# *SOME NEUTRINO PAIR PRODUCTION PROCESSES IN STARS*

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Two processes of neutrino pair production in stars are examined, those occurring during recombination of ionized atoms and those due to $\beta$-interaction of electrons and positrons with nuclei. The reaction cross sections and neutrino energy losses are calculated. Energy emission from stars by neutrino pairs produced as a result of atom recombination is significant in white stars if their central temperature reaches 5—10 keV. $\beta$-interaction of electrons and positrons with nuclei (electron and positron capture of nuclei, $\beta$-decay) is accompanied by enormous losses of energy carried off by the neutrinos (up to $\sim 10^{18}$ erg/g-sec) at temperatures sufficient for electron-positron pair production. The process may be of interest for studying stars in their last stage of evolution and supernova bursts.

MANY neutrino-antineutrino pair production processes have been considered recently [1-5] from the point of view of astrophysics. The main hypothesis underlying the neutrino pair production reactions is the possibility of direct interaction between electrons and neutrinos [6].

In the present article we consider two new neutrino pair production processes, by recombination of ionized atoms and by $\beta$ interaction of electrons and positrons with nuclei. We calculate the cross sections of the processes and the neutrino energy losses, and indicate the possible role of the processes in astrophysics.

## 1. NEUTRINO PAIR PRODUCTION BY RECOMBINATION OF ATOMS

The matter inside the stars is in a partially or completely ionized state. Under these conditions, if a weak neutrino-electron interaction exists, the electrons can go from the free to the bound state with emission of a neutrino pair $\nu\bar{\nu}$ (the analog of the inverse photoeffect in electrodynamics):

$$e^-_{\text{free}} \to e^-_{\text{bound}} + \nu + \bar{\nu}.$$

At high temperatures the electron has a short lifetime: the photoeffect returns the bound electron to the continuous-spectrum state. Since matter and radiation in a star are in thermodynamic equilibrium, and the rate of recombination of the atoms with $\nu\bar{\nu}$ pair emission is many orders of magnitude smaller than the rate of the electromagnetic processes, the number of free electrons states of the atom remains constant.

In calculating the cross section of the process we confine ourselves to the case of recombination on the K shell of the atom, which makes the main contribution (as in the photoeffect). Calculations pertaining to the conditions in stars are carried out in the nonrelativistic approximation.

The density of the neutrino-electron interaction Lagrangian is taken in the form

$$L = -2^{-1/2} G\,[\bar{\psi}_e \gamma_\mu (1+\gamma_5)\psi_\nu]\,[\bar{\psi}_\nu \gamma_\mu (1+\gamma_5)\psi_e].$$

Assuming that the kinetic energy of the electron is much larger than the ionization potential of the K electron

$$E_e \gg I = \alpha^2 Z^2 mc^2/2,$$

we take the electron wave function in the initial state in the form of a plane wave $e^{ipx}u(p)$, $p = (\mathbf{p}, E)$, neglecting the influence of the Coulomb field (the assumption that $E_e \gg I$ coincides with the condition that the Born approximation be valid).

The wave function of the K electron for light nuclei, when $\alpha Z/2 \ll 1$, can be written accurate to $\alpha Z/2$ in the form [7]

$$\psi_K = \pi^{-1/2} a^{-3/2} \exp(-r/a)\, u_0, \quad a = \hbar^2/me^2 Z,$$

$u_0$ — constant bispinor with components

$$u_{0_{\sigma\Pi = 1/2}} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \qquad u_{0_{\sigma\Pi = -1/2}} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}.$$

