SOVIET PHYSICS JETP VOLUME 18, NUMBER 2 FEBRUARY, 1964

THEORY OF PHOTODISINTEGRATION OF LIGHT NUCLEI WITH EMISSION OF FAST DEUTERONS

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Submitted to JETP editor December 29, 1962

J. Exptl. Theoret. Phys. (U.S.S.R.) **45**, 532-540 (September, 1963)

A characteristic feature of the (γ, d) reaction—the shift of the threshold for fast deuteron emission—is examined within the framework of the shell model of light nuclei.

RECENT experiments dealing with the (γ, d) reaction on light nuclei [1-4] disclosed several singularities not possessed by the (γ, p) and (γ, n) reactions. Of particular interest is the sharp suppression of the fast-neutron yield near the reaction threshold [1]. The threshold is shifted, as it were, away from the kinematic value: the magnitude of the shift is approximately equal to the binding energy of the nucleons in the residual nucleus. While this phenomenon can be explained in the case of the odd-odd nuclei Li^6 and B^{10} with the isospin T selection rules, by assuming that the main contribution to the cross section is made by the dipole absorption of the gamma quanta, these rules do not suffice for the other nuclei, for they allow in this case transitions to the lowest states of the residual nucleus.

The authors have shown elsewhere [5] that one of the causes of these singularities is the appearance of a supermultiplet structure in the levels of the light nuclei. The purpose of the present paper is a further clarification of the mechanism of the (γ, d) reactions. All the calculations have been made in the Born approximation, assuming dipole absorption of the γ quanta. The wave functions of the ground and excited states of the nuclei are considered within the framework of the shell model; oscillator radial functions are assumed.

The starting point of the proposed formalism is an accurate expression for the Born amplitude of the photodisintegration of the nucleus into two arbitrary nucleon clusters (Sec. 1). Section 2 is devoted to a calculation of the probabilities of transitions with residual nuclei formed in states of "normal" parity (i.e., equal to the ground-state parity). In Sec. 3 we consider the special case of the (γ, d) reaction in even-even nuclei.

[1] As is customary in the literature, we define as the threshold for the emission of particles of a given energy the minimum incident-particle energy at which the emission becomes energetically feasible.

1. AMPLITUDE OF PHOTODISINTEGRATION OF A NUCLEUS INTO TWO NUCLEON CLUSTERS IN THE DIPOLE APPROXIMATION

The effective nuclear photodisintegration cross section is determined in first-order perturbation theory by the square of the modulus of the matrix element of the transitions from state Ψ_i into state Ψ_f, in the form

$$M_{if} = \int \Psi_f^* (1, 2, \ldots, A) \sum_{l=1}^{A} \mathbf{s}\nabla_l \tau_l \Psi_i (1, 2, \ldots, A)\, dV, \quad (1)$$

where $\mathbf{s}$ —photon polarization vector, ∇ —gradient operator in the variables of the l-th nucleon, and τ_l —proton projection operator. We consider below the matrix element for the photodisintegration of a nucleus into two clusters containing A_1 and A_2 nucleons with relative-motion wave function $f(R_{A_1A_2})$.

Antisymmetrization of the wave function in the final state Ψ_f is by the usual method

$$\Psi_f (1, 2, \ldots, A) = (C_{A_1}^{A})^{-1/2} \sum_P (-1)^P f(R_{A_1A_2}) \Psi_1 (1, \ldots, A_1) \times \Psi_2 (A_1 + 1, \ldots, A), \quad (2)$$

where $C_{A_1}^{A}$ are the binomial coefficients and P all possible permutations of the particles between the clusters. The wave functions of clusters Ψ_1 and Ψ_2 are assumed antisymmetrical. The summation in (2) is over all permutations of the particles between the clusters.

