PHOTOPRODUCTION OF π MESONS ON NUCLEONS AND FERMION REGGE POLES

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The asymptotic behavior of π-meson photoproduction on nucleons at high energies and π-meson production angles $\sim 180°$ is investigated under the assumption of the existence of moving poles in the partial amplitudes. Asymptotic values of the differential cross section and polarization of recoil nucleons are derived. Assuming unitarity in the u-channel, it is shown that in the high-energy limit and at scattering angles close to 180° relation (1) holds for the differential cross sections.

1. INTRODUCTION

The asymptotic behavior of scattering at high energies can be related to the mass spectrum of particles and resonances [1,2] by making use of the hypothesis of the moving poles of the scattering amplitude as functions of the orbital momentum. In accordance with this hypothesis, the asymptotic behavior of the backward scattering (scattering angle $\sim 180°$) at high energies is determined in processes of the type $\gamma + N \rightarrow \gamma + N$, $\gamma + N \rightarrow N + \pi$, $\pi + N \rightarrow \pi + N$ and the like by the fermion poles, that is, poles whose trajectories describe different fermion families. For example the nucleon and the resonance in the πN system with quantum numbers $T = 1/2$, $F_{5/2}$ belong to one fermion family—they lie on the same trajectory [2].

The question of fermion poles was considered in many papers [2-5]. Frautschi, Gell-Mann, and Zachariasen discussed the possibility of experimentally verifying the assumption that the nucleon is a fermion Regge pole. Gribov [4], assuming that the invariant amplitudes satisfy dispersion relations in momentum transfer, and using purely kinematic considerations, has shown that the poles of scattering amplitudes with different parity coincide when the square of the energy in the c.m.s. of the crossed u-channel tends to zero, and are complex conjugate when $u < 0$.

Such a character of fermion-pole trajectories leads to oscillatory behavior of the scattering amplitudes at high energies, but these oscillations manifest themselves neither in the cross section nor in the polarization. Only the correlation quantities oscillate. For example, in meson-nucleon scattering, the oscillating quantity is the polarization of the recoil nucleons, if the initial nucleon is polarized.

In the present paper we consider the asymptotic behavior of the photoproduction of pions on nucleons at angles $\sim 180°$. This process is related to πN scattering through unitarity. In turn, unitarity in the same u-channel relates the photon-nucleon scattering amplitudes to the amplitudes of photoproduction of pions on nucleons. Simultaneous consideration of the processes $\gamma + N \rightarrow \gamma + N$, $\gamma + N \rightarrow N + \pi$, $\pi + N \rightarrow \pi + N$, which are related by unitarity, together with the assumption that the asymptotic behaviors of these processes at high energies and at scattering angles close to 180° are determined by the fermion pole, enables us to conclude that in the limit under consideration the differential cross sections averaged over the polarizations are connected by the relation

$$\left(\frac{d\sigma}{d\Omega_{\gamma\pi}}\right)^2 = \frac{d\sigma}{d\Omega_{\gamma\gamma}}\,\frac{d\sigma}{d\Omega_{\pi\pi}}. \qquad (1)$$

Similar relations hold also between the cross section of different processes in forward scattering [6], but in these relations, at best, one cross section is not observable. On the other hand relation (1) relates asymptotic cross sections of observable processes.

2. INVARIANT PHOTOPRODUCTION AMPLITUDE

Let p_1 and p_2 be the 4-momenta of the nucleons in the initial and final states, and let k and q be the corresponding 4-momenta of the photon and the meson. Then the invariants of the process assume the form

$$s = -(p_1 + k)^2, \quad u = -(p_1 - q)^2, \quad t = -(p_1 - p_2)^2.$$

As shown in [7], the photoproduction amplitude can be written in invariant fashion as a sum of four tensor quantities, multiplied by suitable scalar functions of the invariants (1). The latter, however, have different asymptotic behaviors, so that it is highly inconvenient to operate with them. We therefore write the amplitude in a somewhat modified form:

$$T = \sum_{i=1}^{4} M_i A_i, \quad M_1 = i\gamma_5 \hat{\varepsilon}\hat{k},$$
$$M_2 = -i\gamma_5 \left(p_2 \cdot \varepsilon + \frac{m^2 - u}{t-1} q \cdot \varepsilon\right),$$
$$M_3 = -\frac{\gamma_5}{2}\left(\hat{\varepsilon} + \frac{2}{1-t}\hat{k} q \cdot \varepsilon\right).$$
$$M_4 = -\gamma_5 [\hat{\varepsilon}(m^2 - u) + \hat{k} 2 p_2 \cdot \varepsilon], \quad \hat{a} = \gamma_i a_i, \tag{2}$$

where ϵ —photon polarization vector, m —nucleon mass, and the meson mass is set equal to unity. We have left out the Dirac spinors from (2). A_i are scalar functions of the invariants, connected with the invariant amplitudes A, B, C, and D introduced by Chew and Frautschi [3] in the following manner:

$$A_1 = A - 2mD, \quad A_2 = (1-t)B,$$
$$A_3 = (1-t)(C+D), \quad A_4 = D. \tag{3}$$

All tensors M_i in (2), and consequently the functions A_i in (3), are of the same order as $s \to \infty$ ($t \to -\infty$).

3. AMPLITUDE IN THE u-CHANNEL

To find the asymptotic value of the scattering amplitude in the s channel ($s > 0$, $u < 0$) for large s and finite u, using the hypothesis that there exist principal Regge poles, it is necessary first to change over to the c.m.s. of the u channel ($u > 0$, $s < 0$, $\mathbf{p}_1 = -\mathbf{q}$, $\mathbf{p}_2 = -\mathbf{k}$). The amplitude thus obtained must then be expanded in partial waves f_l corresponding to transitions between states with definite quantum numbers, in our case parity. Further, in analogy with the procedure described in Gribov's papers [8], it is necessary to introduce the functions f_l^+ and f_l^-, which are analytic in l and which coincide with the physical partial amplitudes f_l for even and odd l, respectively. The upper indices, plus or minus, determine the so-called signature [2], which plays the role of a new quantum number. Following this, going from summation over l to integration and assuming that the principal (most remote) singularities of $f_l^{\pm}$ in l are poles, we can represent this integral in the form of a sum of residues at these poles (Regge poles). For each value of the signature there are two sets of poles with positive and negative parity. Each of these sets describes two types of transitions—from a state with total spin ½ into a state with total spin ½ and 3/2. When the arguments $z \simeq -s/2qk$ of the Legendre polynomials in the residues approach $-\infty$ (see, for example, [2]), each term of the sum over the residues acquires a factor z^{l_n}, and consequently the value of the sum will be determined by the term with the largest value of l_n —the principal Regge pole. Continuing analytically the resultant expression into the region $u < 0$ we obtain the required asymptotic value.

The expansion in partial waves is most conveniently carried out for amplitudes with definite helicity $\langle \lambda_n | T | \lambda_n', \lambda_\gamma \rangle$ [9]. In this connection, we introduce in the u-channel the following four helicity amplitudes, which determine the photoproduction process:

$$\psi_1 = \left\langle \frac{1}{2} \middle| T \middle| \frac{1}{2}, 1 \right\rangle, \quad \psi_2 = \left\langle -\frac{1}{2} \middle| T \middle| \frac{1}{2}, 1 \right\rangle,$$
$$\psi_3 = \left\langle \frac{1}{2} \middle| T \middle| -\frac{1}{2}, 1 \right\rangle, \quad \psi_4 = \left\langle -\frac{1}{2} \middle| T \middle| -\frac{1}{2}, 1 \right\rangle. \tag{4}$$

Expansion of these amplitudes in partial waves has the form (see [9])

$$\langle \lambda_n | T | \lambda_n', \lambda_\gamma \rangle = \sum_{l=0}^{\infty} (l+1) f_{\lambda'\lambda}^l \, d_{\lambda'\lambda}(\vartheta),$$
$$\lambda = \lambda_n, \; \lambda' = \lambda_n' - \lambda_\gamma, \quad l = j - \tfrac{1}{2}; \tag{5}$$

where ϑ —scattering angle in the u-channel. The functions $d_{\lambda'\lambda}(\vartheta)$ are defined and their properties described in [9].

In connection with our purpose, we must set up such combinations of the amplitudes (4), which contain only partial waves corresponding to transitions with definite parity. With the aid of the formulas of [9] it is easy to check that the partial function of the form $f_i^l \mp f_k^l$ corresponds 1) to transitions between states with parities $(-1)^l$ and $-(-1)^l$ respectively.