In calculating the matrix elements it is convenient to use the Fierz transformation. The transformed matrix element has the form

$$M = 2\pi \frac{G}{\sqrt{2\pi a^3}} \delta(\varepsilon + \omega_\nu + \omega_{\bar{\nu}} - E)$$
$$\times \int [\bar{u}_0 \gamma_\mu (1+\gamma_5) u(p)]\, [\bar{u}(k_\nu) \gamma_\mu (1+\gamma_5) u(k_{\bar{\nu}})]$$
$$\times \exp[-\eta r + ir(\mathbf{p} - \mathbf{k}_\nu - \mathbf{k}_{\bar{\nu}})]\, dV,$$
$$\eta = 1/a = \sqrt{2mI}, \qquad \varepsilon \approx mc^2 - I,$$

$\omega_\nu$, $\omega_{\bar\nu}$, $k_\nu$, $k_{\bar\nu}$ — energy and momentum of the neutrino and antineutrino; $\epsilon$ — energy of K-electron.

The square of the matrix element is averaged over the initial polarizations of the electron and is summed over the final polarizations of the electron and the emitted neutrino and antineutrino. In the nonrelativistic approximation, assuming that $|p| \ll m$, $\hat{p} \sim \gamma_4 E$, $q = p - k_\nu - k_{\bar\nu}$, we get

$$\sum |M|^2 = 256\,(2\pi)^2 \frac{G^2\eta^5}{(\eta^2+q^2)^4}\frac{E}{m}\,\omega_\nu\omega_{\bar\nu}\,\delta\,(\omega_\nu+\omega_{\bar\nu}-E_e-I).$$

We measure the angles from the direction of the initial electron momentum $\mathbf{p}$. The angle between the antineutrino momenta $\mathbf{k}_{\bar\nu}$ and the electron will be denoted by $\theta$, the angle between the neutrino momenta $\mathbf{k}_\nu$ and the electron by $\vartheta$, and the angle between the planes in which the vectors $\mathbf{p}$, $\mathbf{k}_\nu$ and $\mathbf{p}$, $\mathbf{k}_{\bar\nu}$ lie will by $\varphi$.

After simple calculations we obtain for the differential cross section of the process the expression

$$d\sigma = \frac{128}{(2\pi)^3}\frac{G^2\eta^5}{v} \times \frac{\omega_{\bar\nu}^2(E_e+I-\omega_\nu)^2\,d\omega_\nu\,d(\cos\theta)\,d(\cos\vartheta)\,d\varphi}{(\eta^2+p^2+k_\nu^2+k_{\bar\nu}^2-2pk_{\bar\nu}\cos\theta-2pk_\nu\cos\vartheta+2k_\nu k_{\bar\nu}\cos\chi)^4},$$

v — velocity of the initial electron and $\chi$ — angle between the momenta of the neutrino and antineutrino. In the nonrelativistic approximation the maximum of $d\sigma$ is reached when $\theta = \vartheta = 0$, i.e., when the neutrino and antineutrino are emitted in the direction of the initial electron momentum.

For the sake of simplicity, small quantities in the denominator can be neglected in the calculation of the total cross section. From the energy conservation law $E_e + I = \omega_\nu + \omega_{\bar\nu}$ and $I = \eta^2/2m$ we have

$$p^2 + \eta^2 = 2m\,(\omega_\nu + \omega_{\bar\nu});$$

$$|\mathbf{p}| \ll m, \quad |\mathbf{k}_\nu| < |\mathbf{p}|, \quad |\mathbf{k}_{\bar\nu}| < |\mathbf{p}|.$$

Therefore the term $2k_\nu k_{\bar\nu}\cos\chi$ can be omitted, and the remaining expression can be expanded in a series with only the first term retained.

Integration yields for the total cross section of the recombination of one electron on the K shell

$$\sigma = \frac{64}{15\pi^2}\frac{G^2\eta^5}{v}\frac{(E_e+I)^5}{(\eta^2+p^2)^4}.$$

If we recognize that $\eta = \sqrt{2mI} = \alpha Zmc$, then

$$\sigma = \frac{16\sqrt{2}}{15\pi^2}\frac{G^2m^2}{\hbar^4}\frac{c}{v}I^{5/2}(E_e+I) = \sigma_0 Z^5\frac{c}{v}(E_e+I),$$

$$\sigma_0 = \frac{4}{15\pi^2}\alpha^5\frac{G^2m^2}{\hbar^4} = 0{,}76\cdot 10^{-56}\ \text{cm}^2,$$

$E_e$ and I are in $mc^2$ units. The total cross section for recombination on the K shell depends essentially on the atomic number: $\sigma \sim Z^5$ (as in the photoeffect).

