THEORY OF RESONANCE EXCITATION OF WEAKLY DECAYING ELECTROMAGNETIC WAVES IN METALS

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It is shown that electromagnetic excitations (quasi-particles) of various types exist in metals located in a strong magnetic field. In metals with unequal electron and "hole" concentrations, spiral waves exist with a quadratic spectrum. Magnetohydrodynamic waves exist in metals with equal carrier concentrations. The wavelengths of these excitations are large in comparison with the Larmor radius, and small in comparison with the mean free path.

In metals with a single type of carrier, there exist quasi-particles with a discrete spectrum, the wavelength of the particles being smaller than the Larmor radius. In all cases, the dependence of the natural frequency, attenuation, and polarization of the excitations on the magnetic field strength **H**, the angle between the wave vector **k** and **H**, and on other parameters has been investigated. The existence of quasi-particles is not connected with any singularities of the electron energy spectrum.

The existence of a series of new resonance effects due to quasi-particle excitation by an external magnetic field or by ultrasound is predicted. Resonance occurs when the frequency of the external field coincides with one of the natural frequencies. A slowly decaying electromagnetic wave arises in the metal, and the ultrasonic impedance and absorption coefficient experience resonance oscillations. The distribution of the electromagnetic field in the volume of the metal, and also the dependence of the impedance tensor and the ultrasonic absorption coefficient on the magnitude and orientation of the magnetic field are also studied.

INTRODUCTION

In most cases, electromagnetic waves are very rapidly damped in metals (at distances of the order of the skin depth). The presence of a constant magnetic field **H** leads to the appearance of characteristic low-frequency oscillations of the electromagnetic field. Thus, Konstantinov and Perel'[1] have shown that a radio wave can penetrate to considerable depth in a magnetic field **H** perpendicular to the surface. Buchsbaum and Holt[2] have discovered Alfvén waves in bismuth under similar conditions. The propagation of electromagnetic waves in metals in these cases is determined by the transverse part of the conductivity tensor, for which spatial dispersion plays no role.

According to Azbel',[3] for cyclotron resonance at very high frequencies in metals with non-quadratic dispersion of the carriers, weakly damped spikes of fields and currents must exist.

In the present work it is shown that electromagnetic waves can propagate in metals at arbitrary angles to the direction of the magnetic field when the spatial dispersion is significant. The general reason for the existence of different electromagnetic waves in metals is that the motion of the electrons in a plane perpendicular to the magnetic field **H** is finite. As a consequence of this, the antihermitian part of the conductivity tensor is large in comparison with the hermitian part. Therefore the effective dielectric constant of the metal is real and positive, which indicates the existence of characteristic electromagnetic excitations (quasi-particles).

As can be expected from general considerations, the dispersion law of the electromagnetic quasi-particles is determined only by the spectrum of the electron conductivity. The kinetic characteristics of the electrons (relaxation time) do not enter into the dispersion law and can affect only the damping of the excitations.

It is shown below that in metals with an unequal concentration of electrons and "holes," there are spiral waves with a quadratic spectrum. In metals with identical concentrations of carriers, magnetohydrodynamic waves can exist. The wavelength of all these excitations is large in comparison with the Larmor radius, and small in comparison with the mean free path of the carriers. In metals with a single group of carriers, there are excitations with discrete wave vectors and frequencies whose wavelengths are much smaller than the Larmor radius.

In all cases in which there are weakly damped quasi-particles, the metals can possess an anomalous transparency. In these phenomena, which accompany the variable electromagnetic field, another type of resonance should be observed. We investigated here a number of new resonance effects, associated with the excitation of characteristic electromagnetic waves in metals.

DISPERSION EQUATION FOR ELECTROMAGNETIC WAVES IN A METAL AND ITS SOLUTION

1. The propagation of plane monochromatic waves in a metal is determined by the Maxwell equations

$$k^2\mathbf{E} - \mathbf{k}(\mathbf{kE}) = 4\pi i\omega c^{-2}\mathbf{j}, \tag{1.1}$$

$$j_\alpha = \sigma_{\alpha\beta}(\mathbf{k}, \omega, \mathbf{H}) E_\beta + j_\alpha^{\text{ext}}. \tag{1.2}$$

Here $\mathbf{E}$ is the transverse electric field, $\mathbf{j}$ is the current density, $\sigma_{\alpha\beta}$ the conductivity tensor with account of spatial and temporal dispersion and dependence on the constant magnetic field $\mathbf{H}$, $\mathbf{j}^{\text{ext}}$ is the density of external currents, $\mathbf{k}$ and ω are the wave vector and the wave frequency, $\mathbf{E}$ and $\mathbf{j}$ $\sim e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}$. We have neglected the displacement current, from which it follows from (1.1), that

$$\mathbf{kj} = 0, \tag{1.3}$$

which is identical to the condition of quasi-neutrality of the metal $\rho' = 0$ (ρ' is the uncompensated volume charge density).

We choose the system of coordinates xyz such that the z axis is directed along the magnetic field $\mathbf{H}$, while the x axis is transverse to $\mathbf{k}$ and $\mathbf{H}$. In what follows, we shall also use a system of coordinates $x\eta\zeta$, where $\zeta \parallel \mathbf{k}$. The angle between the vectors $\mathbf{k}$ and $\mathbf{H}$ is Φ.

The spectrum and the damping of the characteristic oscillations of the field are determined from the homogeneous set of equations (1.1)—(1.3) for $\mathbf{j}^{\text{ext}} = 0$. Eliminating the longitudinal component of the electric field E_ζ from (1) by means of (1.3), we get

$$D_{\alpha\beta}E_\beta \equiv (\delta_{\alpha\beta} - i4\pi\omega k^{-2}c^{-2}\tilde{\sigma}_{\alpha\beta}) E_\beta = 0 \qquad (\alpha, \beta = x, \eta), \tag{1.4}$$

$$E_\zeta = -(\sigma_{\zeta x}E_x + \sigma_{\zeta\eta}E_\eta)/\sigma_{\zeta\zeta}, \tag{1.5}$$

where $\tilde{\sigma}_{\alpha\beta}$ is the "renormalized" two-dimensional conductivity tensor

$$\tilde{\sigma}_{\alpha\beta} = \sigma_{\alpha\beta} - \sigma_{\alpha\zeta}\sigma_{\zeta\beta}/\sigma_{\zeta\zeta}. \tag{1.6}$$

The dispersion equation which determines the spectrum and the damping of the electromagnetic waves is obtained by setting equal to 0 the determinant of the system (1.4) $D \equiv \det D$;

$$D \equiv 1 - (4\pi\omega/k^2c^2)^2 \det \tilde{\sigma}_{\alpha\beta} - i\, 4\pi\omega k^{-2}c^{-2}\, \mathrm{Sp}\, \tilde{\sigma}_{\alpha\beta} = 0. \tag{1.7}$$

The elements of the conductivity tensor $\sigma_{\alpha\beta}$, with account of the spatial and temporal dispersion in the presence of a constant magnetic field $\mathbf{H}$, have the form (see, for example, [4]):

$$\sigma_{\alpha\beta}(\mathbf{k}, \omega, \mathbf{H}) = \frac{2e^2}{h^3}\int_0^\infty d\varepsilon\, \frac{m}{\Omega}\frac{\partial f_0}{\partial\mu}\int_{-\infty}^{+\infty} dp_z \int_0^{2\pi} d\tau v_\alpha(\varepsilon, p_z, \tau)$$
$$\times \int_{-\infty}^{\tau} d\tau' v_\beta(\varepsilon, p_z, \tau')$$
$$\times \exp\left\{\frac{1}{\Omega}\int_{\tau}^{\tau'} [\nu - i\omega + i\mathbf{kv}(\varepsilon, p_z, \tau'')]\, d\tau''\right\}, \tag{1.8}$$

where h is Planck's constant, e is the electron charge, $\epsilon(\mathbf{p})$ is the energy, $\mathbf{p}$ is the quasi-momentum, $\mathbf{v} = \partial\epsilon/\partial\mathbf{p}$ is the velocity, $\Omega = eH/mc$ is the cyclotron frequency, $m = (2\pi)^{-1}\partial S(\epsilon, p_z)/\partial\epsilon$ is the effective mass of the electron, $S(\epsilon, p_z)$ is the cross section area of the constant-energy surface $\epsilon(\mathbf{p}) = \epsilon$ in the plane $p_z = \text{const}$, $f_0(\epsilon - \mu)$ is the Fermi distribution function, μ is the chemical potential of the electrons, $\tau = \Omega t$ is the dimensionless period of the electron in orbit in the magnetic field, and ν is the frequency of collisions of electrons with scatterers.

The dependence of the conductivity tensor (1.8) on $\mathbf{k}$, ω, and $\mathbf{H}$ is very complicated, and the dispersion equation (1.7) cannot be solved in general form. We shall investigate Eq. (1.7) by using asymptotic expressions for the conductivity tensor (1.8) in different limiting cases. By using the spectrum of electromagnetic excitations in a metal, we obtain those regions of values of ω and $\mathbf{H}$ in which the corresponding asymptotic expressions of the tensor $\sigma_{\alpha\beta}$ are valid.

2. Let us first consider the simplest limiting case, in which the electromagnetic wavelength is large in comparison with the radius of the orbit of the electron in the magnetic field;

$$kR \ll 1, \tag{2.1}$$

where $R = cp_F/eH$; p_F is the characteristic momentum on the Fermi surface. Here we shall assume a strong spatial inhomogeneity along H:

$$|k_z l^*| \gg 1, \quad (2.2)$$

where $k_z = k \cos \Phi$, while

$$l^* = v/(\nu - i\omega) \quad (2.3)$$

plays the role of the effective path length of the electron in the variable field, v is the maximum Fermi velocity in the direction H. We note that (2.1) and (2.2) assume the inequality $\Omega \gg |\nu - i\omega|$ to be satisfied.

For closed electron trajectories, all the conclusions remain valid for arbitrary form of the Fermi surface. Therefore, we shall consider a quadratic and isotropic dispersion law for electrons, $\epsilon(p) = p^2/2m$.

