REAL PART OF THE AMPLITUDE FOR ELASTIC pp FORWARD SCATTERING

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The real part of the spinless amplitude for pp forward scattering is determined from results on Coulomb scattering of protons on protons in the 150—660 MeV energy range. The results are compared with those obtained from phase shift analysis and from the dispersion relations.

MEASUREMENTS of the parameters of elastic pp-scattering in the small-angle region, where the effect of Coulomb inference is quite strong, make it possible to determine directly some amplitudes of the 0° pp-scattering matrix. This possibility was discussed earlier[1,2]. In this paper we present the results of a determination of the real part of the spin-independent pp-scattering amplitude at 0° in the energy region 150—660 MeV. To this end, we use experimental data on the differential cross section of pp-scattering, $\sigma_{pp}(\theta)$ at energies 147 MeV[3], 380 MeV[4,5], 435 MeV[6], 460 MeV[7-9], 560 MeV[7,10], and 660 MeV[7,9,11].

The nucleon-nucleon scattering matrix which is invariant under space reflection and time inversion can be written, assuming charge independence of the nuclear forces, in the form (see, for example, [12])

$$M_N(\theta) = A(\theta) + B(\theta)\,\sigma_{1n}\sigma_{2n} + C(\theta)\sin\theta\,(\sigma_{1n}+\sigma_{2n}) + E(\theta)\,\sigma_{1p}\sigma_{2p} + F(\theta)\,\sigma_{1q}\sigma_{2q}. \qquad (1)$$

In the case of proton-proton scattering, it is also necessary to take into account the Coulomb interaction. We write the elastic pp-scattering matrix in the presence of Coulomb interaction in the form

$$M(\theta) = M_N(\theta) + M_C(\theta), \qquad (2)$$

where $M_N(\theta)$ is the nuclear scattering matrix, which has the form (1), while $M_C(\theta)$ is the Coulomb scattering matrix[1)]. The antisymmetrized Coulomb matrix is given in the nonrelativistic approximation[2)] by

$$M_C(\theta) = \tfrac{1}{4}[f_C(\theta) - f_C(\pi-\theta)](3+\sigma_1\sigma_2) + \tfrac{1}{4}[f_C(\theta) + f_C(\pi-\theta)](1-\sigma_1\sigma_2), \qquad (3)$$

$$f_C(\theta) = -\frac{\eta}{2k\sin^2(\theta/2)}\exp\left[-i\eta\ln\sin^2\frac{\theta}{2}\right]. \qquad (4)$$

Here $\eta = e^2/\hbar v$, v is the velocity in the laboratory coordinate system (l.s.), and $\hbar k$ and θ are the momentum and scattering angle in the center-of-mass system (c.m.s.).

Recasting the matrix $M_C(\theta)$ in the form (1), we obtain

$$A_C(\theta) = f_C(\theta) - \tfrac{1}{2}f_C(\pi-\theta), \quad C_C(\theta) = 0,$$
$$B_C(\theta) = E_C(\theta) = F_C(\theta) = -\tfrac{1}{2}f_C(\pi-\theta). \qquad (5)$$

For small scattering angles θ we can write

$$A_C(\theta) \approx f_C(\theta).$$

The remaining Coulomb amplitudes in this angle region will be small and can be neglected. Then the differential cross section for the elastic scattering of an unpolarized beam can be written in the form

$$\sigma(\theta) = \tfrac{1}{4}\,\mathrm{Sp}\,M^+M = \sigma_C(\theta) + 2\,\mathrm{Re}\,f_C^*(\theta)\,A(\theta) + \sigma_N(\theta). \qquad (6)$$

Here $\sigma_C(\theta)$ and $\sigma_N(\theta)$ are the differential cross sections of the Coulomb and nuclear scattering, respectively, while the term $2\,\mathrm{Re}\,f_C^*(\theta)A(\theta)$ is the interference between them. At small angles, where the contribution of a Coulomb interference is appreciable, the amplitude $A(\theta)$ can be as-

[1)]The matrix $M_N(\theta)$ contains some contribution from the Coulomb interaction. Because of this, the obtained values of $A_R(0)$ are subject to some uncertainty, estimated not to exceed 10 percent at 660 MeV.