The energy transferred to the neutrino pair consists of the kinetic energy of the initial electron and the binding energy released when the electron goes from the free to the bound state:

$$\omega_\nu + \omega_{\bar\nu} = E_e^{\text{st}} + I.$$

The total energy radiated from a unit volume of matter per unit time is given by

$$q = \int \sigma(E_e)\,(E_e+I)\,vN\,dn,$$

where N — number of free states on the K shell of the atom, and dn — energy distribution of the free electrons. For a Maxwellian electron distribution the number of free states on the K shell is

$$N = 2\frac{N_0}{A}\rho\,(1-f) = 2\frac{N_0}{A}\left\{1-\left[1+\exp\left(-\frac{\mu+I}{kT}\right)\right]^{-1}\right\},$$

$$\exp\left(-\frac{\mu}{kT}\right) = 2\left(\frac{mkT}{2\pi^2\hbar^2}\right)^{3/2}\frac{A}{N_0Z\rho},$$

where $N_0$ — Avogadro's number, $\rho$ — density of matter, T — temperature, $\mu$ — chemical potential, and A — atomic weight.

As a result we obtain after integration with a Maxwellian electron distribution

$$q = \frac{60\sigma_0}{mc}N_0^2\frac{Z^6}{A^2}\rho^2(kT)^2(1-f)\left[1+\frac{1}{5}\frac{I}{kT}+\frac{1}{30}\left(\frac{I}{kT}\right)^2\right].$$

Since $kT \gg I$, the last two terms in the square brackets can be neglected. Substituting the values of the constants in the formula and expressing the temperature in keV, we obtain ultimately for the neutrino energy losses

$$q = 1{,}54\cdot 10^{-8}Z^6A^{-2}\rho^2T^2\,(1-f)\ \text{erg/cm}^3\,\text{sec}$$
$$f = [1 + 320\,AT^{3/2}/Z\rho]^{-1}.$$

For a degenerate electron gas we consider the case when $E_F \gg kT$. The number of free states on the K shell is

$$N = 2\frac{N_0}{A}\rho\,(1-f) \approx 2\frac{N_0}{A}\rho\exp\left(-\frac{E_F}{kT}\right), \quad \mu \approx E_F.$$

The neutrino energy losses are

$$q \approx \int_0^{p_\Phi}\sigma vE_eN\frac{p^2\,dp}{\pi^2\hbar^3} = \frac{\sigma_0Z^5N_0}{14\pi^2m^3\hbar^3A}\rho p_\Phi^7\exp\left(-\frac{E_F}{kT}\right),$$

$p_f$ — limiting Fermi momentum, $p_f = (3\pi^2\hbar^3N_0Z\rho/A)^{1/3}$. Expressing the temperature in keV, we obtain

$$q \approx 1{,}45\cdot 10^{-13}Z^4(Z/A)^{10/3}\rho^{10/3}\exp(-E_F/T)\ \text{erg/cm}^3\,\text{sec}$$

In matter of high density, when the distance between the nuclei becomes of the order of the Bohr radius $r_a$, collective interaction effects cause the K shell to "smear out" and the energy level to broaden. Therefore the formulas given above for the neutrino energy losses are valid so long as $r_{nuc} > r_a$ ($r_{nuc}$ — characteristic distance between the nuclei).

