ON THE THEORY OF GIGANTIC OSCILLATIONS IN ULTRASONIC ABSORPTION

S. V. GANTSEVICH and V. L. GUREVICH

Institute of Semiconductors, Academy of Sciences, U.S.S.R.

Submitted to JETP editor February 21, 1953

J. Exptl. Theoret. Phys. (U.S.S.R.) **45**, 587-594 (September, 1963)

A theory of gigantic oscillations in the absorption of ultrasound by metals in a magnetic field is developed for the case in which the absorption of the ultrasound phonon may involve electron transitions for which the Landau quantum number changes. The angle between the magnetic field and the wave vector of the photon is assumed to be arbitrary, but not too close to a right angle. Only assumptions of a very general nature are made regarding the conduction electron spectrum. Expressions are obtained for the periods of oscillation and for the height and shape of the oscillation peaks near the absorption maxima. It is found that for transitions involving a change in the Landau quantum number, the oscillations are not rigorously periodic with respect to the inverse magnetic field. Deviations from periodicity are investigated; usually they are small, but special cases are possible in which the oscillations become aperiodic and the distance between the individual peaks may increase appreciably.

THE effect of gigantic oscillations of the absorption of ultrasound by metals in a magnetic field was predicted theoretically by Gurevich, Skobov, and Firsov [1] and was observed experimentally by Korolyuk and Prushchak. [2] This is a quantum effect, which can occur under the conditions

$$\zeta \gg \hbar\Omega \gg kT, \tag{1}$$

where ζ is the Fermi energy, Ω the Larmor frequency, k the Boltzmann constant, and T the temperature.

The effect was studied in [1] in regions of the magnetic field which, roughly speaking, satisfy the inequality $\kappa R < 1$, where κ is the wave vector of the ultrasound and R is the Larmor radius of the conduction electrons. The present work is devoted to a study of the case $\kappa R > 1$.

1. We begin with a qualitative consideration of the effect for arbitrary character of the electronic spectrum and an arbitrary (but sufficiently different from right) angle between the direction of the magnetic field and the wave vector κ. We assume that the trajectories of the electrons in quasimomentum space are closed, although in this case the Fermi surface itself can apparently be open. When the mean free path of the conduction electrons L is sufficiently large, the absorption of sound can be regarded as the process of the absorption of a sound quantum (phonon) by the conduction electron. From the laws of conservation of energy and quasimomentum, the following relation follows for such processes:

$$\varepsilon_n(p_z) + \hbar\omega(\varkappa) = \varepsilon_{n+l}(p_z + \hbar\varkappa_z). \tag{2}$$

Here n is the Landau quantum number—the number of the discrete electron level, p_z is the projection of the quasi-momentum of the electron on the z axis—the direction of the magnetic field, ω is the ultrasonic frequency.[1)]

Metals are characterized by $n \gg 1$ and l a small number on the order of several times unity. But when $n \gg l$ the quasi-classical analysis yields

$$\varepsilon_{n+l}(p_z) = \varepsilon_n(p_z) + \hbar\Omega l, \tag{2a}$$

where $\Omega = eH/mc$, $m = m(\epsilon, p_z)$ is the effective mass of the conduction electrons on the given trajectory, the connection of which with the electron spectrum in the absence of the magnetic field was established in [3]. Taking (2a) into account, and carrying out an expansion in terms of the small quantity $\hbar\kappa_z$, one can rewrite (2) in the form

$$l\Omega + \tilde{v}_z\varkappa_z = \omega, \tag{3}$$

where $\tilde{v}_z = \partial\epsilon_n/\partial p_z = (v_z)_{n+l,n+l}$ is the mean velocity of the electron in the direction of the magnetic field.

For a given H this equation determines one or several values of p_z^l, which belong to electrons taking part in the sound absorption. The electrons

[1)]For simplicity we shall not take into account in these qualitative discussions, the spin part of the energy of the electron in the magnetic field. It will be considered below in the construction of the quantitative theory.

with different p_z will not take part in the absorption in the absence of scattering. We note that the constant $\hbar$ does not enter into (3). Therefore, this condition can also be introduced from classical considerations. It was obtained by one of the authors [4] in the classical theory of sound absorption in metals in a magnetic field, and has the sense of a resonance condition which is satisfied for electrons falling in a plane of equal phase of the sound wave during the time of the revolution.