Since Ψ_f is antisymmetrical, we separate in the operator $\sum_l^A \mathbf{s}\nabla_l \tau_l$ the terms acting on the internal variables of the clusters,

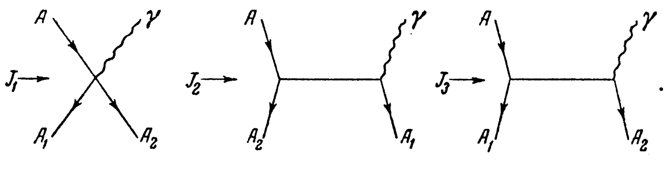
$$\mathbf{r}_{k,\,A_{1(2)}-k} = \mathbf{r}_k - \frac{1}{(A_{1(2)}-1)} \sum_{i \neq k}^{A_{1(2)}} \mathbf{r}_i, \qquad k = 1, 2, \ldots, A_{1(2)},$$

and the relative-motion variable $\mathbf{R}_{A_1A_2}$; we then represent M_{if} in the form

$$M_{if} = \sqrt{C^A_{A_1}}\, J = \sqrt{C^A_{A_1}}\,(J_1 + J_2 + J_3)$$
$$= \sqrt{C^A_{A_1}} \Big\{ \Big(\frac{Z_1}{A_1} - \frac{Z_2}{A_2} \Big) \int f^* \Psi_1^* \Psi_2^* \, (\mathbf{s} \nabla_{R_1 R_2}) \, \Psi_i \, dV$$
$$+ \int f^* \Psi_1^* \Psi_2^* \sum_l^{A_1} \mathbf{s} \nabla_{l,\,A_1 - l}\, \tau_l \Psi_i \, dV$$
$$+ \int f^* \Psi_1^* \Psi_2^* \sum_l^{A_2} \mathbf{s} \nabla_{l,\,A_2 - l}\, \tau_l \Psi_i \, dV \Big\}. \qquad (3)$$

The amplitude J_1, which is proportional to the classical dipole moment of the system, corresponds to the interaction of radiation with rigid clusters having charges Z_1 and Z_2 and masses A_1 and A_2. The amplitudes J_2 and J_3, usually left out of the calculations, take account of the polarizabilities of the clusters; the dipole operators contained in J_2 and J_3 act only on the internal variables of the clusters and not on the relative motion variable $\mathbf{R}_{A_1A_2}$. These amplitudes correspond to a unique photodisintegration exchange mechanism, which can be regarded as a virtual breakup of the nucleus A into two clusters A_1 and A_2 with subsequent transfer of one of the clusters, under the influence of the quantum, into the final state Ψ_1 or Ψ_2.

By way of illustration, we set each of the amplitudes J_1, J_2, and J_3 in correspondence with the following diagrams,



(4)

The meaning of the diagrams is explained by relation (3), which holds true only for the case of electric dipole absorption. If the higher multipoles are taken into account, the values of the individual amplitudes and the relations between them obviously are different. However, if the photodisintegration is considered in the region of the threshold of emission of deuterons with energy on the order of 15 MeV (i.e., E_γ = 35—45 MeV), there is no need for taking these higher multipoles into account.

The integrals J_1, J_2, and J_3 were calculated by the shell-model technique described in [6].

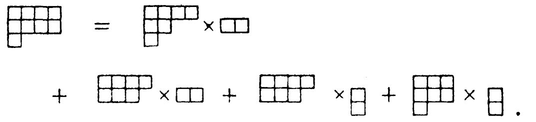
2. CROSS SECTION OF THE (γ, d) REACTIONS ON p-SHELL NUCLEI WITH PRODUCTION OF RESIDUAL NUCLEI IN LOWER STATES OF "NORMAL" PARITY

We consider in this section several specific examples, without aiming at a complete and exact description of the (γ, d) reaction. We are interested only in the energy dependence of the yield of fast deuterons near the kinematic threshold. More accurately speaking, the problem is the following. As will be shown below, various reasons connected with the nuclear structure greatly suppress transitions to the ground state of the residual nucleus with emission of fast deuterons. To explain the shift of the threshold it is therefore necessary to indicate the energy position of the group of levels closest to the ground state, levels with an excitation probability considerably in excess of the probability for producing a nucleus in the ground or weakly-excited state.