In order of magnitude we have $r_{nuc} \sim (A/N_0\rho)^{1/3}$ and $r_a = 0.53 \times 10^{-8} Z^{-1}$ cm. From the requirement $r_{nuc} > r_a$ follows a condition for the applicability of the formulas for the neutrino energy losses: $\rho_{max} < 10AZ^3$. When $\rho \geq \rho_{max}$ the expressions for q give the order of magnitude of the result.

The neutrino energy losses in recombination depend essentially on the atomic number ($q \sim Z^4$), so that the energy losses in a star increase sharply with increasing concentration of the heavy elements.

A comparison of the recombination process with other neutrino production processes shows[8] that recombination gives rise to the main energy loss of the star at temperatures up to 20—30 keV and at densities up to $10^5$ g/cm³. Bremsstrahlung of the neutrino pairs turns out to be comparable with recombination only for the reaction on hydrogen:

$$q_{rec}/q_{br} \approx 56Z^3T^{-2,5} \qquad (T \text{ in keV}).$$

A calculation of the neutrino luminosity of a star due to recombination, analogous to that carried out in [2], shows that in stars with temperature 5—10 keV in the center, Z = 12, and average density $10^3$—$10^4$ g/cm³ the radiation of energy from the star in the form of neutrino pairs exceeds the optical radiation. Densities of this magnitude are found in white dwarfs. If the temperature in the center of the star reaches 5—10 keV (a value obtained by calculation for some models), then the white dwarfs may be the objects in which neutrino radiation due to recombination plays a major role and determines the main energy loss.

Of course, this still leaves the main question of the existence of direct interaction between the neutrino and the electron.

## 2. NEUTRINO PAIR PRODUCTION IN $\beta$ INTERACTION BETWEEN NUCLEI AND ELECTRONS OR POSITRONS

Gamow and Schoenberg[9] first called attention to the role of $\beta$ interaction in energy radiation from the stars. They considered two opposite reactions: the decay of $\beta$-active nuclei and the formation of such nuclei upon collision between fast electrons with stable nuclei.

$$_ZN^A + e^- \rightarrow {}_{Z-1}N^A + \nu, \qquad {}_{Z-1}N^A \rightarrow {}_ZN^A + e^- + \bar{\nu}.$$

The cross section for electron capture of a stable nucleus increases with temperature, and the rate of $\beta$ decay of unstable nuclei remains constant. As a result, $\beta$-active nuclei accumulate and the number of stable nuclei decreases: at a temperature $kT \geq Q$ (Q — decay energy) the number of stable pairs $n(Z) \rightarrow 0$, while the number of $\beta$-active nuclei $n(Z-1)$ tends to the initial concentration of the stable nuclei $n_0(Z)$. At temperatures $kT \sim Q$ this leads to "saturation" of the energy carried away by the neutrino: $q \sim Qn_0(Z)\lambda$, where $\lambda$ is the $\beta$-decay constant. By virtue of this, the Gamow-Schoenberg process is less effective at high temperatures than the processes resulting from electron-neutrino interaction ($e^- + \gamma \rightarrow e^- + \nu + \bar{\nu}$, $e^+ + e^- \rightarrow \nu + \bar{\nu}$).

The situation changes radically if account is taken of the fact that at high temperatures, when kT becomes comparable with the electron rest energy $mc^2$, production of electron-positron pairs sets in [10]. The presence of positrons opens up a new channel for the reaction: positron capture of an unstable nucleus. The competition between the $\beta$ decay and positron capture of the unstable nucleus causes the latter to predominate because its rate increases with the temperature. Since at the temperatures $kT \sim Q$ the cross sections for electron capture of a stable nucleus and for positron capture of an unstable nucleus are approximately equal, and their rates exceed that of the $\beta$ decay of the unstable nucleus, the number of stable and unstable nuclei is approximately evenly distributed, and amounts to $\sim n_0(Z)/2$.