Thus, on the one hand, the quasi-momenta of the electrons taking part in the sound absorption must satisfy the condition (3). On the other hand, they must belong to the electron states in the region of diffuseness of the Fermi distribution, of thickness kT.[2)] For $\hbar\Omega \ll kT$, when the classical theory constructed in [4] is valid, the second requirement does not impose any additional limitations on the obtainable values of p_z, inasmuch as in this case, electron states with all p_z, from $-p_F$ to p_F, are absent from the region of diffuseness of the Fermi distribution; here p_F is the Fermi momentum in the given direction. But for $\hbar\Omega \gg kT$ intervals of permitted and forbidden values of p_z are observed in the region of diffuseness of the Fermi distribution; this places additional limitations on the attainable values of the quasi-momentum of the electrons taking part in the absorption.

In order to understand the state of affairs, let us turn to Fig. 1. In it are drawn curves for the

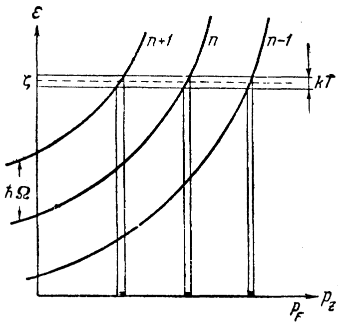

FIG. 1.

dependence of the energy of the electron on p_z for several n close to the Fermi level. These curves are intersected by a band of width kT which corresponds to the region of diffuseness of the Fermi distribution. By virtue of the condition $\hbar\Omega \gg kT$, the width of the band is much less than the distance between the curves. By projecting the portions of the curves intersected by the band on the abscissa, we see that in the region of diffuseness of the Fermi distribution there are intervals of allowed (the thick sections) and the forbidden (thin sections) values of p_z. The distance between the curves in the drawing changes with change in H; consequently, the intervals of allowed values of p_z are shifted. If none of the solutions of Eq. (3), p_z^l, fall in this range, then a strong increase takes place in the sound absorption—the gigantic oscillations.

Thus, the condition for gigantic oscillations is determined by the interplay of two factors: by the locations of the allowed intervals of p_z, which depend on H, and the solutions of the equation which, for $l \neq 0$, also depend on H. We begin with the study in detail of the second of these factors.

Equation (3) can be rewritten in the following form:

$$\frac{leH}{c\varkappa_z} = m\left(\frac{w_p}{\cos\theta} - \tilde{v}_z\right) \equiv g(p_z), \tag{4}$$

where w_p is the phase velocity of sound and θ is the angle between the vectors κ and **H**. With the help of the formulas of quasi-classical quantization, [5]

$$S(\varepsilon, p_z) = \frac{2\pi\hbar eH}{c}(n+\gamma), \quad m = \frac{1}{2\pi}\frac{\partial S}{\partial\varepsilon}, \quad m\tilde{v}_z = -\frac{1}{2\pi}\frac{\partial S}{\partial p_z},$$

where S is the cross-section area of the surface, ϵ = const in the plane p_z = const, and $0 \le \gamma \le 1$, the right side of (4) can be represented in the form

$$g(p_z) = \frac{1}{2\pi}\left[\left(\frac{\partial S}{\partial p_z}\right)_\varepsilon + \left(\frac{\partial S}{\partial\varepsilon}\right)_{p_z}\frac{w_p}{\cos\theta}\right]. \tag{4a}$$

We note that inasmuch as the Fermi velocity is much greater than the sound velocity, while the angle θ differs appreciably from a right angle, the second component in (42) is much smaller than the first.

The left side of Eq. (4) is proportional to the product lH, while the right side is a function of p_z alone for $\epsilon = \zeta$. But this function is bounded, and for sufficiently large H can have solutions only with $l = 0$, which are determined by the equation g = 0. Inasmuch as the second component in (42) is much smaller than the first, these solutions correspond to cross sections of the Fermi surface that are close to extremal.