In the case of a single group of carriers, the asymptotic character of the tensor $\sigma_{\alpha\beta}$ has in the set of coordinates xyz the form

$$\sigma_{\alpha\beta} = \frac{nec}{H}\begin{pmatrix} (\nu - i\omega)\Omega^{-1} + \frac{3}{8}\pi|k_z|R\,\mathrm{tg}^2\Phi; & -1; & -\mathrm{tg}\,\Phi \\ 1; & (\nu - i\omega)/\Omega; & 0 \\ \mathrm{tg}\,\Phi; & 0; & 3(\nu - i\omega)/\Omega(k_z R)^2 \end{pmatrix}, \quad (2.4)^*$$

where n is the electron concentration.

The two-dimensional tensor $\tilde{\sigma}_{\alpha\beta}$ (1.6) in the axes $x\eta$ is equal to

$$\tilde{\sigma}_{\alpha\beta} = \frac{nec}{H|\cos\Phi|}\begin{pmatrix} (\nu - i\omega)\Omega^{-1}|\cos\Phi| + \frac{3}{8}\pi kR\sin^2\Phi; & -1 \\ 1; & (\nu - i\omega)/\Omega|\cos\Phi| \end{pmatrix}. \quad (2.5)$$

Substituting the expression for $\tilde{\sigma}_{\alpha\beta}$ in (1.7), and neglecting small terms, we represent the dispersion equation in the form

$$1 - (k^2 cH\cos\Phi/4\pi\omega\, ne)^2 + i(\tilde{\sigma}'_{xx} + \tilde{\sigma}'_{\eta\eta})/|\sigma_{x\eta}| = 0, \quad (2.6)$$

where $\tilde{\sigma}' = \mathrm{Re}\,\tilde{\sigma}$.

Equation (2.6) determines the spectrum and damping of the elementary electromagnetic excitations in metals in the case under consideration. The excitation spectrum is determined by the antihermitian part of the tensor $\tilde{\sigma}_{\alpha\beta}$, while the damping is determined by the hermitian part ($\tilde{\sigma}'_{xx}$ and $\tilde{\sigma}'_{\eta\eta}$). The smallness of the hermitian part in comparison with the antihermitian guarantees the relatively small damping. These excitations have evidently Bose statistics and a quadratic spectrum:[1)]

$$\omega \equiv \omega' - i\omega''$$
$$= k^2 v_a^2 \Omega^{-1}|\cos\Phi|\,[1 - i(\nu/\Omega + \tfrac{3}{16}\pi kR\sin^2\Phi)], \quad (2.7)$$

where

$$v_a = H/\sqrt{4\pi nm} \quad (2.8)$$

is the Alfvén velocity.

By substituting (2.7) and (2.8) in (1.4) and (1.5), it is not difficult to show that the electric field of the wave rotates in the xy plane:

$$E_y = iE_x/\cos\Phi, \qquad E_z = 0. \quad (2.9)$$

Elementary excitations of the same type for $kl \ll 1$ are very well known in magnetoactive plasma.[5] For the special case $\Phi = 0$, for which the spatial dispersion in metals is also unimportant, they were obtained by Konstantinov and Perel'.[1] The difference of the solution of (2.7) from the corresponding expression in the case $kl \ll 1$ lies in the presence of an additional damping, proportional to $kR\sin^2\Phi$, brought about by spatial dispersion (Landau damping). We note that this damping depends essentially on the angle Φ and vanishes for $\mathbf{k} \parallel \mathbf{H}$. For transverse propagation $\mathbf{k} \perp \mathbf{H}$, excitations of this type are absent.

The region of applicability of Eq. (2.7) is determined by the conditions (2.1) and (2.2). By expressing k in terms of ω by means of (2.7), we can rewrite it in the form

$$1 \ll v_a v^{-1}|\Omega\omega^{-1}\cos\Phi|^{1/2} \ll \Omega/|\nu - i\omega|. \quad (2.10)$$

It follows from the first inequality of (2.10) that the maximum frequency of excitations of this type is proportional to the cube of the magnetic field.

3. Let us investigate the propagation of electromagnetic waves in metals with equal concentrations of electrons and "holes" ($n_1 = n_2 = n$) in the case in which, in addition to (2.1), the following conditions are satisfied:

$$|k_z|\,v_s \ll \omega \ll \Omega_s \qquad (s = 1, 2), \quad (3.1)$$

where the index 1 refers to electrons and the index

*tg = tan.

1)As a curious fact, we note that the contribution to the heat capacity from thermal excitations of this type is equal to $A\gamma_e T$, where $\gamma_e T$ is the electron heat capacity, 2 to "holes." In this case, the spatial dispersion is small and the asymptotic character of the conductivity tensor has the form

$$\sigma_{\alpha\beta} = \frac{nec}{H} \sum_{s=1,2} \begin{pmatrix} \gamma_s a_{xx}^{(s)}; & \gamma_s a_{xy}^{(s)}; & a_{xz}^{(s)} \\ \gamma_s a_{yx}^{(s)}; & \gamma_s a_{yy}^{(s)}; & a_{yz}^{(s)} \\ -a_{xz}^{(s)}; & -a_{yz}^{(s)}; & a_{zz}^{(s)}/\gamma_s \end{pmatrix}, \tag{3.2}$$

where $\gamma_s = (\nu_s - i\omega)/\Omega_s$, while the dimensionless matrices $a_{\alpha\beta}^{(s)}$ depend only on the anisotropy of the Fermi surface and the orientation of the magnetic field relative to the crystallographic axes of the metal. If the magnetic field is directed along the three-fold or higher order axis, then all the nondiagonal elements $a_{\alpha\beta}$ are equal to 0.

The hermitian part of the tensor $\sigma_{\alpha\beta}$ (3.2) determines the damping of the excitations due to electron scattering. In the case

$$\nu_s \ll \omega \tag{3.3}$$

it is small in comparison with the antihermitian part, which determines the spectrum. Neglecting the scattering by the carriers, which gives the relative damping of the excitations of the order of ν/ω, we rewrite the tensor $\sigma_{\alpha\beta}$ in the form

$$\sigma_{\alpha\beta} = \frac{nec}{H} \begin{pmatrix} -i\omega a_1/\Omega; & -i\omega a_{12}/\Omega; & a_{13} \\ -i\omega a_{12}/\Omega; & -i\omega a_2/\Omega; & -a_{32} \\ -a_{13}; & a_{32}; & i\Omega a_3/\omega \end{pmatrix}, \tag{3.4}$$

where $\Omega = eH/c(m_1 + m_2)$, while the quantities $\{a\}$ are connected in obvious fashion with the elements of the matrices $a_{\alpha\beta}^{(s)}$.

The two-dimensional tensor $\tilde{\sigma}_{\alpha\beta}$ in the system of coordinates $x\eta$ is easily expressed in terms of the elements of the matrix (3.4):

$$\tilde{\sigma}_{\alpha\beta} = -i(\omega/\Omega)(nec/H) A_{\alpha\beta},$$

$$A_{\alpha\beta} = \begin{pmatrix} a_1 + a_{13}^2/a_3; & (a_{12} + a_{13}a_{32}/a_3)\,|\cos\Phi|^{-1} \\ (a_{12} + a_{13}a_{32}/a_3)\,|\cos\Phi|^{-1}; & (a_2 + a_{23}^2/a_3)\cos^{-2}\Phi \end{pmatrix}. \tag{3.5}$$

In the calculation of $\tilde{\sigma}_{\alpha\beta}$, we have assumed that $|\cos\Phi| \gg \omega/\Omega$.

Solution of the dispersion equation (1.7), in which $\tilde{\sigma}_{\alpha\beta}$ are determined by the expressions (3.5), gives the spectrum of electromagnetic excitations in the case under consideration. These excitations have a linear dispersion law and represent magnetohydrodynamic waves in the anisotropic metal:

$$\omega = kv_\pm,$$

$$v_\pm = v_a (2 \det A_{\alpha\beta})^{-1/2} [A_{xx} + A_{\eta\eta} \pm \sqrt{(A_{xx} - A_{\eta\eta})^2 + 4A_{x\eta}^2}]^{1/2}, \tag{3.6}$$

where v_a is determined by Eq. (2.8).

If the spectrum of the carriers is isotropic or if the magnetic field is directed along a symmetry axis of higher order, then all the nondiagonal elements of the matrix $a_{\alpha\beta}$ are equal to 0. In this case,

$$a_1 = a_2 = a, \quad A_{xx}^{(0)} = a, \quad A_{\eta\eta}^{(0)} = a\cos^{-2}\Phi, \quad A_{x\eta}^{(0)} = 0 \tag{3.6a}$$

and the phase velocities of the magnetohydrodynamic waves are determined by the formulas:

$$v_+^{(0)} = v_-^{(0)}|\cos\Phi|^{-1} = a^{-1/2} v_a. \tag{3.7}$$

The wave with the phase velocity $v_-^{(0)}$ is similar to an Alfvén wave in a plasma. The electric field in it is polarized along the y axis. The wave with phase velocity $v_+^{(0)}$ is the analog of the fast magnetoacoustic wave, in which the electric field is directed transverse to **k** and **H** (the x axis).

In magnetohydrodynamics, there is also a slow magnetoacoustic wave. In the limit of a strong magnetic field, when the Alfvén velocity is large in comparison with the sound velocity, the phase velocity of this wave is of the order of the velocity of sound. Owing to the Fermi statistics of the current carriers in the metal, the Fermi velocity must play the role of the sound velocity here. Therefore, in the limiting case under consideration, $|k_z| v_s \ll \omega$, this wave is absent (the asymptotic conductivity tensor (3.2) cannot be used to obtain the slow magnetoacoustic waves).