The rate of electron and positron capture is proportional to $\sim (kT)^2 \exp(-Q/kT)$, while the number of electrons and positrons is proportional to $\sim (kT)^3 \exp(-mc^2/kT)$, so that the energy transferred to the neutrino pairs depends on a high power of the temperature $\sim (kT)^6 \exp\{-(Q+mc^2)/kT\}$ and there is no "saturation" of energy.

In the presence of positrons, the process includes one direct and two inverse reactions:

$$_ZN^A + e^- \rightarrow {}_{Z-1}N^A + \nu, \tag{1}$$

$$_{Z-1}N^A + e^+ \rightarrow {}_ZN^A + \bar{\nu}, \tag{2}$$

$$_{Z-1}N^A \rightarrow {}_ZN^A + e^- + \bar{\nu}. \tag{3}$$

If the nucleus $_ZN^A$ is stable while the nucleus $_{Z+1}N^A$ is $\beta^+$-active, then analogous reactions occur, except that the electron and positron change places.

The cross section for the capture of an electron (positron) by a nucleus can be readily obtained by using the experimental values of the relative half-life tf of the unstable nucleus[11-12]:

$$\sigma = \frac{2\pi^2 \ln 2}{tf} \frac{\hbar^3}{m^3 c^3} \frac{(E \pm Q)^2}{v},$$

E — electron (positron) energy in $mc^2$ units and v — relative electron velocity. The "plus" sign pertains to the cross section for the capture of the unstable nucleus, while the "minus" sign pertains to the capture of the stable nucleus (in the latter case $\sigma = 0$ when $E \leq Q$).

The rate of the reactions (1)—(3) and the energy transferred to the neutrino pairs in a medium with temperature T depend on the concentration of the stable and $\beta$-active nuclei. The equilibrium concentration of either nucleus is determined from the condition that the number of the direct and inverse reactions be equal:

$$n(Z) \int \sigma(1)\, v\, dn = n(Z \pm 1)\left[\lambda + \int \sigma(2)\, v dn\right],$$

$$n(Z) + n(Z \pm 1) = n_0(Z),$$

dn — equilibrium distribution of the electron (positron) energy.

The energy carried away by the neutrino per unit volume of the medium and per unit time is determined by

$$q = \int [\sigma(1)(E - Q)\, n(Z) + \sigma(2)(E + Q)\, n(Z \pm 1)] \times v\, dn + a\lambda Q n(Z \pm 1),$$

$$a = \tfrac{2}{3} \text{ for } kT \ll mc^2, \quad a = \tfrac{1}{2} \text{ for } kT \gg mc^2.$$

For the case when $kT \gg E_F$

$$q = \frac{2 \ln 2}{tf} mc^2 \left(\frac{kT}{mc^2}\right)^6 [I_1 n(Z) + I_2 n(Z \pm 1)] + a\lambda Q n(Z \pm 1),$$

$$I_1 = \int_{(mc^2+Q)/kT}^{\infty} \left(x - \frac{Q + mc^2}{kT}\right)^3 \frac{x\sqrt{x^2 - (mc^2/kT)^2}\, dx}{1 + e^x},$$

$$I_2 = \int_{mc^2/kT}^{\infty} \left(x + \frac{Q - mc^2}{kT}\right)^3 \frac{x\sqrt{x^2 - (mc^2/kT)^2}\, dx}{1 + e^x}.$$

Simple formulas for neutrino energy losses are obtained in two limiting cases:

1) when $kT \ll mc^2$, $kT \ll Q$

$$q \approx \frac{12 \ln 2}{tf} n_0(Z)\, mc^2 \left(\frac{Q + mc^2}{kT}\right)^2 \left(\frac{kT}{mc^2}\right)^4 \exp\left(-\frac{Q + mc^2}{kT}\right)$$

$$= \frac{4 \cdot 10^{18}}{Atf} \rho \left(\frac{Q + mc^2}{mc^2}\right)^2 \left(\frac{kT}{mc^2}\right)^4 \exp\left(-\frac{Q + mc^2}{kT}\right) \text{erg/cm}^3\text{sec},$$