This concretization of the problem allows the calculations to be made under several simplifying assumptions which would not hold in either the analysis of the yield of fast deuterons or in the calculation of the total cross section curve in the high-energy region. Foremost among these approximations is the use of the Born approximation with account of the distortion of the deuteron wave. Further, we can confine ourselves to partial transitions to states of "normal" parity ($|s^4p^n\rangle \to |s^4p^{n-2}\rangle$). Account of the transitions to states with opposite polarity, corresponding to the excitation of one of the nucleons of the residual nucleus in a higher shell (for example, $|s^4p^n\rangle \to |s^4p^{n-3}\,2s(1d)\rangle$), which is essential in the calculation of the total cross-section curve, leads in our case only to an additional increase of the investigated effect. Finally, to describe the ground and excited states of the p-shell nuclei we use the shell functions of the LS coupling scheme. The use of the more exact functions of the intermediate coupling scheme, which is technically more cumbersome, will apparently not lead to a significant change in the results.

<u>The reaction $Be^9(\gamma, d)$</u>. The spatially-symmetrical pairs in the wave functions of the Be^9 ground state (shell configuration $|s^4p^5\ {}^{22}P\rangle$ with Young tableau [441]) make contributions to the integrals J_1 and J_2; the antisymmetrical pairs contribute to the integral J_3. It is known [7] that the lower levels of the residual Li^7 nucleus (shell configuration $|s^4p^3\rangle$), with Young tableau [421], lie 7—8 MeV above the states with Young tableaux [43].

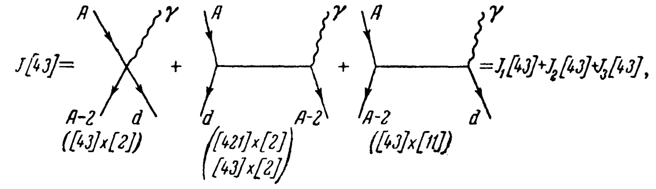
It is convenient to carry out a fractional-parentage decomposition of the configuration $|s^4p^5\ {}^{22}P\rangle$ with Young tableau [441], separating the symmetrical and antisymmetrical pairs and indicating the complete Young tableaux of the residual nucleus [8]:



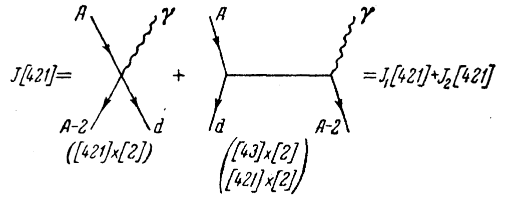
(5)

(the cages are the standard symbols for the Young tableaux in the permutation group).

The total photodisintegration amplitudes J[43] and J[421], corresponding to the production of Li^7 in states $|s^4p^3[43]\rangle$ and $|s^4p^3[421]\rangle$, are written in the form



(6)



(in this case the amplitude J_2 describes a transition via a virtual state of positive parity, corresponding to the shell configuration s^3p^4). In the parentheses under the diagrams are indicated the states of the seven and two particles from the decomposition (5) with the corresponding Young tableaux, which are genealogically separated in the wave function of the ground state and contribute to the total amplitude $J[\lambda]$.

The statistical weight of the states $[421] \times [2]$ in decomposition (6) is higher than the statistical weight of the states $[43] \times [2]$, and transitions with production of Li^7 in states $|s^4p^3[421]\rangle$, due to the amplitude J_1, predominate in expressions (7) over the transitions with Li^7 produced in states $|s^4p^3[43]\rangle$ (as already noted earlier [5]).

Two lower levels of Li^7, $|s^4p^3\ {}^{22}P\rangle$ and $|s^4p^3\ {}^{22}F\rangle$, belong to the Young tableau [43] [7]. Calculations show that transitions to the ${}^{22}F$ level are approximately one-third as strong as transitions to the ${}^{22}P$ level, owing to the specific nature of the fractional parentage of the state Be^9 $|s^4p^5[41]\ {}^{22}P\rangle$; in addition, the following relation is approximately satisfied for the transition to the ${}^{22}P$ level

$$J_2([43]\ {}^{22}P) \approx -J_3([43]\ {}^{22}P).$$

Interference of the integrals $J_1([43]\ {}^{22}P)$ and $J_2([43]\ {}^{22}P) + J_3([43]\ {}^{22}P)$ decreases somewhat the cross section of the transition from the diagram $J_1([43]\ {}^{22}P)$. The diagram $J_2[421]$ makes a small contribution to the amplitude $J[421]$ and influences little the total probability of transitions to levels with configuration $|s^4p^3[421]\rangle$.