In the special case $\mathbf{k} \parallel y$ ($|\cos\Phi| \ll \omega/\Omega$), the "renormalized" conductivity tensor $\tilde{\sigma}_{\alpha\beta}$ has the form

$$\tilde{\sigma}_{\alpha\beta} = \frac{nec}{Ha_2} \begin{pmatrix} -i\omega\Omega^{-1}(a_1a_2 + a_{12}^2); & a_{13}a_2 + a_{12}a_{23} \\ -(a_{13}a_2 + a_{12}a_{23}); & i\Omega\omega^{-1}(a_2a_3 + a_{23}^2) \end{pmatrix}. \tag{3.8}$$

The Alfvén wave is absent in this case and the phase velocity of the fast magnetoacoustic wave, v_0, is given by the formula

$$v_0 = v_a \{a_1 + a_2^{-1} [a_{12}^2 + (a_{13}a_2 + a_{12}a_{23})^2 (a_2a_3 + a_{23}^2)^{-1}]\}^{-1/2}. \tag{3.9}$$

It follows from a comparison of (3.6) with (3.1) and (3.3) that the region of applicability of the solution (3.6) is determined by the conditions

$$\nu_s \ll \omega \ll \Omega_s, \tag{3.10}$$

$$v_s \ll v_\pm. \tag{3.11}$$

In ordinary metals ($n \sim 10^{22}\ \mathrm{cm}^{-3}$) the condition $v_s \ll v_a$ can be satisfied only in very strong magnetic fields, of the order of several million oersteds. In bismuth, $n \sim 10^{17}\ \mathrm{cm}^{-3}$ and the condition

$v_s < v_a$ is already satisfied at $H > 1$ kOe.[2)]

4. We now discuss the question of the possibility of propagation of magnetohydrodynamic waves in metals with $n_1 = n_2$ in the case of comparatively small magnetic fields, in which the following conditions are satisfied:

$$v_a \ll v_s, \qquad \nu_s \ll \omega \ll kv_s \ll \Omega_s. \tag{4.1}$$

Magnetoacoustic waves in a plasma have the following spectrum for $v_a \ll w_0$ (w_0 is the sound velocity): $\omega_+ = kw_0$; $\omega_- = kv_a$, where the plus sign denotes the fast wave and the minus the slow one.

Inasmuch as the role of the sound velocity in a degenerate electron-hole plasma of a metal is played by the Fermi velocity, it is evident that the fast magnetoacoustic wave does not satisfy the conditions (4.1) (for this wave, $\omega \sim kv$). Therefore, only the Alfvén and the slow magnetoacoustic waves can be obtained by means of the asymptotic expression for $\sigma_{\alpha\beta}$ in the case (4.1).

The asymptotic expression for the "electron" part of the tensor $\sigma_{\alpha\beta}$ in the case (4.1) has the form (2.4). The expression for the hole part of $\sigma_{\alpha\beta}$ can be obtained from (2.4) by replacement of the electron characteristics by the "hole" ones. Inasmuch as the nondiagonal elements of the "electron" and "hole" parts differ only in sign, then the total tensor $\sigma_{\alpha\beta}$ is seen to be diagonal in the approximation under consideration. We emphasize that the equating to zero (smallness) of all the nondiagonal elements of $\sigma_{\alpha\beta}$ is a consequence of the compensation condition $n_1 = n_2$ and the strong spatial dispersion $|k_z| v_s \gg \omega$. The conclusion that the tensor $\sigma_{\alpha\beta}$ is diagonal in the case (4.1) is valid also for an arbitrary law of dispersion of the carriers, if the magnetic field is directed along the axis of symmetry of the crystal.

Because the diagonal character of the conductivity tensor and the condition (1.3) $\mathbf{k}\cdot\mathbf{j} = 0$, the z component of the electric field in the Alfvén wave is negligibly small: $|E_z/E_y| \sim |\sigma_{yy}/\sigma_{zz}| \ll 1$. Therefore the spectrum of this wave is determined by the quantity $\tilde{\sigma}_{\eta\eta} = \sigma_{yy}\cos^{-2}\Phi$ and has the form

$$\omega = |k_z| v_a - i\nu/2, \qquad \nu = (m_1\nu_1 + m_2\nu_2)/(m_1 + m_2). \tag{4.2}$$

The spectrum of the second wave is determined by the element $\tilde{\sigma}_{xx}$ the value of which is determined by spatial dispersion for $\Phi \sim 1$. The wave vector in this wave is imaginary, which leads to a rapid damping of the excitations. The absolute value of the "phase velocity" has the order v_a^2/v. For $\Phi = 0$, the Landau damping is absent and the spectrum of the slow magnetoacoustic wave is identical with the spectrum of the Alfvén wave: $\omega = kv_a$.

For strong anisotropy of the dispersion law of the carriers in metals, a situation is possible in which the maximal Fermi velocities of the different carriers in the direction of the magnetic field H are quite different: $v_1 \gg v_2$. In this case the spatial dispersion can be shown to be significant for one group of carriers and unimportant for the other:

$$|k_z| v_2 \ll \omega \ll |k_z| v_1 \ll \Omega_1. \tag{4.3}$$

We shall study this case in a simple model in which there are two groups of carriers of different sign ($n_1 = n_2 = n$) with quadratic and isotropic spectrum, and the mass of the "holes" is large in comparison with the mass of the "electrons": $m_1 \ll m_2$. Of course, this model is not realized in existing metals; however, the qualitative conclusions on the possibility of propagation of electromagnetic waves and their dispersion law remain valid, even for strong anisotropy of the real Fermi surface.

The asymptotic character of the electron part of the conductivity tensor has the form (2.4) upon satisfaction of conditions (4.3) and (3.10). The asymptotic expression for the "hole" part of $\sigma_{\alpha\beta}$ differs from the statistical conductivity tensor in a strong magnetic field ($\nu_2 \ll \Omega_2$) only in that ν_2 is replaced by $\nu_2 - i\omega$. The hermitian part of the total conductivity tensor, which determines the damping of the excitations, is small in comparison with the antihermitian part. Taking it into account that $v_1/\Omega_1 = v_2/\Omega_2$ in the isotropic model we have assumed, the antihermitian part of the tensor $\sigma_{\alpha\beta}$ can be written in the form:

$$\sigma^{(a)}_{\alpha\beta} = \frac{nec}{H}\begin{pmatrix} -i\omega/\Omega_2; & 0; & -\operatorname{tg}\Phi \\ 0; & -i\omega/\Omega_2; & 0 \\ \operatorname{tg}\Phi; & 0; & i\Omega_2\omega^{-1}[1-(\omega/k_z w)^2] \end{pmatrix}, \tag{4.4}$$

where $w^2 = v_1 v_2/3$. The Hall elements σ_{xy} and σ_{yx} are equal to zero as a consequence of the compensation condition $n_1 = n_2$.

If we do not consider angles Φ close to $\pi/2$, then the "renormalized" two-dimensional tensor $\tilde{\sigma}_{\alpha\beta}$ is diagonal:

$$\tilde{\sigma}_{xx} = -i\omega\frac{c^2}{4\pi v_a^2\cos^2\Phi}\left[1-\left(\frac{\omega}{kw}\right)^2\right]\left[1-\left(\frac{w}{k_z w}\right)^2\right]^{-1},$$

$$\tilde{\sigma}_{\eta\eta} = \tilde{\sigma}_{yy}/\cos^2\Phi, \qquad \tilde{\sigma}_{x\eta} = \tilde{\sigma}_{\eta x} = 0. \tag{4.5}$$

Because of the diagonal character of the tensor

[2)]Evidently the linear increase in the surface impedance of bismuth in a magnetic field $H > 3$ kOe, observed by Aubrey and Chambers,[6] is connected with excitation of these waves.

$\tilde{\sigma}_{\alpha\beta}$, the dispersion equation (1.7) divides into two equations:

$$k^2 = 4\pi i\omega c^{-2}\tilde{\sigma}_{\alpha\alpha} \qquad (\alpha = x, \eta). \tag{4.6}$$

As above, the equation containing $\tilde{\sigma}_{\eta\eta}$ gives the Alfvén wave with the spectrum (4.2). The equation with $\tilde{\sigma}_{xx}$ gives two magnetoacoustic waves with the phase velocities:

$$\omega/k = w_\pm \equiv \{\tfrac{1}{2}(w^2 + v_a^2) \pm [\tfrac{1}{4}(w^2 + v_a^2) - w^2 v_a^2 \cos^2\Phi]^{1/2}\}^{1/2}. \tag{4.7}$$

The dependences of the phase velocities of the magnetohydrodynamic waves on the magnetic field are given in Fig. 1.

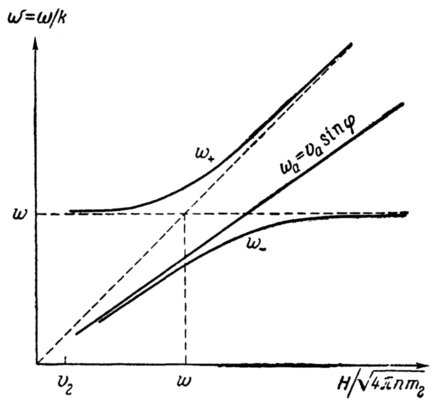

FIG. 1. Dependence of the phase velocities of magnetohydrodynamic waves on the magnetic field: w_+ is the velocity of the fast magnetoacoustic wave, w_- that of the slow magnetoacoustic wave, and w_a that of the Alfvén wave.

It follows from Eq. (4.7) that the quantity w plays the role of the sound velocity in an electron-hole plasma of a metal. However, in contrast with magnetohydrodynamics, the Alfvén and magnetoacoustic waves in the model under consideration exist in the region of strong spatial dispersion $\omega \ll |k_z| v_1$. In conclusion, we note that the conditions (4.3) can be rewritten in the form

$$v_2 \ll v_a \ll v_1 \ll v_a\Omega_1/\omega. \tag{4.8}$$

5. Up to now, we have considered electromagnetic waves whose wavelength is large in comparison with the dimensions of the electron orbits. We now study the problem of the possibility of propagation in a metal of electromagnetic excitations with wavelengths less than the Larmor radius: $kR \gg 1$.

In this case the asymptotic character of the conductivity tensor $\sigma_{\alpha\beta}$ is such that the diagonal element of σ_{xx} transverse to **k** and **H** is real and, in general, very large in magnitude. Here the nondiagonal elements are small in comparison with σ_{xx}, and large in comparison with σ_{yy} and σ_{zz}. Weakly damped excitations obviously do not exist under these conditions.