2) when $kT \gg mc^2$, $kT \gg Q$

$$q \approx \frac{240 \ln 2}{tf} n_0(Z)\, mc^2 \left(\frac{kT}{mc^2}\right)^6 = \frac{0.8 \cdot 10^{20}}{Atf} \rho \left(\frac{kT}{mc^2}\right)^6 \text{erg/cm}^3\text{sec}.$$

The neutrino energy losses are significant for nuclei with low energy threshold Q and relative half-life tf. Processes with formation of $\beta^-$-active nuclei (electron capture of a stable nucleus) are most effective in the nuclei $H^1$, $Li^6$, and $Fe^{56}$ while those with production of $\beta^+$-active nuclei in $B^{11}$, $N^{14}$, $F^{19}$, $Na^{23}$, etc.

Owing to the high capture energy threshold, the nuclei of helium and helium reaction products have low effectiveness.

At temperatures 100—1000 keV the neutrino energy losses constitute in this process $\sim 10^6$—$10^{18}$ erg/g-sec (i.e., $10^{39}$—$10^{51}$ erg/sec for the mass of the sun). In this temperature region the main process is formation of neutrino pairs upon annihilation: $e^+ + e^- \rightarrow \nu + \bar{\nu}$. The energy losses in this case are equal to[13] $q \approx 3.6 \times 10^{22} (kT/mc^2)^9$, $kT \gg mc^2$. Comparing this process with annihilation, we obtain

$$q/q_{e^+ + e^-} \sim 10^{-5} \rho A^{-1} (mc^2/kT)^3 \quad (tf \sim 3 \cdot 10^3),$$

i.e., at densities $10^7$—$10^8$ g/cm$^3$ and at $\sim 1000$ keV, the neutrino energy losses of both processes coincide in order of magnitude (at such densities $E_F \sim kT$, so that the estimate is crude).

The high temperatures and densities considered above can occur in stars differing in their last stage of evolution, during the stage of gravitational compression, and in supernova bursts. Under such conditions, the $\beta$ interaction of the electrons and positrons with the nuclei can be just as important as the annihilation $e^+ + e^- \rightarrow \nu + \bar{\nu}$ [13], and in the absence of the latter it can play the major role.


[1] B. M. Pontecorvo, JETP **36**, 1615 (1959), Soviet Phys. JETP **9**, 1148 (1959).

[2] G. M. Gandel'man and V. S. Pinaev, JETP **37**, 1072 (1959), Soviet Phys. JETP **10**, 764 (1960).

[3] H. Y. Chiu and R. Stabler, Phys. Rev. **122**, 1317 (1961).

[4] V. I. Ritus, JETP **41**, 1285 (1961), Soviet Phys. JETP **14**, 915 (1962).

[5] S. G. Matinyan and N. N. Tsilosani, JETP **41**, 1681 (1961), Soviet Phys. JETP **14**, 1195 (1962).

[6] R. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

[7] A. I. Akhiezer and V. B. Berestetskiĭ, Kvantovaya elektrodinamika (Quantum Electrodynamics), 2d. ed. Fizmatgiz, 1959.

[8] V. S. Pinaev, Voprosy kosmogonii (Problems of Cosmogony), AN SSSR, 1963.

[9] G. Gamow and M. Schoenberg, Phys. Rev. **59**, 539 (1941).

[10] L. D. Landau and E. M. Lifshitz, Statisticheskaya fizika (Statistical Physics), Gostekhizdat, 1951.

[11] A. M. Feingold, Revs. Modern Phys. **23**, 10 (1951).

[12] B. S. Dzhelepov and L. N. Zyryanova, Vliyanie elektricheskogo polya atoma na $\beta$ raspad (Effect of Electric Field of Atom on $\beta$ Decay), AN SSSR, 1956.

[13] H. Y. Chiu, Ann. of Physics **16**, 321 (1961).

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98