collisions at 10 BeV. We calculated the values of β_k for each of the 124 stars with $n_s = 4$, and obtained the following values for the mathematical expectations and variances of β_k:

$$\nu(\beta_1) = -0.48 \pm 0.06; \quad \sigma^2(\beta_1) = 0.45 \pm 0.08;$$

$$\nu(\beta_2) = -0.06 \pm 0.08; \quad \sigma^2(\beta_2) = 0.92 \pm 0.14. \qquad (14)$$

We see that the momentum conservation law lowers the mathematical expectation and the standard deviation of β_1 as compared with (3). The decrease is especially strong when neutral particles are absent and the n_s secondary charged particles have the same transverse momentum. The transverse momentum conservation can then be written in the form:

$$\beta_1 = -\sqrt{\frac{n_s}{n_s-1}}. \qquad (15)$$

The quantity on the right-hand side represents the smallest possible value of β_1. As mentioned in Section 1, β_1 and β_2 can be considered independent if n_s is large and the angles φ_i are isotropic in azimuth and statistically independent. From among a number of trials corresponding to the above situation, the momentum conservation law selects the events satisfying (15). Since β_1 and β_2 are independent, the distribution of β_2 in the selected trials will remain unchanged.

This discussion leads to the natural assumption that for large n_s the momentum conservation law does not affect the distribution of β_2, even if its effect on β_1 is as large as possible.

The result (14) permits us to assume that for the 85 jets with $n_s \geq 8$ investigated in the following we can neglect the effect of the momentum conservation law on the β_2 distribution.

3. EXPERIMENTAL DATA

In area scanning of 37 Ilford G-5 plates, irradiated in the stratosphere during the 1955 Italian expedition, we found and measured 85 jets produced by singly-charged cosmic-ray particles which satisfied the selection criteria

$$n_h + n_g \leqslant 5, \quad n_s \geqslant 8. \qquad (16)$$

No limitation as to the energy were made in selecting the showers. Some of the measured jets could have been produced by neutral particles, since one of the secondary particle tracks could have been erroneously identified as the track of the primary proton. The fraction of such showers was, however, very small since the experimental value $\bar{\beta}_1$ was negative for all jets. Jets with bad geometry were not measured. For each jet the accuracy of angle measurements was fully satisfactory to obtain a precise azimuth-angle distribution.

We determined the primary-particle energy E for each jet using the Castagnoli formula, and calculated β_k according to Eq. (1). Results of the analysis are given in Table II. The small value of $\bar{\beta}_1$ has led us to conclude that the azimuthal isotropy and the statistical independence of the angles φ_1 do not hold for the 85 jets analyzed. The values of β_1 and β_2 are in a good agreement

Table II

Shower characteristics	Quantity	$\bar{\beta}$	$\lvert\bar{\beta}\rvert\sqrt{n}$
$n_h + n_g \leqslant 5$; $8 \leqslant n_s \leqslant 22$; $E \leqslant 6600$ BeV	β_1	−0.24	2.2
$\bar{E} = 25$ BeV; $n = 8.5$	β_2	−0.04	0.4

Note: $\lvert\bar{\beta}_k\rvert\sqrt{n}$ is the difference between the quantity (4) and its mathematical expectation in terms of standard deviation (5). $\bar{E}$ is the median energy. Maximum energy was found to equal 6600 BeV.

with the assumption that the main reason for their violation is the momentum conservation law, which lowers the mathematical expectation of β_1 but does not affect the distribution of β_2 for $n_s \geq 8$. The distribution of 85 showers with respect to β_1 and β_2, which confirms this assumption, is shown in Fig. 2.

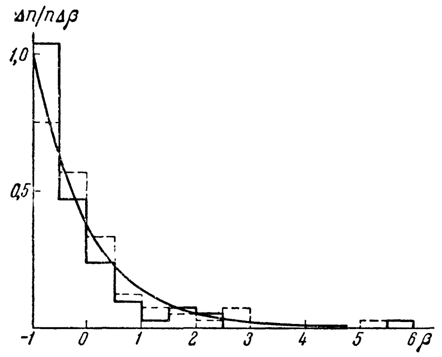

FIG. 2. Distribution of 85 jets with respect to β_1 (solid line) and β_2 (dotted line). The curve represents the density distribution of the quantity (6).

If there were a clearly expressed tendency of the mesons to be complanar, $\bar{\beta}_2$ would be positive. It should be noted that the value $\bar{\beta}_2 = -0.04$ found for 85 jets is compatible with the assumptions made in deducing Eqs. (9) only if $a < 0.16$, since the difference $\nu(\bar{\beta}_2) - \sigma(\bar{\beta}_2)$ is greater than -0.04 for any $a > 0.16$. If we divide the showers into groups according to energy E and multiplicity n_s, the value of $\bar{\beta}_2$ for each group differs from zero by not more than 1.1 standard deviation (5). The reason for this large difference between our jets and those measured in Prague and mentioned in the beginning of the article is not clear.[3)]

In order to study the interaction of cosmic-ray particles with heavy nuclei of the emulsion, we investigated 117 jets produced by singly-charged particles, which satisfied the following selection criterion

$$n_h + n_g \geqslant 8, \quad n_s \geqslant 10. \qquad (17)$$

In 21 plates out of 37 we did not use any energy selection criterion. In the remaining 16 plates we measured only the jets with a sufficiently narrow angular distribution of relativistic particles about the direction of the primary particle in the laboratory system.