If the field is decreased, then for H less than some H_1 (H_{-1}), solutions with $l = 1$ are possible (also $l = -1$). The left side of Eq. (4) depends linearly on H and l, and for consideration of the

[2)]For $\hbar\omega \ll kT$, only those electrons take part in the absorption whose energy is in the range of diffuseness of the Fermi distribution. For the ultrasonic frequencies experimentally attainable at the present time ($\omega \lesssim 10^{10}$ sec^{-1}), this inequality is satisfied down to temperatures $\sim 1°$K. Therefore, we shall assume below that it does take place.

problem of the generation of new solutions, it is sufficient to investigate Eq. (4), for example, for $l = \pm 1$. Here two cases can be distinguished, depending on the behavior of the function $g(p_z)$ (Figs. 2a, 2b).

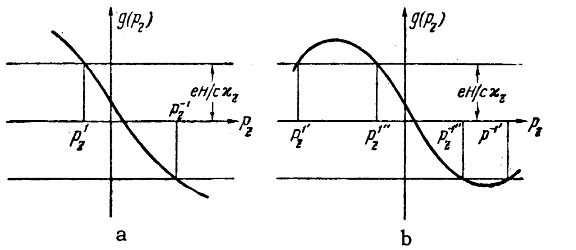

FIG. 2. The function $g(\zeta, p_z)$ for cases a and b. The horizontal straight lines are $\pm eH/c\kappa_z$ away from the abscissa. The points of intersection of these lines and the curve $g(p_z)$ give the solution of Eq. (3) for the given value of H and $l = \pm 1$.

Case a: the function $g(p_z)$ is monotonic inside the interval $[-p_F, p_F]$, reaching a maximum and a minimum on its boundaries. In this case, for each l, there is a single solution of (4), p_z^l, and p_z^l falls off in absolute value for decrease in the field (Fig. 2a).

Case b: the function $g(p_z)$ also has extrema inside the interval (or the limiting momentum in the direction of the field is generally absent, i.e., the Fermi surface is open in this direction). In this case, the solution with $l = 1$ or $l = -1$ appears for the first time. Upon further decrease in the field, two roots appear, the distance between which is increased as H decreases (Fig. 2b).

We note that in the second case Eq. (4) can also have simple roots (solutions of the type a), if the Fermi surface is closed in the direction of the magnetic field. If the Fermi surface is open in the direction of the magnetic field and the boundary momentum p_F is absent, then solutions of type a are also absent. The first case corresponds to a convex Fermi surface and the second to a nonconvex one.

The very instant of the appearance of solutions of type a is difficult to establish by experiment, inasmuch as the corresponding matrix element of the electron-phonon transition is small, while the contribution of the electrons with $l = \pm 1$ in the absorption coefficient is proportional to the square of this matrix element. Therefore, the corresponding oscillations should first have a small amplitude, which then increases with further change of the magnetic field. It was shown earlier [4] that singular periodic changes in the absorption coefficient $\Gamma(H)$ are connected in the classical case with solutions of type a. These changes are the so-called increments of the first type, which are accompanied by discontinuities in the derivatives of the absorption coefficient with respect to the magnetic field.

Conversely, the moment of appearance of solutions of type b can easily be noted, inasmuch as the corresponding matrix elements are far from small. In the classical case, the so-called increments of the second type of function $\Gamma(H)$ are connected with solutions of the type b. The existence of these increments was first shown in [4] and they were later studied in the work of Kaner, [6] and observed in the experiment of Galkin and Korolyuk. [7]

We shall first explain under what circumstances gigantic oscillations can be regarded as periodic, and what their period is. We shall assume that the absorption coefficient is a maximum for some value of the field $H = H_n$. This means that for some p_z^l and any n, the following equality is satisfied:

$$S(\zeta, p_z^l) = 2\pi e\hbar (n + \gamma) c^{-1} H_n. \tag{5}$$

Now we let the magnetic field decrease to the next value H_{n+1}, for which the condition of gigantic oscillations is again satisfied. The solution of Eq. (4) itself depends on H; H undergoes an increase upon change in the field, which we denote δp_z^l. The condition of type (5) is written for the new value of the field in the following fashion:

$$S(\zeta, p_z^l + \delta p_z^l) = 2\pi e\hbar (n + 1 + \gamma) c^{-1} H_{n+1}. \tag{6}$$