However, σ_{xx} is an oscillating function of the magnetic field. The physical nature of these oscillations is the same as in the case of "geometric" resonance for absorption of ultrasound in metals.[7] The fundamental contribution to σ_{xx} is made by electrons moving in phase with the wave, for which the resonance condition is satisfied:

$$\omega - k_z v_z = N\Omega \qquad (N = 0, \pm 1, \pm 2, \ldots). \tag{5.1}$$

Here the contribution of electrons with different N depends primarily on the relation between the diameter of their orbit and the wavelength. If an odd number of half wavelengths is contained in the diameter of the orbit, then such electrons make a large contribution to σ_{xx}. In the case of an even number of half wavelengths, the electrons do not make a contribution to σ_{xx}. For values of $|\cos\Phi|$ of the order of unity, the condition (5.1) is satisfied for different N and the different groups of electrons always include some that make a large contribution to σ_{xx}. Therefore, the relative amplitude of the oscillations of σ_{xx} is small in this case. For values of Φ close to $\pi/2$,

$$|\Phi - \pi/2| \equiv \varphi < (kR)^{-1}, \tag{5.2}$$

the condition (5.1) can be satisfied only for $N = 0$. In this case the oscillations of the geometric resonance are not small,[8] and the value of σ_{xx} can be close to zero, which leads to slowly damped spikes of the field in the volume of the metal.[4] It will be shown below that electromagnetic excitations with a discrete spectrum can exist in the metal for $\sigma_{xx} = 0$.

Small values of φ are of interest for another reason. The value of the element σ_{zz} is proportional to $\sin^{-2}\varphi$. Therefore, for sufficiently small φ, the value of σ_{zz} becomes large in comparison with σ_{xz}, and the tensor $\tilde{\sigma}_{\alpha\beta}$ is diagonal. The imaginary part of the element is negative in this case. Therefore, excitations with a discontinuous spectrum exist in a metal in which the electric field is polarized along **H**.

We investigate the asymptotic nature of the tensor $\sigma_{\alpha\beta}$ for large kR in the case

$$|k_z| R < 1 \ll |k_z l^*|. \tag{5.3}$$

We first limit ourselves to the case of an isotropic quadratic spectrum of the electrons, inasmuch as all the conclusions remain valid for arbitrary form of the Fermi surface for closed trajectories. Direct computation shows that the "renormalization'' of the tensor $\sigma_{\alpha\beta}$, which arises upon elimination of the longitudinal field E_ζ from Maxwell's equations (1.1), leads only to a small correction. Moreover, inasmuch as $\varphi < (kR)^{-1}$, one cannot distinguish between the coordinates xyz and $x\eta\zeta$.

By use of the inequalities (5.3), the tensor conductivity (1.8) can be reduced to the following from in the case of a single group of carriers:

$$\tilde{\sigma}_{\alpha\beta} = \frac{3}{2}\frac{ne^2}{m}\int_0^\pi \frac{\sin\theta\, d\theta}{\nu + i(k_z v\cos\theta - \omega)} \times \begin{pmatrix} J_1^2(u)\sin^2\theta; & iJ_0(u)J_1(u)\sin\theta\cos\theta \\ -iJ_0(u)J_1(u)\sin\theta\cos\theta; & J_0^2(u)\cos^2\theta \end{pmatrix}, \quad (5.4)$$

where $J_N(u)$ is the Bessel function of order N; $u = kR\sin\theta$.

It is not difficult to obtain the expressions for $\tilde{\sigma}_{x\eta}$ and $\tilde{\sigma}_{\eta\eta}$ by taking the asymptote of the corresponding Bessel functions and computing the integral over θ by the method of stationary phase. This gives

$$\tilde{\sigma}_{\eta\eta} = \frac{3}{2}\frac{ne^2}{m(k_z v)^2}\frac{\nu - i\omega}{kR},$$

$$\tilde{\sigma}_{\eta x} = -\tilde{\sigma}_{x\eta} = \frac{3}{2\sqrt{\pi}}\frac{ne^2}{m|k_z|v}\frac{\cos(2kR - \pi/4)}{(kR)^{3/2}}. \quad (5.5)$$

The asymptotic value of $\tilde{\sigma}_{xx}$ depends on the relative rate of change of two rapidly changing functions

$$J_1^2(kR\sin\theta) \text{ и } D(\theta) = [\nu + i(k_z v\cos\theta - \omega)]^{-1}.$$

For value of $\cos\theta = w/k_z v$, the function $D(\theta)$ has a maximum, the width of which is $\Delta\theta \sim (k_z l)^{-1}$. The characteristic interval of change in the functions is of the order of $1/kR$. Therefore, in the case

$$\varphi > \nu/\Omega, \qquad \omega \lesssim \nu \quad (5.6)$$

one can replace the function $D(\theta)$ by $\pi\delta(k_z v\cos\theta)$ and $\tilde{\sigma}_{xx}$ takes the form

$$\tilde{\sigma}_{xx} = 3\frac{ne^2}{m|k_z|v}\frac{\cos^2(kR - 3\pi/4)}{kR}. \quad (5.7)$$

The inequality (5.6) expresses the condition that the scatter of radii ΔR of those electrons which give a fundamental contribution to σ_{xx} is small in comparison with the length of the electromagnetic wave. Actually, in the case (5.3), the fundamental contribution to σ_{xx} comes from electrons close to the central cross section of the Fermi surface, for which

$$(\Delta p_z/p_F) \sim (k_z l)^{-1}.$$

The scatter of radii of these ''effective'' electrons $\Delta R \sim R(\Delta p_z/p_F)^2$. Therefore, for $k\Delta R \approx (\nu/\Omega\varphi k_z l) \ll 1$, all these electrons are under the same conditions relative to the wave. The most effective interaction with the electromagnetic field occurs on those parts of the orbit in which the electrons move along planes of equal phase of the wave A and B (Fig. 2). The phase difference at the two points A and B is determined by the relation between the diameter of the electron orbit and the length of the electromagnetic wave. Therefore the conductivity σ_{xx} oscillates strongly for change in kR.

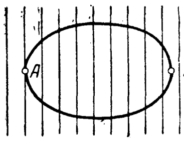

FIG. 2

It follows from (5.7) that for the values $kR = \alpha_n \equiv \pi(n + 1/4)$ the value of σ_{xx} vanishes (n is an integer). Here the hermitian part of the tensor $\tilde{\sigma}_{\alpha\beta}$ is seen to be small in comparison with the antihermitian one, which accounts for the existence of excitations with discrete wave vectors and frequencies. The dispersion equation D = 0 (1.7) in this case has the form

$$1 - (4\pi\omega_n/k_n^2c^2)^2\sigma_{x\eta}^2(\alpha_n) - i4\pi\omega_n k_n^{-2}c^{-2}\tilde{\sigma}_{\eta\eta}(\alpha_n) = 0. \quad (5.8)$$

Substituting Eqs. (5.5) for $\tilde{\sigma}_{x\eta}$ and $\tilde{\sigma}_{\eta\eta}$ with $kR = \alpha_n$ in Eq. (5.8), it is not difficult to obtain its solution with respect to ω:

$$\omega_n = \tfrac{2}{3}\pi v_a^2 v^{-2}\alpha_n^4\,[\Omega\varphi(\pi\alpha_n/2)^{1/2} - i\nu]. \quad (5.9)$$

The transverse part of the electric fields in these excitations is circularly polarized: $E_\eta = iE_x$, $E_\zeta > E_\eta$.

The region of applicability of the solution (5.9) is limited by the conditions

$$\nu/\Omega < \varphi < 1/\alpha_n, \qquad 1 \ll \alpha_n^4 \ll (v/v_a)^2, \quad (5.10)$$

which involve the inequalities (5.3) and (5.6). In the opposite limiting case,

$$\varphi \ll \nu/\Omega \quad (5.11)$$

the nondiagonal elements of the tensor $\tilde{\sigma}_{\alpha\beta}$ are negligibly small and the dispersion equation (1.7) splits into two equations of the form (4.7). As a consequence of the realness of σ_{xx}, a wave with polarization along the x axis cannot be propagated. The second equation yields if conditions (3.10) are satisfied a weakly damped wave with a discontinuous spectrum:

$$\omega = (^2/_3)^{1/2} (kR)^{3/2} |k_z| v_a - i\nu/2. \quad (5.12)$$

The electric field in this wave is directed along the η axis.

The region of applicability of (5.12) is determined by the conditions (3.10), (5.11), and (5.3). Eliminating the wave vector k from (5.3) by means of (5.12), one can rewrite it in the form

$$1 < (v_a \Omega / v\omega)^{2/5} \ll \Omega/\omega. \quad (5.13)$$

THEORY OF HIGH-FREQUENCY SURFACE IMPEDANCE

6. In the present section we investigate the resonance excitation of weakly damped waves by an external electromagnetic field, and consider their effect on the high-frequency characteristics of metals. The fundamental quantity characterizing the high frequency properties of metals is the surface impedance tensor $Z_{\alpha\beta}$ which connects the total current in the metal **J** with the tangential components of the field $\mathscr{E}_\alpha$ on the surface:

$$Z_{\alpha\beta} = \partial \mathscr{E}_\alpha (0)/\partial J_\beta = 4\pi i \omega c^{-2} \partial \mathscr{E}_\alpha(0)/\partial \mathscr{E}'_\beta (0). \quad (6.1)$$

The prime denotes the derivative with respect to the inward normal to the surface.

