EMISSION OF RADIO WAVES UPON MODULATION OF AN INTENSE BEAM OF LIGHT IN A MEDIUM

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The emission of radio waves from an intense modulated beam of light passing through a medium is considered. The emission is caused by the change of the mean nonlinear polarization of the medium when the beam intensity or polarization is changed. The effect is estimated for isotropic media to which an external field is applied and for anisotropic media with ordered atomic fields.

INTRODUCTION

THE emission of radio waves from an intense modulated light or radio beam passing through a medium can be associated with different nonlinear processes. The emission created by the rapid transfer of a focus of striction polarization of the medium by the inhomogeneous field of a modulated beam has been considered previously.[1] In the present article are considered the effects associated with the nonlinear dependence of the polarization on the field.

We shall be interested in cases for which, after averaging over the carrier frequency, an average polarization of the medium remains because of nonlinearity. When the beam is modulated this polarization changes and causes the emission of radio waves.

We shall illustrate the difference between radiation arising from striction polarization and from the effect of the nonlinear polarizability using as an example an isotropic anharmonic oscillator with a cubic nonlinear term. The equation for the displacement **r** which determines the polarization in the combined field of the beam and an external constant field, viz:*

$$\ddot{\mathbf{r}} + \omega_0^2\mathbf{r} + \gamma\dot{\mathbf{r}} + \frac{\alpha}{m}\mathbf{r}^3 = \frac{e}{m}\mathbf{E}(\mathbf{r}, t) + \frac{e}{mc}[\dot{\mathbf{r}}, \mathbf{H}] + \frac{e}{m}\mathbf{E}_0$$

is greatly simplified by averaging over the high carrier frequency of the field. In fact, the mean values of the derivatives with respect to time vanish, and on the right-hand side of the equation we have

$$\frac{e}{m}\left\{\mathbf{E}(0, t) + (\mathbf{r}\nabla)\mathbf{E} + \frac{1}{c}[\dot{\mathbf{r}}\mathbf{H}]\right\}_{\text{av}} = \frac{e}{m}\nabla(\mathbf{r}\mathbf{E})_{\text{av}}$$

*$[\mathbf{r}, \mathbf{H}] = \mathbf{r} \times \mathbf{H}$.

(using the well known relation $(\mathbf{r}\cdot\nabla)\mathbf{E} = \nabla(\mathbf{r}\cdot\mathbf{E}) - \mathbf{r} \times \text{curl}\,\mathbf{E}$, putting $\text{curl}\,\mathbf{E} = -c^{-1}\dot{\mathbf{H}}$, and combining the two components in the derivative $\partial[\mathbf{r} \times \mathbf{H}]/\partial t$, which vanishes after averaging). Therefore,

$$\omega_0^2\mathbf{r}_{\text{av}} = -\frac{\alpha}{m}(\mathbf{r}^3)_{\text{av}} + \frac{e}{m}\nabla(\mathbf{r}\mathbf{E})_{\text{av}} + \frac{e}{m}\mathbf{E}_0.$$

The radiation from the striction polarization is associated with the last term and occurs even in media with linear polarization ($\alpha \to 0$) and also in nonlinear isotropic media without an external constant field (in this case $(\mathbf{r}^3)_{\text{av}} \to 0$). On integrating over the entire volume of the beam field, the total dipole moment of these radiators vanishes; this demonstrates the multipole character of the striction radiation.

In contrast, the component with the nonlinear term gives the total dipole moment of the radiators, which shows the presence of radiation also in a range of wavelengths exceeding by far the transverse dimensions of the modulated beam. We shall consider this form of radiation in more detail.

1. THE MEAN NONLINEAR POLARIZATION OF A MEDIUM IN A BEAM

Different media display nonlinear properties differently. The general form of the variation of polarization is

$$\mathbf{P} \sim \varkappa_0\{1 + \xi_1 E/E_a + \xi_2 (E/E_a)^2 + \ldots\}\,\mathbf{E},$$

where E_a is a quantity of the order of atomic field strengths, $E_a \sim e/a^2 \sim 10^6$ cgs esu, and ξ are coefficients. For isotropic coupling $\xi_1 = 0$.

In what follows we shall principally consider isotropic media with $\kappa = \kappa_0\{1 + \xi(E/E_a)^2\}$, since they are encountered most frequently on a large scale (gases, liquids, etc.). Certain forms of anisotropy (uniaxial) can be included in this development by supposing that the ordered atomic field acts as an external field (the nonlinear quasistatic polarization in anisotropic media in a beam was recently considered in a paper by Bass et al[3]).