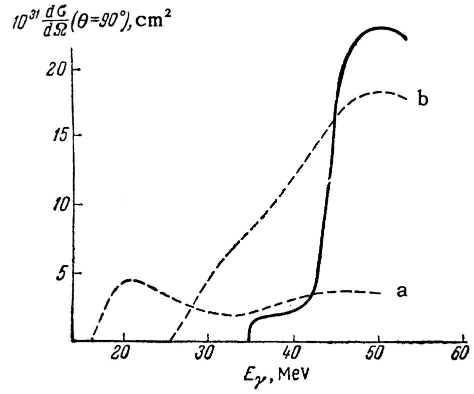
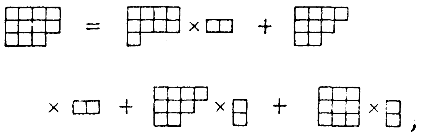


FIG. 1

Figure 1 shows the integral curve of the yield of deuterons with $E_d \geq 15$ MeV, obtained by summing all the energetically feasible transitions. Dashed curves a and b are the cross sections of the reaction $Be^9(\gamma, d)Li^7$ with production of Li^7 in states $|s^4p^3[43]\rangle$ and $|s^4p^3[421]\rangle$, respectively.

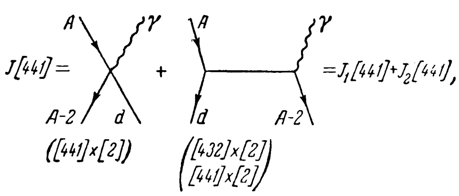
The reaction $B^{11}(\gamma, d)$. In the spectrum of the Be^9 levels, as follows from the work of French et al [9], the four lowest states $|s^4p^5\ {}^{22}P\rangle$, $|s^4p^5\ {}^{22}D\rangle$, $|s^4p^5\ {}^{22}F\rangle$, and $|s^4p^5\ {}^{22}G\rangle$ are characterized by a Young tableau [441], the higher states $|s^4p^5\ {}^{24}P\rangle$ and $|s^4p^5\ {}^{24}D\rangle$ by a tableau [432], etc.

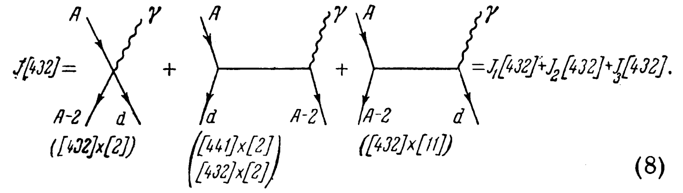
Decomposition of the B^{10} ground state $|s^4p^7\ {}^{22}P\rangle$ with tableau [442], with separation of the symmetrical and antisymmetrical pairs,



(7)

shows that a contribution to the amplitudes of the transitions J[441] and J[432], when Be^9 is produced in the states $|s^4p^5[441]\rangle$ and $|s^4p^5[432]\rangle$, is made by the following diagrams:





(8)

The fractional parentage of the B^{11} state $|s^4p^7[443]\,^{22}P\rangle$ is such that the transition to the Be^9 state $|s^4p^5[441]\,^{22}D\rangle$ is strictly forbidden ($J_1([441]\,^{22}D) = J_2([441]\,^{22}D) = 0$). The orbital angular momentum selection rule forbids the transition to the state $|s^4p^5[441]\,^{22}G\rangle$. The role of the integrals $J_1[441]$ and $J_1[432]$ for each pair of transitions to the levels $|^{22}P[441]\rangle$, $|^{24}P[432]\rangle$ and $|^{22}F[441]$, $|^{24}D[432]\rangle$ is approximately the same, but the contribution of the integrals $J_2[432] + J_3[432]$ to the total amplitude $J[432]$ is appreciably larger than the contribution of the integral $J_2[441]$ to the amplitude $J[441]$, inasmuch, as shown by calculation, the integrals $J_2[432]$ and $J_3[432]$ are contained in $J[432]$ with identical signs.