The 117 showers were divided into two groups, according to the number of strongly ionizing particles $n_h + n_g$. The results for these groups are given in Table III (rows 1 and 2) and in Fig. 3. To find the variation of the azimuth-angle distribution of the secondary particles with the primary proton energy, out of 50 jets with $n_h + n_g > 15$ we selected the showers with large values of the Lorentz factor γ_c of the center-of-mass system, obtained according to the Castagnoli formula (row 3, Table III). The values $\bar{\beta}_1$ given in rows 2 and 3 of Table III indicate the existence of clear-cut correlations in the azimuth-angle distribution of secondary particles, which cannot be explained by the effect of the momentum conservation law. The values of $\bar{\beta}_1$ and $\bar{\beta}_2$ agree with the assumption of an asymmetrical density distribution

Table III

Shower characteristics	$\bar{\beta}_1$	$\bar{\beta}_2$	a	$\bar{\beta}'_1$	$\bar{\beta}''_1$
$8 \leqslant n_h + n_g \leqslant 15$ $10 \leqslant n_s \leqslant 35$ $\bar{n}_s = 14$ $\gamma_c \leqslant 10.0$ $\bar{\gamma}_c = 3.5$ $n = 67$	−0.23 (1.9)	+0.02	—	−0.03	+0.11
$15 < n_h + n_g \leqslant 35$ $11 \leqslant n_s \leqslant 41$ $\bar{n}_s = 20$ $\gamma_c \leqslant 8.6$ $\bar{\gamma}_c = 2.7$ $n = 50$	+0.59 (4.2)	−0.04	0.35±0.06	+0.02	+0.54 (3.9)
$n_h + n_g > 15$ $\gamma_c > 2.7$ $\bar{n}_s = 21$ $\bar{\gamma}_c = 4.1$ $n = 25$	+0.66 (3.3)	+0.09	$0.36^{+0.08}_{-0.09}$	+0.10	+0.60 (3.0)

Note: $\bar{n}_s$ and $\bar{\gamma}_c$ are median values. The numbers in brackets denote the value of $|\bar{\beta}|\sqrt{n} > 1$.

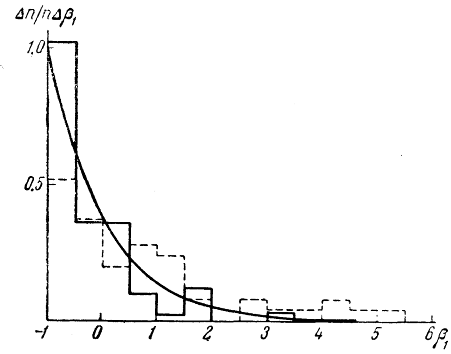

FIG. 3. Distribution of jets with respect to β_1. The solid histogram refers to 65 jets with $n_h + n_g$ = 8–15, the dotted one to 50 jets with $n_h + n_g > 15$. The curve represents the same as in Fig. 2.

[3)]Applying the new method to showers measured in Prague, we obtained $\bar{\beta}_1 = -0.13$ and $\bar{\beta}_2 = +1.11$ for n = 16, which confirms our earlier conclusion.[2,3]

$$p(\varphi) = \frac{1}{2\pi}(1 + a \cos\varphi), 0 \leqslant a \leqslant 1, \quad (18)$$

and the statistical independence of the emission angles φ_i ($i = 1,2,\ldots n_s$) of secondary particles. Equations (9) describe the mathematical expectation and the dispersion of β_1 and $\nu(\beta_2) = 0$. Using Eqs. (9) and assuming that the parameter a is the same for all showers, we found the value of a from the experimental values of $\overline{\beta}_1$ (see Table III).

As mentioned in the beginning of Section 3, the large value of $\overline{\beta}_1$ for jets with $n_h + n_g > 15$ may be due to an incorrect identification of the primary particle track. With increasing Lorentz factor γ_c, the probability of mistaking the track of a secondary particle for the primary proton track decreases and the spurious azimuthal asymmetry should become smaller. The values of β_1 and a do not decrease, however, with increasing γ_c (compare rows 2 and 3 of Table III). To confirm that the effect is not spurious, we determined for each jet the half-angle $\theta_{1/2}$, and calculated β_1 separately for the particles with angles of emission $\theta < \theta_{1/2}$ (β_1') and with $\theta > \theta_{1/2}$(β_1''). It can be seen from Table III that the azimuthal asymmetry effect is due to the particles of the wide cone, which is not compatible with the assumption that the primary particle was wrongly identified.

The effect is apparently due to intranuclear cascades in non-central collisions of fast particles with heavy nuclei.[4)] For decreasing $n_h + n_g$, the contribution of secondary interactions inside the target nucleus to the azimuth angle distribution of secondary particles decreases,[5)] and can even become smaller than the effect of the momentum conservation law. However, from the absence of an azimuthal asymmetry we cannot conclude that there are no secondary interactions producing jet particles, since only the nucleons of the target nucleus which lie on the prolongation of the primary-particle trajectory take part in these interactions.

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[4] Castagnoli, Cortini, Franzinetti, Manfredini, and Moreno, Nuovo cimento **10**, 1539 (1953).

[5] S. N. Bernshtein, Teoriya veroyatnosteĭ (Theory of Probability), ONTI, 1934.

[6] G. I. Kopylov, Preprint, Joint Inst. Nuc. Res. R-259 (1959).

[4)] We call a collision non-central if the center of the nucleus does not lie on the prolongation of the trajectory of the primary particle.

[5)] We assume that the values $n_h + n_g$ can fluctuate strongly for the same impact parameter, depending on the degree of development of the intranuclear cascade. The mean values of the impact parameter for collisions with $n_h + n_g$ equal 8 to 15 and collisions with $n_h + n_g > 15$ can therefore be similar.

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