To find $Z_{\alpha\beta}$, it is necessary to solve Maxwell's equations:

$$\mathscr{E}''_\alpha (\zeta) = -4\pi i \omega c^{-2} j_\alpha (\zeta) \qquad (\alpha = x, \eta). \quad (6.2)$$

The ζ axis is directed along the inward normal to the surface, while the η axis coincides with the projection of the constant magnetic field **H** on the surface of the metal $\zeta = 0$. It is convenient to solve Eq. (6.2) in the Fourier representation. Continuing the vector of the electric field $\mathscr{E}_\alpha(\zeta)$ in even fashion in the region outside the metal, $\zeta < 0$, we seek it in the form

$$\mathscr{E}_\alpha(\zeta) = \frac{1}{\pi} \int_0^\infty dk E_\alpha (k) \cos k\zeta. \quad (6.3)$$

Equations (6.2) for the Fourier components E(k) are algebraic

$$k^2 E_\alpha(k) + 2\mathscr{E}'_\alpha (0) = \frac{4\pi i \omega}{c^2} \sum_{\beta=1}^{3} \sigma_{\alpha\beta} (\mathbf{k}, \omega, \mathbf{H}) E_\beta(k) \qquad (\alpha = x, \eta),$$

$$\sum_{\beta=1}^{3} \sigma_{\zeta\beta} (\mathbf{k}, \omega, \mathbf{H}) E_\beta = 0, \quad (6.4)$$

where the wave vector $\mathbf{k} \parallel \zeta$. We neglect changes in the conductivity operator as a consequence of collisions of the electrons with the surface of the metal. In all the cases considered below, these changes are not essential (account of the electron collisions with the surface leads only to the appearance in $\sigma_{\alpha\beta}$ and $Z_{\alpha\beta}$ of numerical factors of the order of unity (see, for example, [3]).

After elimination of the longitudinal field E_ζ from Eqs. (6.4), they take on the form

$$D_{\alpha\beta} E_\beta = -2k^{-2} \mathscr{E}'_\alpha (0), \quad (6.5)$$

where $D_{\alpha\beta}$ is determined by Eq. (1.4).

Solution of Eqs. (6.5) with respect to **E** and its subsequent integration over k lead to the following expressions for electric field $\mathscr{E}$ in the volume of the metal and for the impedance tensor

$$\mathscr{E}_\alpha (\zeta) = T_{\alpha\beta} (\zeta) \mathscr{E}'_\beta (0), \qquad Z_{\alpha\beta} = 4\pi i \omega c^{-2} T_{\alpha\beta} (0), \quad (6.6)$$

$$T_{\alpha\beta} (\zeta) = -\frac{2}{\pi} \int_0^\infty \frac{dk}{k^2} D^{-1}_{\alpha\beta} \cos k\zeta, \quad (6.7)$$

where $D^{-1}_{\alpha\beta}$ is the inverse of the tensor $D_{\alpha\beta}$.

In those cases in which the electric field of the wave in the metal is circularly polarized in the $x\eta$ plane, it is natural to introduce the surface impedance tensor for circularly polarized waves, along with the tensor $Z_{\alpha\beta}$ of (6.6). The elements of this tensor are determined by the formulas

$$\mathscr{E}_\pm (0) = Z_\pm J_\pm + Z'_\pm J_\mp, \qquad \mathscr{E}_\pm = \mathscr{E}_x \pm i\mathscr{E}_\eta \quad (6.8)$$

and are connected with the elements of the tensor $Z_{\alpha\beta}$ by the simple relations

$$Z_\pm = \tfrac{1}{2} (Z_{xx} + Z_{\eta\eta}) \pm i Z_{x\eta}, \qquad Z'_\pm = \tfrac{1}{2} (Z_{xx} - Z_{\eta\eta}). \quad (6.9)$$

As was shown in Sec. 2, in the case (2.10), there are electromagnetic excitations in a metal with the quadratic spectrum (2.7). The transverse part of the electric field **E** in these waves is circularly polarized: $E_\eta = iE_x$. Upon incidence on the surface of the metal of an external electromagnetic wave with frequency ω, resonance excitation takes place with the natural oscillations (2.7). Direct calculation gives the following expressions for the elements of the impedance tensor:

$$Z_- = -iZ^*_+ = 4\pi c^{-1} (\omega_0^{-1} \sqrt{\omega \Omega \sin \varphi} - i \tfrac{3}{32} \pi \omega R c^{-1} \cos^2 \varphi),$$

$$Z'_\pm = (i-1) \tfrac{3}{8} \pi^2 \omega R c^{-2} \cos^2 \varphi, \quad (6.10)$$

where $\omega_0 = (4\pi ne^2/m)^{1/2}$ is the plasma frequency.

Thus the element Z_- is almost real, which corresponds to the generation of a wave with polarization (−) in the interior of the metal. The imaginary part of Z_- for $\varphi \neq \pi/2$ is brought about by the damping of this wave as a consequence of spatial dispersion. The wave with polarization (+), on the other hand, experiences total internal reflection, as a consequence of which the element Z_+, in particular, is imaginary. The penetration of the electromagnetic wave into the metal for H perpendicular to the surface ($\varphi = \pi/2$) was first investigated by Konstantinov and Perel'.[1]

In accord with Eqs. (6.10), $Z_\pm$ are proportional to the square root of the frequency of the wave and the magnetic field. The value of Z' is proportional to the frequency, and inversely proportional to the magnetic field.

7. In Sec. 3, it was shown that in metals with different concentrations of electrons and "holes," there exist waves with a linear dispersion law (3.6) and elliptic polarizations upon satisfaction of the conditions (3.10) and (3.11). The relative damping of these excitations is of the order of ν/ω. The external electromagnetic wave with frequency ω excites these natural oscillations in the metal.

In the calculation of the impedance tensor $Z_{\alpha\beta}$ of (6.6), the factor $1/D$ in the integrand can be written in the form

$$1/D = \mathrm{P}\,(1/D') + \pi i \delta\,(D'), \tag{7.1}$$

Here $D = D' + iD''$; the small quantity $D'' \sim D'\nu/\omega$ is neglected on the right hand side of (7.1); the symbol P means that the integral over k is taken in the sense of the principal value.

Substituting (7.1) in (6.6), it is not difficult to show that the principal value of the integral is equal to zero; therefore the tensor $Z_{\alpha\beta}$ in the given approximation is real:

$$Z_{\alpha\beta} = \frac{4\pi v_a^2}{c^2(v_+ + v_-)}\,\frac{1}{\det A_{\alpha\beta}}\begin{pmatrix} A_{\eta\eta} + \sqrt{\det A_{\alpha\beta}}; & A_{x\eta} \\ A_{\eta x}; & A_{xx} + \sqrt{\det A_{\alpha\beta}} \end{pmatrix}, \tag{7.2}$$

where $A_{\alpha\beta}$ and $v_\pm$ are determined by Eqs. (3.5) and (3.6). The imaginary part of the impedance tensor $Z_{\alpha\beta}$ is determined by the damping of the electromagnetic waves as a consequence of the electron scattering and is smaller than (7.2) by a factor ω/ν.

It follows from (7.2) that the reflected and the two transmitted waves in the metal are all elliptically polarized in the general case. The degree of ellipticity is determined by the nondiagonal elements of the tensor $Z_{\alpha\beta}$.

For a magnetic field parallel to the surface ($\varphi \ll \omega/\Omega$), the fast magnetoacoustic wave is excited in the metal with the spectrum (3.9). The surface impedance corresponding to this wave is

$$Z_{xx} = 4\pi v_0/c^2. \tag{7.3}$$

The nondiagonal elements of the tensor $Z_{\alpha\beta}$ are smaller than Z_{xx} by the factor Ω/ω.

In the case of much weaker magnetic fields satisfying the conditions (4.1), the external field of frequency ω excites an Alfvén wave (4.2) in the metal. The corresponding surface impedance is

$$Z_{\eta\eta} = 4\pi v_a c^{-2} \sin\varphi\,(1 - i\nu/2\omega). \tag{7.4}$$

The nondiagonal elements of the impedance tensor are negligibly small in the given approximation, while the value of the element Z_{xx} is determined by the transverse conductivity σ_{xx}. For $\varphi \neq \pi/2$, its value is chiefly determined by the spatial dispersion (Landau damping), as a consequence of which the x component of the electric field is damped.

It should be noted that the special features of the penetration of $\mathscr{E}_x$ into the interior of the metal can depend materially on the character of the reflection of the electrons from the surface. It can be shown that the surface conductivity plays the dominant role in this case, inasmuch as, by virtue of the conditions $n_1 = n_2$ and $\omega \ll kv_S$, the nondiagonal elements of the volume conductivity tensor are extraordinarily small. This problem is very complicated and deserves special investigation.

Upon satisfaction of the conditions (4.8) and (3.10), the external electromagnetic field excites all three magnetohydrodynamic waves in the metal. As a consequence of the diagonal character of $\tilde{\sigma}_{\alpha\beta}$ in the given case, the tensor $Z_{\alpha\beta}$ is also diagonal. The impedance $Z_{\eta\eta}$ is chiefly connected with the Alfvén wave and has the form (7.4). The value of the element Z_{xx} is determined by the magnetoacoustic waves. The real part of the impedance Z_{xx} is large in comparison with the imaginary, and has the form

$$\mathrm{Re}\, Z_{xx} = \frac{4\pi v_a}{c^2}\,\frac{v_a + w\sin\varphi}{w_+ + w_-} \approx \frac{4\pi v_a}{c^2}\begin{cases} \sin\varphi, & v_a \ll w\sin\varphi \\ 1, & v_a \gg w\sin\varphi \end{cases}. \tag{7.5}$$

The imaginary part of Z_{xx} is connected with the damping of the magnetoacoustic waves which, for $\varphi \neq \pi/2$, is brought about by the spatial dispersion.

The behavior of the electric fields in the magnetoacoustic waves close to the surface of the metal is determined by the formula

$$\frac{\mathscr{E}_x^{(+)}(0)}{\mathscr{E}_x^{(-)}(0)} = \frac{w_+^2 - w^2\sin^2\varphi}{w\sin^2\varphi - w_-^2}\,\frac{w_-}{w_+} \approx \frac{v_a}{w\sin\varphi}\begin{cases} \cos^2\varphi, & v_a \ll w\sin\varphi \\ v_a^2/w^2, & v_a \gg w\sin\varphi \end{cases}. \tag{7.6}$$

Finally, for $\mathbf{H} \parallel \zeta$ ($\varphi = \pi/2$), a single magnetoacoustic wave is propagated in the metal, both in the case (4.1) and in the case (4.8). The spectrum and the damping of this wave are identical with the spectrum and damping of the Alfvén wave. Therefore, $Z_{xx} = Z_{\eta\eta}$ (7.4) in this case.