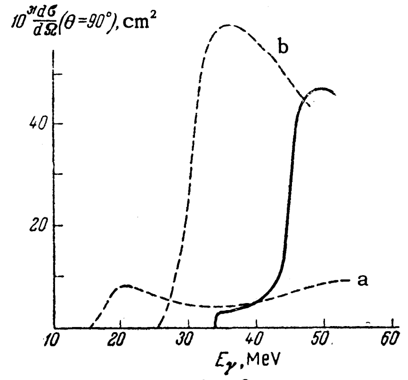


FIG. 2

Figure 2 shows the integral curve for the yield of deuterons with energy $E_d \geq 15$ MeV. The dashed curves a and b show the cross sections of the reaction $B^{11}(\gamma, d)Be^9$ with production of Be^9 in states with Young tableaux [441] and [432], respectively.

3. MECHANISM OF (γ, d) REACTION ON EVEN-EVEN NUCLEI

In this case the amplitude J_1 corresponding to the "direct" photoeffect, vanishes identically. This evokes a search for another, more complicated mechanism of the (γ, d) reaction (for example, the pickup reaction[10]). No account is taken here, however, of exchange effects connected with the polarizability of the scattered nuclei.

As shown in Sec. 1, these effects are described by terms J_2 and J_3 of the total photodisintegration amplitude (6). Our problem is to ascertain the role they play in the case of an even-even nucleus. An attempt to develop a theory of the (γ, d) reaction along these lines was made by Madsen and Henley [11]. Translated into our language, they took account in some manner of only the amplitude J_3, corresponding to the transition of an antisymmetrical nucleon pair onto a symmetrical (deuteron) state. We see that in such an approach the main features of the theory of "direct" emission and pickup theory are retained: according to these theories, the only states of the residual nucleus that can be realized in the (γ, d) reaction are those corresponding to the extraction of a pair of nucleons from the ground state of the target nucleus (i.e., those represented in the pair fractional-parentage expansion of the function of this state). As will be shown below, an account of the amplitude J_2 leads to more complicated states of the residual nucleus.

Let us consider first transitions to levels of "normal" parity. In practice we can deal with only two pairs of the p-shell nuclei— C^{12} and O^{16} (Be^8 is unstable). The ground states of these nuclei in the LS coupling are described by shell configurations $|s^4p^8[44]\,^{11}S\rangle$ and $|s^4p^{12}[4444]\,^{11}S\rangle$. The spins of the ground states of C^{12} and O^{16} are equal to $S_i = 0$, while the spin of the deuteron is $S_d = 1$. Therefore, according to the rule for the addition of momenta, the spins of the residual nuclei are $S_f = S_d = 1$, It is easy to show[12] that the only allowed states of "normal" parity of the residual nuclei B^{10} and N^{14}, for which the amplitudes J_2 and J_3 differ from zero, are the states $|s^4p^6[433]\,^{33}P\rangle$ and $|s^4p^{10}[4433]\,^{33}P\rangle$.

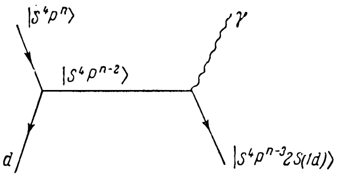
The cross section of the γ, d reaction with production of B^{10} and N^{14} in states $|s^4p^6\rangle$ and $|s^4p^{10}\rangle$ is proportional to the square of the modulus of the expression

$$J_2 + J_3 \sim \int_0^\infty \varphi_d R_{00} r^2\,dr \int_0^\infty R_{00} R_{11} r^3\,dr - \int_0^\infty \varphi_d R_{11} r^3\,dr, \qquad (9)$$