[2)]At 660 MeV with $\theta = 5°$ the relativistic amplitude of the Coulomb scattering differs in magnitude from the nonrelativistic one by approximately 1 percent, while at 10° the difference is approximately 3 percent, but with increasing angle θ the contribution of the scattering to the cross section is itself greatly reduced.

sumed constant, accurate to terms of order θ^2, and equal to its value at zero A(0).

For the pp-scattering cross section we have the following relations, which follow from the general theory of angular distributions of nuclear reactions [13]:

$$\sigma_N(\theta) = \sum_{l=0}^{l_{max}} a_{2l} \cos^{2l} \theta. \qquad (7)$$

Separating in the amplitude A(0) the real and imaginary parts $A_R(0)$ and $A_I(0)$, we rewrite (6) in the form

$$\sigma(\theta) = \left[\frac{\eta}{2k \sin^2(\theta/2)}\right]^2 + \frac{\eta A_I(0)}{k \sin^2(\theta/2)} \sin\left(\eta \ln \sin^2 \frac{\theta}{2}\right) - \frac{\eta A_R(0)}{k \sin^2(\theta/2)} \cos\left(\eta \ln \sin^2 \frac{\theta}{2}\right) + \sum_{l=0}^{l_{max}} a_{2l} \cos^{2l} \theta. \qquad (8)$$

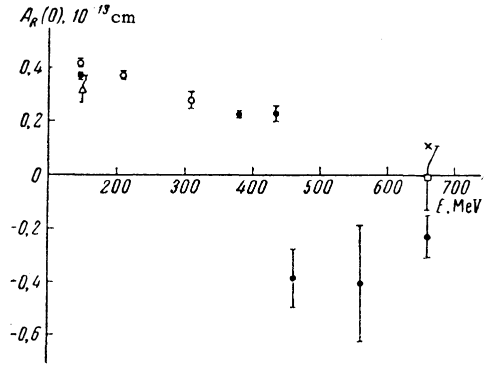
The imaginary part $A_I(0)$ is determined in accordance with the optical theorem from the total scattering cross section σ_t:

$$A_I(0) = k\sigma_t/4\pi. \qquad (9)$$

The real part $A_R(0)$ together with the coefficients a_{2l} can be obtained from (8) by least squares. This procedure was carried out for the experimental values of $\sigma_{pp}(\theta)$ at the energies indicated above, where measurement data for the small-angle region are available. The experimental points that make a contribution $\chi_i^2 > 9$ to the sum of the square deviations $\chi^2 = \sum_{i=1}^{n} (\epsilon_i/\Delta_i)^2$ were discarded in the calculations. Calculations for each energy were carried out for different values of l_{max}. The results of the calculations are listed in the table, which shows also the values of $v^2 = \chi^2/(n-m)$, where n is the number of experimental points and m is the number of the determined parameters.

The calculation of $A_R(0)$ should be confined to the value of l_{max} beyond which the rapid decrease of v^2 stops. It is possible to stop at $l_{max} = 2$ for 147, 435, and 460 MeV and at $l_{max} = 3$ for 380, 560, and 660 MeV. Such a choice of l_{max} is determined, of course, not by considerations of the number of the partial waves effectively participating in the scattering, but by the accuracy of the processed experimental data. Increase of l_{max} by unity changes $A_R(0)$ in all cases by an amount equal to the error.

The c.m.s. values of $A_R(0)$ calculated in this manner, together with the error in their determination, are shown in the figure as functions of the kinetic energy of the incident proton in l.s. The figure shows also the values calculated from the



Dependence on the amplitude $A_R(0)$ (c.m.s.) on the kinetic energy of the incoming proton E (l.s.). ● – present work; △ – [1]; o – calculated from the phase shifts obtained in [14,15]; x – from the phase shift obtained in [19]; □ – from the phase shift obtained in [20].