8. We now investigate the excitation of electro-

magnetic waves whose wavelength is small in comparison with the dimensions of the electron orbits.

If the values of ω, H, and φ satisfy the inequalities (3.10), (5.11), and (5.13), then the external field excites a wave (5.10) in the metal, the electric field in which is directed along the η axis. The component of the electric field $\mathscr{E}_x$ experiences spikes in the interior of the metal.[4] As a consequence of the smallness of the nondiagonal elements of $\tilde{\sigma}_{\alpha\beta}$ the impedance tensor $Z_{\alpha\beta}$ is diagonal. The element Z_{xx} which corresponds to the component $\mathscr{E}_x$ was computed in the work of one of the authors.[4] The expression for the element $Z_{\eta\eta}$, corresponding to a slowly damped wave, has the form

$$Z_{\eta\eta} \approx 4e^{-3\pi i/10}\omega\lambda/\pi c^2, \qquad \lambda = 2\pi\,(\delta_0^3 R v\,\varphi^2/\omega)^{1/5}, \qquad (8.1)$$

where λ is the length of the weakly damped wave in the metal and $\delta_0 = (c^2 p_F/6\pi ne^2\omega)^{1/3}$ is the penetration depth of the electromagnetic field for the anomalous skin effect and H = 0. The effective damping length of the wave is $L \sim \lambda\omega/\nu$.

We call attention to the possibility of the following effect.3) In a plate whose thickness d is large in comparison with the wavelength λ and small in comparison with the damping distance L, the wave vector is "quantized": $k_N = \pi N/d$ (N = integer). Resonant oscillations of the impedance should be observed upon a change in the magnetic field or frequency; these are brought about by the coincidence of the wave vector k with one the eigenvalues k_N. Here a standing electromagnetic wave arises in the plate.

This effect is possible in principle in all cases in which there are weakly damped electromagnetic excitations in the metal. By this means, Libchaber and Veilex[10] discovered spiral waves in crystals of InSb, similar to those considered in Sec. 2. The experiments were carried out in a magnetic field perpendicular to the surface ($\mathbf{k} \parallel \mathbf{H}$) in the absence of spatial dispersion.

If the magnetic field **H** satisfies the conditions (5.10), then oscillations with the discrete spectrum (5.9) can propagate in metals with one group of conduction electrons. When the frequency of the external field coincides with that of one of the natural frequencies ω_n (5.9), a weakly damped electromagnetic wave is excited in the metal and the impedance has a resonance maximum.

We first consider the region of nonresonant values of the magnetic field, when neither of the frequencies ω_n coincides with the frequency of the external field ω. As was noted above, the conductivity $\tilde{\sigma}_{xx}$ is an oscillating function of kR and decreases rapidly in a narrow range of values of kR close to α_n. However, in spite of this fact, the fundamental contribution to the field and to the impedance in the nonresonant region is made by those regions of values of k where $\tilde{\sigma}_{xx}$ is large in comparison with $\tilde{\sigma}_{x\eta}$ and $\sigma_{\eta\eta}$. Therefore the quantity D in this case can be represented approximately in the form

$$D \approx (1 - i4\pi\omega\tilde{\sigma}_{xx}/k^2c^2)\,(1 - i4\pi\omega\tilde{\sigma}_{x\eta}^2/k^2c^2\tilde{\sigma}_{xx}). \qquad (8.2)$$

Substituting $\tilde{\sigma}_{\alpha\beta}$ from (5.5) and (5.7) and D from (8.2) in Eqs. (6.6) and (6.7), and carrying out the integration over k, one can find the electric field in the metal $\mathscr{E}(\zeta)$ and the tensor $Z_{\alpha\beta}$ in the case under consideration.

The distribution of the x component of the field $\mathscr{E}$ in the interior of the metal, and the corresponding impedance Z_{xx}, were investigated earlier.[4] In accord with [4], $\mathscr{E}_x(\zeta)$ is a periodic function of ζ with period 2R. At distances which are multiples of the maximum diameter of the electron orbit, the field $\mathscr{E}_x$ has narrow spikes. In the intervals between the spikes the field is practically absent. The height of the spikes decreases comparatively slowly with increase in their number.

One can also study the function $\mathscr{E}_\eta(\zeta)$ by a method entirely analogous to that used in [4] in the investigation of $\mathscr{E}_x$. Without carrying out the complete analysis here, we only show that the component $\mathscr{E}_\eta$ also has narrow, high spikes for ζ = 2NR (N = 0, 1, 2, ...). The height of the spikes falls off with number as $N^{-3/2}$. The schematic character of the field distribution $\mathscr{E}_\eta$ in the volume of the metal is shown in Fig. 3.

Far from resonance, the elements of the tensor of the impedance $Z_{\alpha\beta}$ have the form

$$Z_{xx} \approx 4\pi\omega\delta c^{-2}\, e^{-\pi i/8},$$
$$Z_{\eta\eta} = 4\pi\omega\delta c^{-2}\,(R/\delta)^{1/4} e^{-3\pi i/5},$$
$$|Z_{x\eta}| \ll |Z_{xx}|, \qquad (8.3)$$

where $\delta = (\delta_0^3 R\varphi)^{1/4}$.

Resonance maxima, corresponding to coincidence of ω with one of the natural frequencies ω_n, are superimposed on this comparatively smooth dependence of the elements $Z_{\alpha\beta}$ on the magnetic field. For a fixed frequency of the external field, the resonance can take place for values of the cyclotron frequency

3)Bass, Blank, and Kaganov,[9] made a detailed analysis of this effect for the case of low frequency spiral waves ($n_1 \neq n_2$) in which the spatial and temperal dispersions are unimportant, $kl \ll 1$, $\omega \ll \nu$.

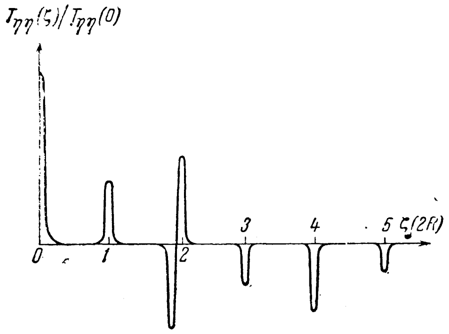

FIG. 3 Schematic diagram of the field distribution in a metal.

$$\Omega = \Omega_n \equiv \Omega_0 \left(n + \tfrac{1}{4}\right)^{-3/2}, \qquad (8.4)$$

where

$$\Omega_0 \approx (\omega\omega_0^2 v^2/\varphi c^2)^{1/3}/\pi\sqrt{2} \qquad (8.5)$$

is the maximal cyclotron frequency for which resonance occurs.

An electromagnetic wave is excited in the metal upon satisfaction of the resonance condition (8.4). For obtaining this resonance part of the electric field, one must replace 1/D in (6.6), (6.7) by

$$i\pi\delta\ [1 - (4\pi\omega\tilde{\sigma}_{x\eta}/k^2c^2)^2]. \qquad (8.6)$$

The principal value of the integral of 1/D is small for large ζ. Therefore,

$$\mathscr{E}^{\mathrm{res}}(\zeta) = -\frac{2}{9}\frac{\cos k_n\zeta}{k_n}\begin{pmatrix} i & 1 \\ -1 & i \end{pmatrix}\mathscr{E}'(0), \qquad (8.7)$$

$$k_n = \pi\left(n + \tfrac{1}{4}\right)\Omega_n/v = \pi\Omega_0 v^{-1}\left(n + \tfrac{1}{4}\right)^{-1/2}. \qquad (8.8)$$

An elementary calculation of the resonance part of the impedance gives the following result:

$$\Delta Z_{\alpha\beta}^{\mathrm{res}} \approx \frac{4\pi\omega}{3}\delta_n c^{-2}\left(n + \tfrac{1}{4}\right)^{1/8}\begin{pmatrix} 1-3i; & -i+0.9 \\ i-0.9; & 1-3i \end{pmatrix}, \qquad (8.9)$$

where δ_n is the value of δ for a resonance value of the magnetic field $H = H_n$; $\delta_n \sim (n + 1/4)^{3/8}$.

Inasmuch as the resonance part of the field (8.7) is circularly polarized, it is useful to introduce the tensor ΔZ^{res} for the circularly polarized waves (6.8) and (6.9):

$$\Delta Z_+^{\mathrm{res}} \approx -i\frac{16}{3}\pi\omega\delta_n c^{-2}\left(n + \tfrac{1}{4}\right)^{1/8}, \quad \Delta Z'_{\mathrm{res}} = 0,$$
$$\Delta Z_-^{\mathrm{res}} \approx (1 - i)\frac{8}{3}\pi\omega\delta_n c^{-2}\left(n + \tfrac{1}{4}\right)^{1/8}. \qquad (8.10)$$

The character of the dependence of the resonance part of the impedance as a function of the inverse magnetic field is shown in Fig. 4. The height of the resonance maxima increases upon decrease in H as $H^{-1/3} \sim n^{1/2}$. This increase is brought about by the fact that the wavelength of the characteristic electromagnetic oscillations increases for an increase in H. The resonance

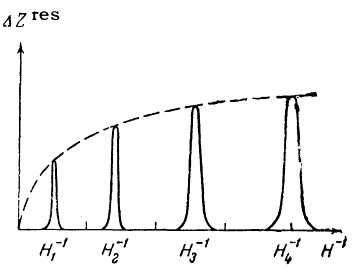

FIG. 4. Dependence of the resonant part of the impedance on the reciprocal of the magnetic field.

maxima are not equidistant in the reciprocal of the field ($H_n^{-1} \sim n^{3/2}$). The width of the maxima is determined by the damping of the electromagnetic excitations: $(\Delta H/H)_n \sim n$.