From (5) and (6), we have

$$\Delta\left(\frac{1}{H}\right) = \frac{1}{H_{n+1}} - \frac{1}{H_n} = \frac{2\pi e\hbar}{cS}\left(1 + \frac{cm\tilde{v}_z}{e\hbar H_{n+1}}\delta p_z^l\right). \tag{7}$$

For $l = 0$, the increment $\delta p_z^0 = 0$, and the oscillations are strictly periodic in the inverse field (in agreement with [1]). The δp_z^l (for $l \neq 0$) are found from Eq. (4). If we assume that the derivative $\partial g/\partial p_z = g'(p_z) \neq 0$, while δp_z^l is small, then

$$\delta p_z^l = el(H_{n+1} - H_n)/c\varkappa_z g'(p_z). \tag{8}$$

Substituting this expression in (7), we get, finally,

$$\Delta\left(\frac{1}{H}\right) = \frac{2\pi e\hbar}{cS}\left[1 + \frac{2\pi eml\tilde{v}_z H}{cS\varkappa_z g'(p_z)}\right]^{-1}. \tag{9}$$

Thus the gigantic oscillations with $l \neq 0$ are generally not strictly periodic. However, inasmuch as many oscillations are generally included in a small interval of change in H, the deviation from periodicity is rather slight. An exception is the case in which the denominator of Eq. (9) is small. In this case the oscillations can be aperiodic, and the distance between neighboring maxima increases.

The latter circumstance is important, since it increases the resolution of the oscillation peaks and can lead, in an especially favorable case, to the observation of gigantic oscillations in normal electron groups.

2. We proceed to the rigorous solution of the problem. Let the electron dispersion law have the form $\epsilon(\mathbf{p})$. One can then carry out their quasiclassical quantization if the energy operator is taken in the form $\epsilon(\mathbf{p} - ec^{-1}\mathbf{A})$,[8] where $\mathbf{p} = -i\hbar\nabla$, $\mathbf{A}$ is the vector potential, which we take in the form $A_x = A_z = 0$, $A_y = Hx$. Then the state of the electron in the magnetic field will be characterized by the quantum numbers n, p_z, $p_y = -eHx_0/c$ and σ (the z projection of the electron spin). The orbital wave function of the electron has the form

$$\psi(\mathbf{r}) = \frac{1}{2\pi\hbar}\exp\left[\frac{i}{\hbar}(p_y y + p_z z)\right]\varphi_{n,\,p_z}(x - x_0). \quad (10)$$

The energy of the electron depends on n, p_z, σ; there is degeneracy in p_y in this approximation.

The interaction operator of the electron with the sound wave has the form

$$V = \frac{1}{2}(Ue^{-i\omega t} + U^{+}e^{i\omega t}), \quad (11)$$

where[3]

$$U = e^{i\varkappa \mathbf{r}}\Lambda_{ik}u^0_{ik}. \quad (12)$$

Here u^0_{ik} is the maximum value of the deformation tensor in the sound wave, $\Lambda_{ik}(\mathbf{p})$ is the tensor of the "deformation potential," whose components generally depend on the quasi-momentum $\mathbf{p}$ (while in the magnetic field one must make here too the substitution $\mathbf{p} \to \mathbf{p} - ec^{-1}\mathbf{A}$).

The absorption coefficient Γ, under neglect of the interaction of electrons with scatterers, is expressed by the following formula:[1] [4]

$$\Gamma = \frac{\pi}{V_0\rho u_0^2 w}\sum_{\alpha\alpha'}\frac{\partial F}{\partial\zeta}|\langle\alpha'|U|\alpha\rangle|^2\,\delta(\omega_{\alpha\alpha'} + \omega). \quad (13)$$

Here ρ is the density of the crystal, V_0 is its volume, u_0 is the amplitude of the sound vibration, w is the group velocity of sound, α is the set of electron quantum numbers in a magnetic field, $F(\epsilon_\alpha)$ is the Fermi function, and $\omega_{\alpha\alpha'}$ is the transition frequency:

$$\hbar\omega_{\alpha\alpha'} = \varepsilon_\alpha - \varepsilon_{\alpha'}.$$

[3]In the quasiclassical approximation, to which we restrict ourselves, the order of the non-commuting operators $\Lambda_{ik}(p - ec^{-1}A)$, and $e^{i\varkappa\cdot r}$ does not play a role.