We assume that an electric field $\mathbf{E} = \mathbf{E}_0 + \mathbf{E}_1 \times \sin \omega t$ is applied to the isotropic medium, where $\mathbf{E}_0$ is a constant field either external or ordered internal, and $\mathbf{E}_1$ is the field of the beam, the amplitude of which will be modulated slowly compared with the carrier frequencies. Substituting $\mathbf{E}(t)$ into the expression for $\mathbf{P}(t)$, averaging with respect to ω, and omitting constant terms, we obtain

$$\mathbf{P}_{av}(t) = \varkappa_0 \xi E_a^{-2} \{\tfrac{1}{2}\, \mathbf{E}_0 \mathbf{E}_{10}^2(t) + \mathbf{E}_{10}(t)(\mathbf{E}_0 \mathbf{E}_{10}(t))\},$$

where the amplitude of $\mathbf{E}_{10}(t)$ can change both in magnitude and in polarization.

From the average of the anharmonic equation $\omega^2(\mathbf{r})_{av} = -\alpha m^{-1}(\mathbf{r}_0^3)_{av}$ the dispersion of the mean polarizability of the oscillator can be obtained. Substituting in the right-hand side of the equation the solution for the forced linear oscillations, and averaging, we obtain, for not very dense media,

$$\mathbf{P}_{av} = e\mathbf{r}_{av} = -\frac{\alpha}{m}\left(\frac{e}{m}\right)^3 \frac{{}^1\!/_2 E_{10}^2 \mathbf{E}_0 + \mathbf{E}_{10}(\mathbf{E}_{10}\mathbf{E}_0)}{\omega_0^4\{(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2\}}$$

(for dense media two multipliers appear, distinguishing the effective fields from the external fields).

The formula given shows the growth of the effect as the frequency approaches resonance. We also note that the mean polarization cannot coincide in direction with either $\mathbf{E}_0$ or $\mathbf{E}_{10}$.

For media containing dipole molecules, when an external electric field $\mathbf{E}_0$ is present, the additional change of the dipole moment in the beam is $\dot{\mathbf{P}}(t) = \dot{\kappa}(t)\mathbf{E}_0$, where the change of polarizability $\dot{\kappa}(t)$ is associated with the rapid bulk heating of the medium in the intense beam: $\dot{\kappa}(t) = \kappa \dot{T}(t)\dot{T}^{-1} \sim E_{10}^2$. Such media under an electric field can also show anisotropy associated with the orientation of the dipole molecules.

In anisotropic media even when there is no external field there is a quadratic variation of P_{av} on the amplitude of the wave field $\mathbf{E}_{10}$ (see, for example, [2,3]).

2. THE EMISSION OF RADIO WAVES FROM A MODULATED BEAM

We assume further that the change of $P_{av}(t)$ during modulation of the amplitude $\mathbf{E}_{10}$ is given, and from it we calculate the radio emission field, assuming the medium outside the beam is linear for the emitted radio waves (this assumption is valid because of the small intensity of the radio wave emission) and is isotropic (which is justified even when the beam passes through an anisotropic specimen, if its transverse dimensions are small compared with the radio wavelength at the external receiver).

The emission field from the system of dipoles in the medium is

$$\mathbf{E}(t) = \frac{1}{c^2 R_0}\left[\left[\int d\ddot{\mathbf{P}}\left(t - \frac{R}{c'}\right)\mathbf{n}\right]\right],$$

$$\mathbf{H}(t) = \frac{\sqrt{\varepsilon}}{c^2 R_0}\left[\int d\ddot{\mathbf{P}}\left(t - \frac{R}{c}\right)\mathbf{n}\right],$$

where c' is the propagation velocity of the radio waves. We introduce the dipole moment of the unit length of the beam $\mathbf{P}_1(t, z) = \mathbf{P}_{av}(t, z)S(z)$, where $S(z)$ is the section of the beam, the transverse dimensions of which will be taken as small compared with the wavelength of the waves received. Then one can limit oneself to the dipole radiation from the linear radiators $d\mathbf{P} = \mathbf{P}_1(t, z)dz$, where the z axis is chosen along the radiation beam.

