SOVIET PHYSICS JETP VOLUME 18, NUMBER 2 FEBRUARY, 1964

POLARIZATION PHENOMENA IN SCATTERING OF π-MESONS ON NUCLEONS AND THE FERMION REGGE POLES

M. P. REKALO

Physico-technical Institute, Academy of Sciences, Ukr. S.S.R.

Submitted to JETP editor February 23, 1963

J. Exptl. Theoret. Phys. (U.S.S.R.) **45**, 595-598 (September, 1963)

Polarization phenomena in meson-nucleon scattering are studied for high energies and scattering angles ~ 180°. It is shown that the recoil nucleon polarization does not oscillate at asymptotically high energies. Only correlative quantities oscillate. The magnitude of the oscillation amplitude is estimated.

1. According to the hypothesis of moving poles of the scattering amplitude as a function of the orbital angular momentum, the asymptotic behavior of the pion-nucleon backward scattering [1] (scattering angles ~ 180°) at high energies is determined by the fermion poles, i.e., poles whose trajectories describe various fermion families.

Assuming that the invariant amplitudes satisfy the momentum-transfer dispersion relation, and using purely kinematic considerations, Gribov has shown [2] that the poles of scattering amplitudes of different parity coincide when the square of the c.m.s. energy of the crossing u-channel tends to zero, and become complex-conjugate when $u < 0$. This character of fermion-pole trajectories leads to an oscillatory behavior of the scattering amplitudes of high energies, but these oscillations do not appear in the differential scattering cross section averaged over the polarizations of all the particles that participate in the process.

We discuss in the present paper polarization phenomena in high-energy meson-nucleon scattering (scattering angles ~ 180°). It turns out that the polarization of the recoil nucleons, like the cross section, does not oscillate at high energies. Only the correlative quantities oscillate.

2. The pion nucleon scattering amplitude is of the form

$$F(u, t) = a(u, t) + \tfrac{1}{2}(\hat{k} - \hat{k}')\, b(u, t), \tag{1}$$

where $u = (k' + p)^2$ — square of the energy in the u-channel, $t = (p + p')^2$; k, k', and p, p' — momenta of the pions and nucleons, respectively, before and after scattering $(k + p + k' + p' = 0)$.

The connection between the invariant amplitudes a and b and the partial amplitudes $\varphi^j_{\lambda'\lambda}$ in the u-channel is of the form

$$A(u, t) \equiv 2ma(u, t) + (u - m^2 - \mu^2)\, b(u, t) = 2\sum_j \varphi^j_{\lambda'\lambda}(u)\,[P'_{j+1/2}(z) - P'_{j-1/2}(z)],$$

$$B(u, t) \equiv (u + m^2 - \mu^2)\, a(u, t) + m(u - m^2 + \mu^2)\, b(u, t) = 2\sqrt{u}\sum_j \varphi^j_{\lambda'\lambda}(u)\,[P'_{j+1/2}(z) + P'_{j-1/2}(z)], \tag{2}$$

where

$$z \equiv \cos\psi = 1 + 2ut/[u^2 - 2u(m^2 + \mu^2) + (m^2 - \mu^2)^2],$$

ψ — scattering angle in the u-channel.

The helicity amplitudes $\varphi^j_{\lambda'\lambda}$ are related to the partial amplitudes of fixed parity in the following manner:

$$\varphi^j_{\pm\lambda\lambda}(u) = \tfrac{1}{2}[f^j_-(u) \pm f^j_+(u)].$$

If we assume that the nearest singularities on the side of large J of the amplitudes $f^j_\pm$ are poles, then we obtain for the asymptotic values of $A(u, s)$ and $B(u, s)$ with $u < 0$ and $t \to \infty$, i.e., $s \to \infty$ (in the physical region of the s-channel), the following expressions [2]:

$$\operatorname{Im} A(u, s) = \pm\rho(u)\cos(j''\xi + \varphi)\, s^{j'-1/2},$$
$$\operatorname{Re} A(u, s) = \alpha_\pm\rho(u)\cos(j''\xi + \varphi \mp \beta)\, s^{j'-1/2},$$
$$\operatorname{Im} B(u, s) = \pm\sqrt{-u}\,\rho(u)\sin(j''\xi + \varphi)\, s^{j'-1/2},$$
$$\operatorname{Re} B(u, s) = \alpha_\pm\sqrt{-u}\,\rho(u)\sin(j''\xi + \varphi \mp \beta)\, s^{j'-1/2},$$
$$\alpha^2_\pm = (\operatorname{ch}\pi j'' \mp \sin\pi j')/(\operatorname{ch}\pi j'' \pm \sin\pi j'),$$
$$\operatorname{tg}\beta = \operatorname{sh}\pi j''/\cos\pi j', \tag{3*}$$

where $\xi = \ln s$, while $j' = j'(u)$ and $j'' = j''(u)$ are the real and imaginary parts of the function $j = j(u)$, which determines the pole position. The quantities $\rho e^{\pm i\varphi}$ are the residues f^j_- and f^j_+ multiplied by

*ch = cosh, sh = sinh, tg = tan.

some known functions that depend only on u.