[4]Here, as in[1], we neglect the contribution of induction effects and variable electric fields in the sound absorption. The problem of the validity of such a neglect was discussed previously.[4]

Let the wave vector κ have x and z components that differ from zero. The matrix element of the quantity $U = e^{i\kappa\cdot\mathbf{r}}u^0_{ik}\Delta_{ik}$ is

$$\langle\sigma', p'_y, p'_z, n + l|U|\sigma, p_y, p_z, n\rangle = \delta_{\sigma\sigma'}\delta(p_y - p'_y)\,\delta(p_z + \hbar\varkappa_z - p'_z)\,e^{i\varkappa_x x_0}I_{n+l,\,n}, \quad (14)$$

$$I_{n+l,\,n} = \int \overset{*}{\varphi}_{n+l,\,p_z+\hbar\varkappa_z}e^{i\varkappa_x x}u^0_{ik}\Lambda_{ik}\left(p_x, -\frac{eH}{c}x, p_z\right)\varphi_{n,\,p_z}dx \quad (15)$$

(this quantity does not depend on p_y).

Substituting (14) in (13), we get the following expression for the absorption coefficient:

$$\Gamma = \frac{eH/4kT}{4\pi\hbar^2\rho u_0^2 wc}\sum_{n,\,l,\,\sigma}\int dp_z|I_{n+l,\,n}|^2 \times \mathrm{ch}^{-2}\frac{\zeta - \varepsilon_n(p_z) - \mu H\sigma}{2kT}\,\delta(\tilde{\omega} + \omega), \quad (16)^*$$

where the quantity $\tilde{\omega}$ is determined as

$$\hbar\tilde{\omega} = \varepsilon_{n+l}(p_z + \hbar\varkappa_z) - \varepsilon_n(p_z). \quad (17)$$

The condition for vanishing of the argument of the δ function is identical with Eq. (2). Carrying out integration over p_z in (16), we get

$$\Gamma = \frac{eH/4kT}{4\pi\hbar^2\rho u_0^2 wc}\sum_{n,\,l,\,\sigma}\frac{|I_{n+l,\,n}|^2}{|(\partial\tilde{\omega}/\partial p_z)_{n,\,l}|}\mathrm{ch}^{-2}\frac{\zeta - \varepsilon_n(p_z^l) - \mu H\sigma}{2kT}, \quad (18)$$

where the quantity p_z^l is determined from Eq. (2), and

$$\left(\frac{\partial\tilde{\omega}}{\partial p_z}\right)_{n,\,l} = \frac{1}{\hbar}\{v_z(p_z^l + \hbar\varkappa_z)_{n+l,\,n+l} - v_z(p_z^l)_{n,\,n}\} \equiv \frac{\Delta v_z}{\hbar}. \quad (19)$$

We introduce the notation

$$\Gamma_l^{\pm} = \frac{eH/4kT}{4\pi\hbar\rho u_0^2 wc}\sum_n\frac{|I_{n+l,\,n}|^2}{|\Delta v_z|}\mathrm{ch}^{-2}\frac{\zeta - \varepsilon_n(p_z^l) \mp \mu H}{2kT}. \quad (20)$$

The absorption coefficient can be written in the form of a sum of the partial coefficients:

$$\Gamma = \sum_l \Gamma_l^{+} + \sum_l \Gamma_l^{-}.$$

Inasmuch as n is large, the quantities $(v_z)_{n,n}$ and $I_{n+l,n}$ can be found by means of the rules for the calculation of quasiclassical matrix elements (see [9]), if the spectrum and the deformation potential in the absence of the magnetic field are known. It is obvious that for $n \gg l$

$$(v_z)_{n,\,n} = \tilde{v}_z(p_z, \varepsilon_n) \equiv \frac{1}{2\pi}\int_0^{2\pi} v_z(\varepsilon_n, p_z, \varphi)\,d\varphi, \quad (21)$$

where φ is a dimensionless variable proportional to the periods of rotation of the electron in the magnetic field.