Starting from the explicit form of $d_{\lambda'\lambda}(\vartheta)$, we can choose the following combinations:

$$\tau_1^{\pm} = \frac{\pm 1}{\sqrt{2}}\left(\frac{\psi_2}{\cos(\vartheta/2)} \mp \frac{\psi_1}{\sin(\vartheta/2)}\right),$$
$$\tau_2^{\pm} = \frac{\mp 1}{\sqrt{2}}\left(\frac{\psi_3}{\cos(\vartheta/2)} \pm \frac{\psi_4}{\sin(\vartheta/2)}\right). \tag{6}$$

Substituting (5) in (6) and using the explicit form of $d_{\lambda'\lambda}(\vartheta)$ we obtain

1) It can also be shown that $(f_2^l \mp f_1^l)$ corresponds to a transition between states with total spin ½, while $(f_3^l \mp f_4^l)$ corresponds to a transition from a state with spin ½ to a state with spin 3/2.

$$\tau_1^{\pm} = \frac{\pm 1}{\sqrt{2}} \sum_{l=0}^{\infty} \{(f_2^l \mp f_1^l) P'_{l+1} - (f_2^l \pm f_1^l) P'_l\},$$

$$\tau_2^{\pm} = \frac{\pm 1}{\sqrt{2}} \sum_{l=0}^{\infty} \left\{(f_3^l \mp f_4^l) \sqrt{\frac{l}{l+2}} P'_{l+1} - (f_3^l \pm f_4^l) \sqrt{\frac{l}{l+2}} P'_l\right\},$$

$$P'_l = \frac{dP_l(z)}{dz}, \quad z = \cos\vartheta. \tag{7}$$

4. ASYMPTOTIC VALUE OF THE AMPLITUDE

It is easy to obtain by direct calculation the relations between the amplitudes A_i of (3) and τ_i of (6) in the asymptotic region of large z:

$$\tau_1^{\pm} = -M_{\pm} \frac{1}{2qk} \{(m \pm \sqrt{u}) A_2 + A_3\},$$

$$\tau_2^{\pm} = M_{\pm} \left\{\mp \frac{2}{q} [A_1 + (m \mp \sqrt{u}) A_4] \mp \frac{m \mp \sqrt{u}}{\sqrt{u}} A_3\right\} - \tau_1^{\pm},$$

$$M_{\pm} = \frac{1}{16\pi} \frac{u - m^2}{2u} [(m \mp \sqrt{u})^2 - 1] \left\{\frac{(m \pm \sqrt{u})^2 - 1}{u}\right\}^{1/2},$$

$$k = \frac{u - m^2}{2\sqrt{u}},$$

$$q = \frac{1}{2\sqrt{u}} \{[(\sqrt{u} + m)^2 - 1] [(\sqrt{u} - m)^2 - 1]\}^{1/2}. \tag{8}$$

Since the A_i have no singularity of the form $\sqrt{u}$ [10], the expressions for τ_i^+ and τ_i^-, as can be seen from (8), differ from each other in the sign of $\sqrt{u}$. It follows therefore, in particular, that $\tau_1^+ + \tau_1^-$ remains finite as $u \to 0$, whereas $\tau_1^+ - \tau_1^-$ vanishes or becomes infinite. This is possible only if the singularities of the partial waves for τ_1^+ and τ_1^- coincide as $u \to 0$.

Going over in (7) from a sum over l to an integral and using formulas (8), we will determine in accordance with the program described above the contributions from the principal Regge poles to the amplitudes A_i. Denoting the residues of the functions $f_2^l - f_1^l$ and $f_3^l - f_4^l$ by r_1 and r_2, respectively, we obtain the following expression for the contribution from the pole with parity $(-1)^l$

$$2A_2 = \alpha_1 - m\alpha_2 + \beta_2, \qquad A_3 = -m\alpha_1 - \beta_1,$$

$$A_2 = \alpha_1, \qquad 2A_4 = \alpha_2, \qquad \beta_i = \sqrt{u}\, \alpha_i,$$

$$\alpha_1 = 2N^{\pm} r_1^{\pm} \frac{s^l \pm (-s)^l}{\sin \pi l},$$

$$\frac{\alpha_2}{\alpha_1} = -\frac{1}{2k} \left(\frac{m}{\sqrt{u}} + \sqrt{\frac{l}{l+2}} \frac{r_2^{\pm}}{r_1^{\pm}}\right),$$

$$N^{\pm} = \Gamma\left(l + \frac{3}{2}\right) \{\Gamma(l+1)\, 2\sqrt{2\pi u}\, M (qk)^{l-1}\}^{-1}. \tag{9}$$

The plus and minus signs in (9) establish a definite signature. The contribution from the residue with opposite parity, in accordance with the foregoing, is obtained from (9) by reversing the sign of $\sqrt{u}$.