The helicity amplitudes in the s-channel are related to the invariant amplitudes by

$$f_{11} \equiv (\tfrac{1}{2} | F | \tfrac{1}{2}) = [2ma\,(u, s) - (s - m^2 - \mu^2)\, b\,(u, s)] \cos\frac{\theta}{2},$$

$$f_{1-1} \equiv (-\tfrac{1}{2} | F | \tfrac{1}{2}) = s^{-1/2}\, [(s + m^2 - \mu^2)\, a\,(u, s) - m\,(s - m^2 + \mu^2)\, b\,(u, s)] \sin\frac{\theta}{2}. \quad (4)$$

As $s \to \infty$, the amplitudes $a(u,s)$ and $b(u,s)$ are asymptotically of the same order of magnitude, so that (4) can be rewritten

$$f_{11} \underset{(s\to\infty)}{=} -\sqrt{-us}\, b(u, s),$$

$$f_{1-1} \underset{(s\to\infty)}{=} \sqrt{s}\,(a(u, s) - mb(u, s)). \quad (5)$$

Expressing with the aid of (2) the values of a and b in terms of A and B, and substituting the resultant expressions in (5), we get

$$f_{11} = \sqrt{-us}\, \frac{(u + m^2 - \mu^2)\, A - 2mB}{\gamma},$$

$$f_{1-1} = \sqrt{s}\, \frac{2muA - (u + m^2 - \mu^2)\, B}{\gamma}, \quad (6)$$

where $\gamma = -u^2 + 2u(m^2 + \mu^2) - (m^2 - \mu^2)^2$.

3. The scattering amplitude can be represented in the c.m.s. of the s-channel in the following form [3], which is convenient for the calculation of polarization effects:

$$f = f_{11} + i f_{1-1}\, (\cos\Phi\sigma_y - \sin\Phi\sigma_x). \quad (7)$$

With the aid (7) we can obtain the differential scattering cross section

$$d\sigma/d\Omega = s^{-1}\, \mathrm{Sp}\, f^+ f, \quad (8)$$

the polarization $\mathbf{P}$ of the recoil nuclei (unpolarized target)

$$\mathbf{P} d\sigma/d\Omega = s^{-1}\, \mathrm{Sp}\, f^+ \boldsymbol{\sigma} f \quad (9)$$

and the polarization of the recoil nucleons for a polarized target

$$T_{ij} d\sigma/d\Omega = s^{-1}\, \mathrm{Sp}\, f^+ \sigma_i f \sigma_j, \quad (10)$$

where i and j determine the polarization directions of the scattered and target nucleons, respectively.

If the scattering plane coincides with the xz plane, then

$$P_x = P_z = 0, \qquad P_y\, d\sigma/d\Omega = -s^{-2}\, \mathrm{Im}\, f_{11} f^*_{1-1}. \quad (11)$$

It is assumed that the momentum of the incoming pion is directed along the z axis. Substituting in (11) the asymptotic values for f_{11} and f_{1-1}, according to (6), we obtain

$$P_y\, d\sigma/d\Omega = \mp 2u\alpha_\pm \sin\beta\, s^{2j'-1}. \quad (12)$$

We see that, like the cross section, the recoil-nucleon polarization does not oscillate at high energies.