where $R_{nl}(r)$ —functions of the harmonic oscillator and φ_d —radial wave function of the deuteron. For the Hulthén deuteron function[13], the relation $J_2 \approx -J_3$ is approximately satisfied [exact equality takes place with the oscillator deuteron function $\varphi_d = R_{00}(r)$]. The amplitudes J_2 and J_3 have a common integral in the relative-motion variable $R_{A_1A_2}$, and the result obtained will hold for both the plane and the distorted deuteron wave. Thus, although each of the amplitudes J_2 and J_3 corresponding to transitions to "normal" parity levels can assume a sufficiently large value, their addition leads in the case of the even-even nucleus to strong suppression of such transitions.

On the other hand, the structure of the photodisintegration amplitude $J_1 + J_2 + J_3$ points to the possibility of dipole excitations of the residual nuclei, described by configurations $|s^4p^{n-1}2s(1d)\rangle$ or $|s^3p^{n+1}\rangle$. Direct calculation shows that the probability of fast-neutron emission accompanied by dipole excitation of the residual nucleus exceeds greatly the total probability of the transitions to the normal-parity levels. A contribution to the cross section for the production of states of the type $|s^3p^{n+1}\rangle$ is made by both diagrams J_2 and J_3, while only J_2 contributes to $|s^4p^{n-1}2s(1d)\rangle$.

For reasons explained in detail in Sec. 2, we do not consider the highly excited "hole" states $|s^3p^{n+1}\rangle$, and concentrate our attention on the diagram J_2:



(10)

The set of intermediate (virtual) states of the residual nucleus is determined by the pair fractional-parentage expansion of the function of the target-nucleus ground state. In the case of an even-even nucleus these are the states ^{13}D and ^{13}S, which correspond to orbital momenta $L_0 = 0$ and 2 of the relative motion of the reaction products. The fractional parentage of the ground states of even-even nuclei is such that the statistical weight of the D states exceeds the weight of the S states. It is known, on the other hand, that the ground states of Be^{10} and N^{14} can be represented with good accuracy by the component $|s^4p^n\ {}^{13}D\rangle$. Therefore, were we able to neglect the share of the S state, then the entire energy dependence of diagram (10) would be determined by the matrix element of the dipole excitation of the residual nucleus[2)] and could be derived from the experimental curve for the giant photoabsorption resonance of this nucleus.

An account of the S state cannot change the result qualitatively: by virtue of the collective nature of the dipole giant resonance, the positions of the dipole excitation of the states $|s^4p^n\ {}^{13}D\rangle$ and $|s^4p^n\ {}^{13}S\rangle$ cannot differ essentially (the levels themselves are shifted relative to each other by about 2—3 MeV).

4. CONCLUSIONS

In spite of the many crude approximations—neglect of the interaction in the final state, oscillator wave functions, dipole interaction between radiation and the nucleus—some general conclusions can be drawn with respect to the singularities of the (γ, d) reaction on light nuclei.

1. The sharp suppression of the yield of fast deuterons near the kinematic threshold is explained by the fact that the most intense transitions, corresponding to direct emission of photodeuterons in dipole absorption of the γ quanta, lead not to the ground state but to excited states of the residual nucleus. In the calculation of the probabilities of these transitions it is important to take into account the individual structure of the light nuclei, in particular the additional quantum numbers $[\lambda]$ connected with the symmetry properties of the wave functions of the states.

2. The calculations presented above show that the polarization (exchange) terms J_2 and J_3 in the total amplitude of dipole interaction between the γ quanta and the nucleus play a very important role. Within the system of concepts employed in modern physics of photonuclear reactions, a notion that has taken firm hold is that the operator responsible for the direct emission of photodeuterons is directly proportional to the classical dipole moment of the deuteron + residual nucleus system [15]. The fact that the deuteron and the residual nucleus have almost the same e/m ratio makes this moment very small. This leads to the conclusion that the mechanism of direct emission of photodeuterons is small on the whole. The usual alternative indicated is the mechanism whereby the deuterons are emitted via a compound nucleus and pickup reaction.