This new resonance effect is significantly different from the well known cyclotron resonance in a metal.[11] The cyclotron resonance takes place when the frequency of the external field is a multiple of the cyclotron frequency. Here a sharp increase takes place in the high-frequency conductivity, which causes a resonance decrease in the impedance of the metal. In contrast to the cyclotron resonance, the resonance under consideration is possible for much lower frequencies (even for $\omega < \nu$). The physical nature of this effect consists in the resonance excitation of the characteristic, weakly damped electromagnetic oscillations with a discrete spectrum. Here the metal becomes relatively transparent for the electromagnetic field, and this brings about an increase in the impedance.

RESONANCE EXCITATION OF ELECTROMAGNETIC WAVES BY ULTRASOUND IN METALS

9. The interaction of the conduction electrons with acoustic lattice vibrations leads to the appearance of a variable electromagnetic field in the metal. Therefore, we can use ultrasound to excite weakly damped electromagnetic waves. This effect is obviously a resonant one, in which the phase velocities of the electromagnetic and acoustic oscillations coincide.

For the determination of the absorption coefficient of the sound by the conduction electrons in the metal, we write down the kinetic equation for the electron distribution function F:

$$dF/dt + \hat{I}(F) = 0, \qquad (9.1)$$

where d/dt is the total time derivative and $\hat{I}$ is the collision operator.

The dispersion law for the electrons is given in the non-inertial reference frame K′, connected with the lattice moving under the action of the sound wave. The coordinates of the electron in the K′ system are connected with the coordinates in the laboratory system K by the relation

$$\mathbf{r}' = \mathbf{r} - \mathbf{u}(\mathbf{r}, t), \tag{9.2}$$

where $\mathbf{u}(\mathbf{r}, t)$ is the displacement vector in the sound wave. The energy of the electron in the K′ system has the form

$$\varepsilon'(\mathbf{P}', \mathbf{r}', t) = \varepsilon_0 \{\mathbf{P}' - ec^{-1}\mathbf{A}'(\mathbf{r}', t)\} + \Lambda_{ik} u_{ik} - m_0 \mathbf{v}' \dot{\mathbf{u}}, \tag{9.3}$$

Here $\epsilon_0(\mathbf{p})$ is the dispersion law of the electron in the absence of external fields; $\mathbf{P}'$ is the generalized momentum; $\mathbf{v}' = \partial\epsilon'/\partial\mathbf{P}'$ is the velocity of the electron, m_0 the mass of the free electron, u_{ik} the deformation tensor, Λ_{ik} the deformation potential tensor, which satisfies the condition $\langle\Lambda_{ik}\rangle = 0$ (the angle brackets indicate averaging over the Fermi surface). Repetition of the vector indices i and k indicates summation from 1 to 3; the dot indicates the partial time derivative. The vector potential of the electromagnetic field $\mathbf{A}'(\mathbf{r}', t)$ is connected with the vector potential $\mathbf{A}(\mathbf{r}, t)$ in the K system by the relation

$$A_i'(\mathbf{r}', t) = A_i(\mathbf{r}, t) + A_k(\mathbf{r}, t)\, \partial u_k/\partial x_i. \tag{9.4}$$

The scalar potential φ' in the K′ system is set equal to zero. Therefore, the potential φ in the K system is equal to $\dot{\mathbf{u}}\cdot\mathbf{A}/c$. The component $\Lambda_{ik}u_{ik}$ describes the change in the energy of the electron as the result of inhomogeneous deformation of the crystal, and $m_0\mathbf{v}'\cdot\dot{\mathbf{u}}$ is brought about by the Stewart-Tolman effect.

The energy of the electron ϵ' in the K′ system appears in the law of conservation of energy in the collision integral. Therefore the collision integral is made to vanish by the distribution function $f_0(\epsilon' - \mu)$:

$$\hat{I}\{f_0(\varepsilon' - \mu)\} = 0. \tag{9.5}$$

Solution of the kinetic equation (9.1) will be sought in the form

$$F(\mathbf{P}', \mathbf{r}', t) = f_0(\varepsilon' - \mu) + \chi(\mathbf{P}', \mathbf{r}', t)\, \partial f_0/\partial\mu. \tag{9.6}$$

From the condition of electrical quasi-neutrality, it follows that $\langle\chi\rangle = 0$. The function χ satisfies in the approximation linear in u the equation

$$(d/dt + \nu)\chi \equiv \partial\chi/\partial t + \mathbf{v}'\nabla\chi + \Omega\,\partial\chi/\partial\tau + \nu\chi = g, \tag{9.7}$$

where

$$g \equiv d\varepsilon'/dt = \Lambda_{ik}\dot{u}_{ik} + e\mathbf{E}'\mathbf{v}' - m_0\mathbf{v}'\ddot{\mathbf{u}}, \tag{9.8}$$

the electric field $\mathbf{E}' = -c^{-1}\partial\mathbf{A}'(\mathbf{r}', t)/\partial t$ is connected with the electric field $\mathbf{E} = -c^{-1}[\partial\mathbf{A}(\mathbf{r}, t)/\partial t + \nabla(\mathbf{A}\cdot\dot{\mathbf{u}})]$ by the relation

$$\mathbf{E}' = \mathbf{E} + \mathbf{G} \qquad (\mathbf{G} = c^{-1}[\dot{\mathbf{u}}\,\mathbf{H}]). \tag{9.9}^*$$

The component G on the right hand side of (9.9) represents the induction field in the system of coordinates moving with the lattice. The appearance of the induction field G and the ultrasonic absorption associated with it is due to the fact that the conductor deformed by the sound wave intersects the lines of force of the constant magnetic field. V. Gurevich[12] was the first to point out the possibility of induction absorption of ultrasound in metals.

The density of the electrical current is expressed in terms of the function χ in the following way:

$$\mathbf{j} = \frac{2e}{h^3}\int d^3P'\mathbf{v}'F(\mathbf{P}', \mathbf{r}', t) = \frac{2e}{h^3}\int d^3p\,\frac{\partial f_0}{\partial\mu}\mathbf{v}\chi(\mathbf{p}, \mathbf{r}, t). \tag{9.10}$$

The electric field E can be found from Maxwell's equations (1.1) with the total current (9.10) (the current j does not change in the transition from the K′ system to the laboratory system).

The mean density of sound energy Q absorbed by the electrons per unit time is, from [13]:

$$Q = \frac{2}{h^3}\overline{\int d^3p'F(\mathbf{p}', \mathbf{r}', t)\,\dot{\varepsilon}'(\mathbf{p}', \mathbf{r}', t)}, \tag{9.11}$$

where the prime denotes averaging with respect to time. For a plane monochromatic sound wave $\mathbf{u} \sim \exp i(\mathbf{k}\cdot\mathbf{r} - \omega t)$,

$$Q = \operatorname{Re}\int\frac{d^3p}{h^3}\frac{\partial f_0}{\partial\mu}g^*\chi = \int\frac{d^3p}{h^3}\frac{\partial f_0}{\partial\mu}\nu|\chi|^2; \tag{9.12}$$

Here * denotes the complex conjugate. It follows from (9.12) that the quantity Q is identical with the dissipation function.

We now introduce the functions χ_Λ and χ_E, which satisfy the equation (9.7) with $g_\Lambda = \Lambda_{ik}\dot{u}_{ik}$ and $g_E = e\mathbf{E}'\cdot\mathbf{v}$, respectively. Then, neglecting the term $-m_0\mathbf{v}\ddot{\mathbf{u}}$, associated with the Stewart-Tolman effect,[14] we write (9.12) in the form

$$Q = Q_\Lambda + Q_E + Q_I, \tag{9.13}$$

where

$$Q_\Lambda = \operatorname{Re}\int\frac{d^3p}{h^3}\frac{\partial f_0}{\partial\mu}g_\Lambda^*\chi_\Lambda \tag{9.14}$$

is the purely deformation absorption,

$$Q_E = \tfrac{1}{2}\operatorname{Re}\sigma_{\alpha\beta}E_\alpha'^*E_\beta' \tag{9.15}$$

is the Joule heat and

$$Q_I = \operatorname{Re}\int\frac{d^3p}{h^3}\frac{\partial f_0}{\partial\mu}(g_E^*\chi_\Lambda + g_\Lambda^*\chi_E) \tag{9.16}$$

is the interference term. The latter can be ex-

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*$[\dot{\mathbf{u}}\mathbf{H}] = \dot{\mathbf{u}} \times \mathbf{H}$.

pressed in terms of the deformation current

$$\mathbf{j}_\Lambda = \frac{2e}{h^3}\int d^3p\, \frac{\partial f_0}{\partial \mu}\mathbf{v}\chi_\Lambda \qquad (9.17)$$

and the electric field $\mathbf{E}'$:

$$Q_l = \frac{1}{2}\operatorname{Re}\mathbf{E}'^*\,[\mathbf{j}_\Lambda(\mathbf{k}, \omega, \mathbf{H}) + \mathbf{j}_\Lambda(-\mathbf{k}, -\omega, -\mathbf{H})]. \qquad (9.18)$$

10. We now consider absorption of ultrasound in metals in a strong magnetic field, in which the wave vector of the sound $\mathbf{k}$ satisfies the conditions (2.1) and (2.2). In this case, the change in the energy of the electron under the action of the induction electric field $e\mathbf{G}\cdot\mathbf{v}$ is $1/kR$ times larger than the energy change as the result of the deformation interaction $\Lambda_{ik}\dot{u}_{ik}$. Therefore, in the determination of the electric fields, one can neglect the deformation current $\mathbf{j}_\Lambda$ and write the current density $\mathbf{j}$ in the form

$$j_\alpha = \sigma_{\alpha\beta}E'_\beta. \qquad (10.1)$$

The solution of Maxwell's equations (1.1) with the external induction current $\mathbf{j}^{ext} = \hat{\sigma}\mathbf{G}$ has the form

$$E'_\alpha = D^{-1}_{\alpha\beta}G_\beta \qquad (\alpha, \beta = x, \eta), \qquad (10.2)$$

$$E'_\zeta = -(\sigma_{\zeta x}E'_x + \sigma_{\zeta\eta}E'_\eta)/\sigma_{\zeta\zeta}. \qquad (10.3)$$

By using the circularly polarized fields $E'_\pm$ and $G_\pm$ from (6.8), we can rewrite the expression (10.2) in the form

$$E'_\pm = G_\pm\,[1 \pm 4\pi\omega k^{-2}c^{-2}\tilde{\sigma}_{\eta x} - i2\pi\omega k^{-2}c^{-2}(\tilde{\sigma}_{xx} + \tilde{\sigma}_{\eta\eta})]^{-1}. \qquad (10.4)$$

When the phase velocities of the acoustic and electromagnetic waves (2.7) are identical, resonance takes place for the wave E'_-. For the other polarization (+) there is no resonance. Therefore, only circular polarization makes sense for transverse sound. For a longitudinal wave, the resonance induction absorption takes place only when $\mathbf{k}$ and $\mathbf{H}$ are not parallel.