In this approximation, it is somewhat more complicated to find $I_{n+l,n}$, since this matrix element extends between states in which not only the

*ch = cosh.

n are different, but also the p_z. For $n \gg l$, we obtain

$$I_{n+l,\,n} = I_l(\varepsilon_n, p_z, \varkappa_z) = \frac{1}{2\pi}\int_0^{2\pi} \Lambda_{ik}(\varepsilon_n, p_z, \varphi)\, u^0_{ik} \times \exp\left\{i\varkappa\mathbf{r} - i\tilde{v}_z\varkappa_z\frac{\varphi}{\Omega} - il\varphi\right\} d\varphi = \frac{1}{2\pi}\int_0^{2\pi}\Lambda_{ik}u^0_{ik} \times \exp\left[\frac{i\varkappa}{\Omega}\int_0^{\varphi}\mathbf{v}(\varphi')\,d\varphi' - i\tilde{v}_z\varkappa_z\frac{\varphi}{\Omega} - il\varphi\right] d\varphi. \quad (22)$$

Further simplification can be obtained if we take the following circumstance into account: $\cosh^{-2}[(\zeta - \epsilon_n \mp \mu H)/2kT]$ as a function of ϵ_n has a sharp maximum for $\epsilon_n = \zeta \mp \mu H$. Since I and Δv_z are smooth functions of ϵ_n for $n \gg 1$, they can be taken out from under the summation over n, putting $\epsilon_n = \zeta$ in their arguments. Then

$$\Gamma_l^{\pm} = \frac{(eH/4kT)\,|\,I_l(\zeta, p_z^l)|^2}{4\pi\hbar\rho u_0^2\, wc\,|\,\partial(\tilde{v}_z\varkappa_z + l\Omega)/\partial p_z|}\sum_n \mathrm{ch}^{-2}\frac{\zeta - \varepsilon_n \mp \mu H}{2kT}. \quad (23)$$

Knowledge of the sum on the right side of this formula depends on the relation between $\hbar\Omega$ and kT. To be precise, if $\hbar\Omega \ll kT$, then the sum can be replaced by an integral, which yields

$$\sum_n \mathrm{ch}^{-2}\frac{\zeta - \varepsilon_n(p_z^l) \mp \mu H}{2kT} \cong \frac{4kT}{\hbar\Omega}. \quad (24)$$

The absorption coefficient in this (classical) case is equal to

$$\Gamma_l^{cl} = \Gamma_l^+ + \Gamma_l^- = \frac{m\,|\,I_l(\zeta, p_z^l)|^2}{2\pi\hbar^2\rho u_0^2 w\,|\,\Delta v_z\,|}. \quad (25)$$

We now consider the opposite case $\hbar\Omega \gg kT$. In this case, it suffices to keep only one term in the sum over n; the argument of this term is less than any other in absolute magnitude. If, for this, $|\,\zeta - \epsilon_n(p_z^l) \mp \mu H\,| \ll kT$, then

$$\Gamma_l^{\pm} = \Gamma_l^{cl}\hbar\Omega/8kT. \quad (26)$$

In this case, the absorption coefficient is maximized, and gigantic oscillations take place. In the case of the reverse inequality, the absorption coefficient is exponentially small.

Thus the partial absorption coefficients $\Gamma_l^{\pm}$, together with the total coefficient Γ, experience oscillations as a function of H. By means of an analysis similar to that given in [1], it can be established that for $l \neq 0$, the following inequality must also be satisfied as a criterion for the applicability of a given theory, in addition to the inequality (1):

$$\varkappa L\hbar\Omega/\zeta \gg 1, \quad (27)$$

in contrast with the much weaker inequality for $l = 0$ ([1]):

$$\varkappa L\sqrt{\hbar\Omega/\zeta} \gg 1.$$

Here L is the mean free path of the conduction electrons. In the condition for satisfying a given inequality, the theory gives the correct form of the dependence $\Gamma_l^{\pm}(H)$ close to the maximum of each partial absorption coefficient. For a description of the behavior of the absorption coefficient far from the maxima, where it is exponentially small in the given approximation, account of the scattering of the conduction electrons is necessary.