The foregoing conclusion regarding the importance of the exchange processes increases appreciably the scope of applicability of the theory of direct photodisintegration in the (γ, d) reaction, and in particular leads to the following characteristic effect: when fast deuterons are emitted, there is also high probability for the realization of residual-nucleus states which are not represented in the fractional-parentage expansion of the target-nucleus wave function.

3. In the photodisintegration of the even-even nuclei C^{12} and O^{16}, with emission of fast deuterons, the formation of the nuclei B^{10} and N^{14} in states of "normal" parity is appreciably suppressed. The residual-nucleus levels closest to the ground state, to which intense transitions correspond, are those in the region of photoabsorption

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[2)] It can be assumed that account of the distortion of the deuteron wave in the complex optical potential will cause additional suppression of the S state compared with the D state [14].

giant resonance. This leads to a shift in the threshold of the (γ, d) reaction by about 15–20 MeV (compared with the value of order 2 MeV which follows from the usual isotopic spin selection rules), and the emission of the deuterons is accompanied in this case by the emission of protons and neutrons, owing to the decay of the dipole states of the nuclei B^{10} and N^{14}. A high probability of nucleon emission was observed experimentally[4] in the reaction $O^{16}(\gamma, d)$, albeit with scanty statistics.

To check the indicated mechanism further detailed investigation of the characteristics of the reactions $C^{12}(\gamma, d)$ and $O^{16}(\gamma, d)$ is necessary, in parallel with a study of the photoabsorption in the region of giant resonance in B^{10} and N^{14}. As follows from Sec. 3, these reactions should be very close not only in excitation energy but also in such characteristics as the spectrum of the nucleus.

The mechanism of dipole interaction of the residual nucleus in the (γ, d) reaction undoubtedly takes place also in the disintegration of other nuclei. Here, however, it is not decisive in the question of the threshold shift.

The authors are grateful to A. P. Komar and co-workers for furnishing the experimental data [1-4] and to A. M. Baldin for a discussion of the results and for critical remarks.

[1] Volkov, Kulikov, and Chizhov, JETP **42**, 61 (1962), Soviet Phys. JETP **15**, 42 (1962).

[2] Yu. M. Volkov and L. A. Kul'chitskiĭ, JETP **42**, 108 (1962), Soviet Phys. JETP **15**, 77 (1962).

[3] Chizhov, Komar, Kulchitsky, Kulikov, Makhnovsky, and Volkov, Nucl. Phys. **34**, 562 (1962).

[4] Komar, Kulkov, Chizhov, Yavor, and Volkov, JETP **43**, 1657 (1962), Soviet Phys. JETP **16**, 1168 (1963).

[5] V. V. Balashov and V. N. Fetisov, Izv. AN SSSR ser. fiz. **26**, 1188 (1962), Columbia Tech. Transl. p. 1199.

[6] Balashov, Neudachin, Smirnov, and Yudin, JETP **37**, 1385 (1959), Soviet Phys. JETP **10**, 983 (1960).

[7] S. Meshkov and C. W. Ufford, Phys. Rev. **101**, 734 (1956).

[8] H. Jahn and van Wieringen, Proc. Roy. Soc. **A209**, 512 (1951).

[9] French, Halbert, and Pandya, Phys. Rev. **99**, 1387 (1955).

[10] J. Sawicki and W. Czyz, Bull Acad. Polon. Sci. **5**, 631 (1957).

[11] V. A. Madsen and E. M. Henley, Nucl. Phys. **33**, 1 (1962).

[12] Elliott, Hope, and Jahn, Phil. Trans. Roy. Soc. **246**, 241 (1953).

[13] L. Hulthen and M. Sugawara, Handbuch Physik **39**, (1957).

[14] G. M. Shklyarevskiĭ, JETP **39**, 1031 (1961), Soviet Phys. JETP **12**, 717 (1962).

[15] J. S. Levinger, Nuclear Photodisintegration, Oxford University Press, 1960.

Translated by J. G. Adashko
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