Results of determination of $A_R(0)$

E, MeV	l_{max}	$A_R(0)$, 10^{-13} cm	a_{2l}, 10^{-26} cm^2					$v^2 = \frac{\chi^2}{n-m}$
			$l=0$	$l=1$	$l=2$	$l=3$	$l=4$	
147	1	0.348±0.009	0.405±0.007	0,128±0,012				0.61
	2	0.369±0.012	0.420±0.009	−0.022±0.055	0.17±0.06			0.26
	3	0.371±0.016	0.419±0.010	−0.01 ±0.13	0.12±0.34	0.03±0.24		0.28
380	2	0.196±0.010	0.373±0.002	−0,005±0.014	0.11±0.02			2.2
	3	0.227±0.012	0.372±0.002	0.144±0.037	−0.35±0.11	0.34±0.08		1.2
	4	0.256±0.015	0.373±0.002	−0.049±0.069	0.74±0.34	−1.54±0.56	1.0±0.3	0.63
435	2	0.230±0.030	0.356±0.011	−0.01±0.06	0.13±0.07			1.3
	3	0.272±0.037	0.350±0.012	0.28±0.16	−0.66±0.42	0.56±0.30		1.2
460	2	−0.39±0.11	0.342±0.007	0.16±0.05	−0.16±0.08			1.1
	3	−0.45±0.16	0.343±0.007	0.10±0.12	0.05±0.40	0.19±0.34		1.2
560	2	−0.56±0.19	0.307±0.008	0.13±0.09	0.12±0.13			0.48
	3	−0.41±0.22	0.305±0.008	0.32±0.18	−0.52±0.55	0.55±0.46		0.29
	4	−0.46±0.23	0.304±0.009	0.40±0.31	−1.2 ±1.9	1.8 ±3.8	−0.8±2.2	0.33
660	2	−0.435±0.070	0.205±0.003	0.46±0.05	0.05±0.08			4.1
	3	−0.234±0.081	0.203±0.003	0.86±0.10	−1.67±0.36	1.55±0.32		1.3
	4	−0.146±0.091	0.205±0.003	0.60±0.16	0.34±1.00	−2.5 ±1.9	2.4±1.1	0.77

phase shifts, obtained in [14,15] at 147, 210, and 310 MeV. The value obtained in [1] for 150 MeV is also shown.

As can be seen from the figure, the amplitude $A_R(0)$ passes through zero near 450 MeV, and becomes negative at higher energies. It was established earlier [16] that an analogous dependence is possessed by the amplitude averaged over the isotopic spin of the nucleons

$$\overline{A}_R = \frac{3}{4} A_R^1 + \frac{1}{4} A_R^0,$$

where the indices 1 and 0 pertain to the isotopic spin of the two-nucleon system. At 660 MeV we have $A_R^1(0) = (-0.23 \pm 0.08) \times 10^{-13}$ cm. Using the previously obtained [16] value $A_R = (-0.36 \pm 0.04) \times 10^{-13}$ cm, we get $A_R^0 = (-0.74 \pm 0.29) \times 10^{-13}$ cm. The information obtained in this manner can be useful in a simultaneous phase shift analysis of the pp and np scattering data.

It was found in [17] from the dispersion relations that at 660 MeV the real part of the spin-independent amplitude is $A_R(0) = 0.7 \times 10^{-13}$ cm, which does not agree with our value. It must be noted that in [17] the calculation of the energy dependence of the real part of the amplitude was normalized in accordance with data obtained at 3 BeV [18]. Thus, the discrepancy can be due either to the considerable contribution made to the scattering amplitude by terms containing integrals over the unobservable region of the energy, or to the insufficient accuracy of the experimental data at 3 BeV.

The values of $A_R(0)$ were also calculated from the pp scattering phase shifts at 660 MeV [19,20]. The set of phase shifts obtained by Hoshizaki and Machida [19] corresponds to $A_R(0) = 0.11 \times 10^{-13}$ cm. The first set of phase shifts from [20] corresponds to $A_R(0) = (-0.01 \pm 0.12) \times 10^{-13}$ cm. This value has been calculated from the more exact phase shifts obtained in [20] with a value $f^2 = 0.08$ for the pion-nucleon interaction coupling constant. The large values obtained in the present work for $A_R(0)$ in the 460–660 MeV interval may possibly be due to the fact that the differential cross sections for the scattering at small angles have not yet been experimentally investigated in sufficient detail for these energies.

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