In the vicinity of resonance, $|E'_-| \gg |G_-|$ and the Joule losses Q_E play the dominant role in the sound absorption:

$$Q_- = \frac{Q_{max}}{1 + (\omega' - \omega)^2/\omega''^2}, \qquad Q_{max} = \frac{\tilde{\sigma}^2_{x\eta}\,|G_-|^2}{2(\tilde{\sigma}_{xx} + \tilde{\sigma}_{\eta\eta})}, \qquad (10.5)$$

where ω' and ω'' are determined by the formula (2.7). The resonance condition has the form

$$\omega\Omega\,|\cos\Phi| = \omega_0^2 s^2/c^2, \qquad (10.6)$$

where s is the speed of sound.

Resonance induction sound absorption depends primarily on the angle between $\mathbf{k}$ and $\mathbf{H}$. Q_{max} is largest for transverse sound vibrations at small Φ ($\Phi^2 \lesssim 1/kl$). In the given case, $kR \ll 1 \ll k_z l$, the deformation absorption is $Q_\Lambda \sim Q_0\,|\cos\Phi|^{-1}$ (Q_0 is the absorption at $H = 0$[14]) and does not play a role close to resonance:

$$Q_\Lambda/Q_{max} \sim (kR)^2\,[1/kl + \tfrac{3}{16}\pi\sin^2\Phi] \ll 1. \qquad (10.7)$$

As is well known, the ultrasound absorption coefficient Γ is determined by the ratio of the absorbed energy Q to the energy flux W. The latter is made up of the acoustic energy flux and the energy flux of the electromagnetic field. As a result of the sharp increase in the field E' close to resonance, the flux of electromagnetic energy can be larger than the flux of acoustic energy. Here, an electromagnetic wave is propagated in the metal while the sound wave is absent.

11. We now turn to resonance absorption of ultrasound in the case of large kR in the region of angles φ satisfying the inequalities (5.3). In finding the electric field $\mathbf{E}$, one can neglect the induction current $\hat{\sigma}\mathbf{G}$, in comparison with the deformation current $\mathbf{j}_\Lambda$. It is seen that the "renormalization" of the deformation current, which arises in the elimination of the longitudinal field E_ζ, is unimportant. In the case (5.11), where the excitations (5.12) existing in the metal are polarized along the axis, the resonant part of the field E_η is determined by the component of the deformation current $j_{\Lambda\eta}$. Moreover, one can show that this same component of the current is responsible for the resonance excitation of electromagnetic waves with the discrete spectrum (5.9) (at resonance, $j_{\Lambda x} = 0$).

The asymptotic value of $j_{\Lambda\eta}$ (9.17) in the case (5.3), and for a square law of dispersion with $\Lambda_{ik}(\mathbf{p}) = \text{const}$, has the form

$$j_{\Lambda\eta} = \frac{3}{2i}\,\frac{\Lambda_{ik}\dot{u}_{ik}}{ev}\,\frac{\sigma}{kRk_z l}, \qquad \sigma = \frac{ne^2}{mv}. \qquad (11.1)$$

In the case of closed trajectories, the dispersion law and the arbitrary dependence of Λ_{ik} on $\mathbf{p}$ do not change the character of the asymptotic value of $\mathbf{j}_\Lambda$.

The deformation absorption Q_Λ was studied previously[8] for large values of kR. Here we shall compute the Joule losses Q_E which are brought about by excitation of weakly damped electromagnetic waves.

The resonance excitation of quasiparticles with a discrete spectrum takes place when the frequency ω and the wave vector $\mathbf{k}$ of ultrasound coincide with the frequency ω_n and wave vector α_n/R of the electromagnetic wave (5.9). The condition for equality of the wave vectors has the form

$$\omega R/s = \alpha_n. \tag{11.2}$$

Substituting the value of the magnetic field H found from (11.2) in (5.9), we can represent the condition for identity of the frequencies in the form

$$\varphi = \varphi_n \equiv \left(\frac{9}{2\pi^3}\right)^{1/2} \frac{s}{v} \left(\frac{s\omega_0}{c\omega}\right)^2 \alpha_n^{-3/2}. \tag{11.3}$$

Thus at a fixed sound frequency ω the resonance takes place for completely determined values of the angle between **k** and **H** (the resonance angle). Here the value of the magnetic field must satisfy the condition (11.2).

In the vicinity of resonance, $\tilde{\sigma}_{xx} = 0$, and the electric field is

$$E_- = \frac{4\pi s^2}{\omega c^2} j_{\Lambda\eta}\left[1 - \frac{4\pi s^2}{\omega c^2}\tilde{\sigma}_{\eta x} - i\frac{2\pi s^2}{\omega c^2}\tilde{\sigma}_{\eta\eta}\right]^{-1},$$

$$|E_+| \ll |E_-|. \tag{11.4}$$

The absorption coefficient of the ultrasound has the form

$$\Gamma_E \equiv \frac{Q_E}{W} = \Gamma_{max}\left[1 + \left(\frac{3}{\pi^2}\frac{s^2\omega_0^2}{c^2\omega v\alpha_n^4}\right)^2\left(1 - \frac{\varphi}{\varphi_n}\right)^2\right]^{-1}, \tag{11.5}$$

$$\Gamma_{max} = |j_{\Lambda\eta}|^2/2\tilde{\sigma}_{\eta\eta}W \sim \Gamma_\Lambda\,|k_z|\,l. \tag{11.6}$$

The coefficient of the purely deformation absorption Γ_Λ in this case is an oscillatory function of the magnetic field.[8] For resonance, $\varphi = \varphi_n$ the single maximum Γ_E coincides with one of the maxima Γ_Λ, and the resonance peak Γ_E is significantly narrower than the maxima Γ_Λ, while the value of Γ_{max} is $k_z l$ times larger than the value of Γ_Λ.

Resonance excitation of an electromagnetic wave (5.9) by the ultrasound is possible only upon satisfaction of the inequalities (5.10). Eliminating the magnetic field H and the angle φ from them by means of the resonance conditions (11.2) and (11.3), we write (5.10) in the form

$$\alpha_n^2 < (\omega_0 s/\omega c)^2\,\alpha_n^{-1/2} < v/s. \tag{11.7}$$

Here the limitation $\alpha_n^4 \ll (v/v_a)^2$ is a consequence of the first of the inequalities (11.7). Moreover, for resonance in this case it is necessary that the Fermi surface have only one central cross section for a given direction of the magnetic field (the necessary condition for causing σ_{xx} to vanish).

Analysis of the resonance conditions (11.2) and (11.3) and the limitations following from (11.7) show that the resonance must be observed at frequencies of the ultrasound $\omega \sim \nu \sim 10^9\ \text{sec}^{-1}$, magnetic field $H \sim 10^3$–10^4 Oe, and angles φ of the order of a degree. For values of the angle φ satisfying the conditions (5.11) and (5.13), resonance excitation of electromagnetic waves with a continuous spectrum (5.12) is possible. Here the ultrasonic frequency and the magnetic field must satisfy the inequalities $\nu \ll \omega \ll \Omega$.

The condition for resonance is obtained from (5.12) by substitution of ω/s for k, and has the form

$$\Omega = \Omega_{res} \equiv \tfrac{2}{3}\varphi^2\,\omega^3 c^2 v^3/\omega_0^2 s^5. \tag{11.8}$$

The ultrasound absorption coefficient in the neighborhood of resonance is

$$\Gamma_E = \Gamma_{max}\,[1 + (\omega/\nu)^2\,(1 - \Omega/\Omega_{res})^2]^{-1}, \tag{11.9}$$

where Γ_{max} is given by Eq. (11.6).

[1] O. V. Konstantinov and V. I. Perel', JETP **38**, 161 (1960), Soviet Phys. JETP **11**, 117 (1960).

[2] S. J. Buchsbaum and J. K. Galt, Phys. Fluids **4**, 1514 (1961).

[3] M. Ya. Azbel', JETP **39**, 400 (1960), Soviet Phys. JETP **12**, 283 (1961).

[4] E. A. Kaner, JETP **44**, 1036 (1963), Soviet Phys. **17**, 700 (1963).

[5] V. L. Ginzburg, Rasprostranenie elektromagnitnykh voln v plazme (Propagation of Electromagnetic Waves in a Plasma) Fizmatgiz, 1960, Ch. 3.

[6] N. E. Aubrey and R. G. Chambers, Phys. Chem. Solids **3**, 128 (1957).

[7] A. B. Pippard, Phil. Mag. **2**, 1147 (1957).

[8] Kaner, Peschanskiĭ, and Privorotskiĭ, JETP **40**, 214 (1961), Soviet Phys. JETP **13**, 147 (1961).

[9] Bass, Blank, and Kaganov, JETP **45**, 1081 (1963), Soviet Phys. JETP, in press.

[10] A. Libchaber and R. Veilex, Phys. Rev. **127**, 774 (1962).

[11] M. Ya. Azbel' and É. A. Kaner, JETP **32**, 896 (1957), Soviet Phys. JETP **5**, 730 (1957).

[12] V. L. Gurevich, JETP **37**, 71, 1681 (1959), Soviet Phys. JETP **10**, 51, 1190 (1960).

[13] L. D. Landau and E. M. Lifshitz, Statisticheskaya fizika (Statistical Physics) Gostekhizdat, 1951, p. 55.

[14] Akhiezer, Kaganov, and Lyubarskiĭ, JETP **32**, 837 (1957), Soviet Phys. JETP **5**, 685 (1957).

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