If we neglect the change of spectrum of $P_\Omega(z)$ along the beam due to nonlinearity and dispersion of the absorption (the total absorption of light is easily allowed for, since $P_{av} \sim E_{10}^2(z) \sim E_{10}^2(0)e^{-kz}$, where k is the coefficient of absorption and scattering for light), then it is possible to separate out the phase multiplier of the lag between occurrence and arrival of the elementary waves from the different elements of the beam, and to integrate along the z axis. We then obtain

$$\mathbf{E}_\Omega = \frac{\exp(i\Omega R_0/c')}{c^2 R_0}\,\frac{\exp\{(i\chi - k)L\}}{i\chi - k}\,[[\ddot{\mathbf{P}}_{10\Omega}\mathbf{n}]\,\mathbf{n}],$$

where $\chi = \Omega(v_{gr}^{-1} - c'^{-1}\cos\theta)$, v_{gr} is the group velocity of transport of a modulation pulse along a beam, and

$$\ddot{\mathbf{P}}_{10\Omega} = \frac{1}{2\pi}\int_0^\infty \ddot{\mathbf{P}}(t')\, e^{i\Omega t'} dt'$$

is the Fourier component with respect to reduced time calculated from the start of the arrival of modulation, L is the length of the beam, and Ω is the frequency of the waves received.

Actually the spectral intensity of the radiation is

$$\mathscr{E}_\Omega = \sqrt{\varepsilon}\, c\, |E_\Omega|^2 = \frac{\sqrt{\varepsilon}}{c^3}\left(\frac{L}{R_0}\right)^2 \{[\chi L]^2 + [kL]^2\}^{-1}$$
$$\times \{(e^{kL} - 1)^2 + 4e^{kL}\sin^2[\tfrac{1}{2}\chi L]\}\, |\ddot{\mathbf{P}}_{10\Omega}|^2 \sin^2\varphi,$$

where θ is the angle between the direction of the receiver and the beam axis, φ is the angle between the direction of the dipole moment and the beam direction.

For small absorption $kL \to 0$ the spectral density of energy is

$$\mathscr{E}_\Omega = \frac{4\sqrt{\varepsilon}}{c^3}\left(\frac{L}{R_0}\right)^2 \frac{\sin^2\{{}^1/_2\,\chi L\}}{\{{}^1/_2\,\chi L\}^2}\,|\,\ddot{\mathbf{P}}_{10\Omega}\,|^2 \sin^2\varphi.$$

A sharp asymmetry is obvious from the formula when $v_{gr} \gtrsim c'$ under the angle $\theta = \arccos(c'/v_{gr})$; the maximum intensity of radiation is

$$(\mathscr{E}_\Omega)_{max} \approx \frac{\sqrt{\varepsilon}}{c^3}\left(\frac{L}{R}\right)^2 |\,\ddot{P}_{10\Omega}\,|^2 \sin^2\varphi$$

in an angular spread $\Delta\theta \approx \pi c'/\Omega L \sin\theta$.

As an example we evaluate the intensity of the radiation for a single pulse of deep modulation. Since $\ddot{\mathbf{P}}_{10\Omega} = i\Omega\dot{\mathbf{P}}_{10\Omega}$ and for a sharp modulation front $\dot{\mathbf{P}}_{10\Omega} = \mathbf{P}_{10\,max}/2\pi$, in this case $\ddot{\mathbf{P}}_{10\Omega} = i\Omega\mathbf{P}_{10\,max}/2\pi$, where $\mathbf{P}_{10}$ is the amplitude of the emission of the running mean of the dipole moment ($P_{10} \sim \xi k E_a^{-2} E_0 E_{10}^2 s$). Therefore

$$(\mathscr{E}_\Omega)_{max} = \frac{\sqrt{\varepsilon}\,\Omega^2}{\pi^2 c^3}\left(\frac{L}{R}\right)^2 \left(\frac{\xi\varkappa_0}{E_a^2} E_0 E_{10}^2 s\right)^2,$$

and the power in the interval $\Delta\Omega$ in the time $\tau \sim 1/\Delta\Omega$ amounts to $\Delta W_\Omega \sim \mathscr{E}_\Omega(\Delta\Omega)^2$.

We estimate the order of magnitude of the effect. If the constant electric field is provided by atomic systems (an ordered interatomic field—one of the possible causes of natural anisotropy), then $E_0 \sim E_a \sim 10^6$ cgs esu, and for $L/R \sim 0.1$, $\Omega \sim 2 \times 10^{11}\ \text{sec}^{-1}$, $\Delta\Omega \sim 0.1\,\Omega$, $E_{10} \sim 10^{-3} E_a$, $s \sim 1\ \text{cm}^2$, $\kappa \sim 0.1$ we obtain $\Delta W_\Omega \sim 1$W. In the case of an external electric field $E_0 \sim 10^5$ V/cm applied to an isotropic dielectric, we obtain for the same values $\Delta W_\omega = 1\,\mu$W. Although for air the effect in the electric field of the earth is small, the study of the radio emission accompanying a powerful flash or beam modulation presents interest.

It should be noted that in the case of sinusoidal modulation of the beam amplitude the band of radiated waves narrows sharply, which simplifies the conditions at the radio receiver.

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