We can obtain from (10) the connection between the components of T_{ij} (the polarization correlation tensor) and the helicity amplitudes;

$$T_{xx} d\sigma/d\Omega = s^{-1}\, (|f_{11}|^2 - |f_{1-1}|^2 + 2|f_{1-1}|^2 \sin^2\Phi),$$
$$T_{yy} d\sigma/d\Omega = s^{-1}\, (|f_{11}|^2 - |f_{1-1}|^2 + 2|f_{1-1}|^2 \cos^2\Phi),$$
$$T_{zz} d\sigma/d\Omega = s^{-1}\, (|f_{11}|^2 - |f_{1-1}|^2),$$
$$T_{xy} d\sigma/d\Omega = T_{yx} d\sigma/d\Omega = -s^{-1}\, |f_{1-1}|^2 \sin 2\Phi,$$
$$T_{xz}\, d\sigma/d\Omega = -T_{zx}\, d\sigma/d\Omega = -2s^{-1}\, \mathrm{Re}\, f^*_{11} f_{1-1} \cos\Phi,$$
$$T_{yz}\, d\sigma/d\Omega = -T_{zy}\, d\sigma/d\Omega = -2s^{-1}\, \mathrm{Re}\, f^*_{11} f_{1-1} \sin\Phi, \quad (13)$$

where Φ — nucleon azimuthal scattering angle.

Each of the components of the tensor (13) has a structure

$$T_{ij}\, d\sigma/d\Omega = \Omega^{(0)}_{ij}\, (s, u) + f_{ij}\, (\Phi)\, \Omega^{(1)}_{ij}\, (s, u), \quad (14)$$

where $f_{ij}(\Phi)$ are known functions of Φ, while the quantities $\Omega^{(0,1)}_{ij}$ have, after (3) and (6) are substituted in (13), the form

$$\Omega_{ij}\, (s, u) = Q^{(1)}_{ij}(u)\, |A|^2 + Q^{(2)}_{ij}(u)\, \mathrm{Re}\, A^* B + Q^{(3)}_{ij}(u)\, |B|^2, \quad (15)$$

where $Q^{(\alpha)}_{ij}$ are known functions that depend only on u. After simple transformations we get

$$\Omega_{ij}\, (s, u) = (s/m\mu)^{2j'-1} \mathcal{P}^{(0)}_{ij}\, [1 + \Delta_{ij} \cos\,(2j''\xi + 2\varphi + \delta_{ij})]; \quad (16)$$

$$\Delta^2_{ij} = [(\mathcal{P}^{(1)}_{ij})^2 + (\mathcal{P}^{(2)}_{ij})^2]/(\mathcal{P}^{(0)}_{ij})^2,$$

$$\mathrm{tg}\, \delta_{ij} = -\mathcal{P}^{(2)}_{ij} / \mathcal{P}^{(1)}_{ij},$$

$$\mathcal{P}^{(0)}_{ij} = \tfrac{1}{2}(1 + \alpha^2_\pm)\, (Q^{(1)}_{ij} - uQ^{(3)}_{ij}), \quad (16a)$$

$$\mathcal{P}^{(1)}_{ij} = \tfrac{1}{2}(1 + \alpha^2_\pm \cos 2\beta)\, (Q^{(1)}_{ij} + uQ^{(3)}_{ij}) \mp \tfrac{1}{2}\alpha^2_\pm \sin 2\beta\, \sqrt{-u}\, Q^{(2)}_{ij}, \quad (16b)$$

$$\mathcal{P}^{(2)}_{ij} = \pm \tfrac{1}{2}\alpha^2_\pm \sin 2\beta\, (Q^{(1)}_{ij} + uQ^{(3)}_{ij}) + \tfrac{1}{2}\sqrt{-u}\, (1 - \alpha^2_\pm \cos 2\beta)\, Q^{(2)}_{ij}. \quad (16c)$$

We see from (16) that the correlative quantities T_{ij} oscillate with a frequency that increases logarithmically with increasing energy.

The oscillation amplitude Δ_{ij} depends on the pole position, which determines by means of (3) the quantities $\alpha^2_\pm$ and β in (16a), (16b), and (16c). If we assume that when $u \approx -\mu^2$ both quantities j' and j'' are of the order of zero (i.e., $\alpha^2_\pm = 1$, $\tan\beta = 0$) then, as can be readily seen from (16a)–(16c), $\Delta_{ij} \approx 1$. If these estimates are correct, then the amplitude of the oscillations may turn out to be comparable in magnitude with the non-oscillating part of the cross section.

In conclusion I express sincere gratitude to A. I. Akhiezer for continuous interest in the work and for numerous discussions, and to I. Ya. Pomeranchuk for interest in the work.

[1] Frautschi, Gell-Mann, and Zachariasen, Phys. Rev. **126**, 2204 (1962).

[2] V. N. Gribov, JETP **43**, 1529 (1962), Soviet Phys. JETP **16**, 1080 (1963).

[3] M. Jacob and G. C. Wick, Ann. Physics **7**, 404 (1959).

Translated by J. G. Adashko
103