3. In conclusion, we shall consider the question of the possibility of experimental observation of gigantic oscillations, brought about by transitions with change in the Landau quantum number. The inequalities (1) and (27) and the condition $\kappa R \gtrsim 1$ can be simultaneously satisfied in semimetals of the bismuth type or in ordinary metals on anomalously small electron groups (on which the oscillations of the Shubnikov–de Haas and de Haas–van Alphen effects are usually observed).

Actually, let $\omega \sim 10^{10}\ \mathrm{sec}^{-1}$, $\kappa \sim 10^5\ \mathrm{cm}^{-1}$, $T \sim 2°K$, $m \sim 0.1m_0$ (m_0 is the mass of the free electron), $H \sim 10^4$ Oe, $p_F \sim 3 \times 10^{-21}$ gm-cm/sec, $L \sim 10^{-2}$ cm. Then $\zeta \sim 10^{-13}$ erg, $\hbar\Omega \sim 10^{-15}$ erg, $kT \sim 10^{-16}$ erg, $\kappa R \sim 1$, $\kappa L \sim 10^3$, and $\hbar\Omega/\zeta \sim 10^{-2}$.

For the period of the oscillations under such conditions, we have the estimate

$$\Delta(1/H) \sim 10^{-6}\ \mathrm{Oe}^{-1},$$

which is identical in order of magnitude with the data of Korolyuk and Prushchak.[2] The effective mass m can take on values even smaller than $0.1m_0$ (for example, in bismuth). Smaller values of the effective mass allow us to observe the effect for less stringent conditions: since the quantum limit $\hbar\Omega \gg kT$ is achieved for smaller values of the magnetic field, and the ultrasonic frequency for which $\kappa R \sim 1$ can be correspondingly reduced.

For normal electron groups in ordinary metals, the inequalities introduced above are more difficult to satisfy in an experiment.

We note the following important circumstance. Observation of gigantic oscillations on anomalously small electron groups does not at all mean that the corresponding effect is small against the background of absorption brought about by normal electron groups. To the contrary, as is seen from Eq. (23), the amplitude of the oscillations (in order of magnitude) does not depend on the Fermi momentum of the given electron group if only the corresponding Fermi velocity is much larger than the sound velocity. Thus, there is here a significant difference between the gigantic oscillations and, for example, the Shubnikov–de Haas effect.

[1] Gurevich, Skobov, and Firsov, JETP **40**, 786, (1961), Soviet Phys. JETP **13**, 652 (1961).

[2] A. P. Korolyuk and T. A. Prushchak, JETP **41**, 1689 (1961), Soviet Phys. JETP **14**, 1201 (1962).

[3] Lifshitz, Azbel', and Kaganov, JETP **31**, 63 (1956), Soviet Phys. JETP **4**, 41 (1957).

[4] V. L. Gurevich, JETP **37**, 71, 1680 (1959), Soviet Phys. JETP **10**, 51, 1190 (1960).

[5] I. M. Lifshitz and A. M. Kosevich, JETP **29**, 730 (1955), Soviet Phys. JETP **2**, 636 (1956).

[6] E. A. Kaner, JETP **38**, 212 (1960), Soviet Phys. JETP **11**, 154 (1960).

[7] A. A. Galkin and A. P. Korolyuk, JETP **36**, 1307 (1959) and **37**, 310 (1959), Soviet Phys. JETP **9**, 925 (1959) and **10**, 219 (1960).

[8] I. M. Lifshitz and M. I. Kaganov, UFN **69**, 419 (1959), Soviet Phys. Uspekhi **2**, 831 (1960).

[9] I. M. Lifshitz, JETP **32**, 1505 (1957), Soviet Phys. JETP **5**, 1223 (1957).

Translated by R. T. Beyer
102