We shall now show that the contributions from the principal Regge poles can be written in factorized form, which in analogy with pole diagrams we choose in the form

$$T = R\Gamma \cdot \varepsilon\, (i\hat{f} + \beta)\, \gamma_5, \qquad \Gamma_{\mu} = \gamma_{\mu} + \frac{i\lambda}{4m} (\gamma_{\mu}\hat{k} - \hat{k}\gamma_{\mu}),$$
$$f = p_1 - q. \tag{10}$$

Expanding (10) in the tensors (2), we obtain expressions for A_i in terms of the functions R, λ, and β. Equating these expressions to the functions obtained in (9), we arrive at four equations for the three unknowns R, λ, and β. It is easy to verify that one of these equations is the consequence of the other three. As a result we obtain

$$R = -r_1 N \frac{s^l \pm (-s)^l}{\sin \pi l},$$

$$\lambda = -\frac{m}{k}\left[\frac{m}{\sqrt{u}} + \sqrt{\frac{l}{l+2}} \frac{r_4}{r_1}\right), \quad \beta = -\sqrt{u}. \tag{11}$$

Continuing expressions (9) and (11) into the region $u < 0$, we can show, by following verbatim Gribov's arguments [4], that the residues of the poles of different parity are complex conjugate. The expression for the amplitude will therefore consist of the following combinations $(i = 1, 2)$:

$$\alpha_i = R_i^{\pm} \frac{s^l \pm (-s)^l}{\sin \pi l} + (R_i^{\pm})^* \frac{s^{l^*} \pm (-s)^{l^*}}{\sin \pi l^*}, \tag{12}$$

$$\beta_i = \sqrt{u} \left(R_i^{\pm} \frac{s^l \pm (-s)^l}{\sin \pi l} - (R_i^{\pm})^* \frac{s^{l^*} \pm (-s)^{l^*}}{\sin \pi l^*}\right),$$

$$R_1^{\pm} = 2 (r_1 N)^{\pm}, \quad R_2^{\pm} = -2 (r_1 N)^{\pm} \lambda^{\pm}/2m. \tag{13}$$

Introducing the moduli and phases of the expressions (13)

$$2r_1 N = {}^1\!/_2 \rho_1 e^{i\varphi_1}, \quad \lambda/2m = -\rho_2 e^{i\varphi_2}, \tag{14}$$

we obtain for the imaginary and real parts of (12)

$$\operatorname{Im} \alpha_i = \pm P_i^{\mp}(u)\, s^l \cos(l''\xi + \psi_i),$$

$$\operatorname{Im} \beta_i = \pm \sqrt{-u}\; P_i^{\pm}(u)\, s^l \sin(l''\xi + \psi_i),$$

$$\operatorname{Re} \alpha_i = \alpha_{\pm} P_i^{\mp}(u)\, s^l \cos(l''\xi + \psi_i \mp \beta),$$

$$\operatorname{Re} \beta_i = \alpha_{\pm} \sqrt{-u} P_i^{\mp}(u)\, s^l \sin(l''\xi + \psi_i \mp \beta),$$

$$P_1^{\pm} = \rho_1^{\pm}, \quad P_2^{\pm} = \rho_1^{\pm}\rho_2^{\pm}; \quad \psi_1 = \varphi_1, \quad \psi_2 = \varphi_1 + \varphi_2;$$

$$\alpha_{\pm}^2 = \frac{\operatorname{ch} \pi l'' \mp \cos \pi l'}{\operatorname{ch} \pi l'' \pm \cos \pi l'}, \quad \operatorname{tg} \beta = -\frac{\operatorname{sh} \pi l''}{\sin \pi l'}, \quad \xi = \ln s, \tag{15*}$$

l' and l'' are the real and imaginary parts of the function $l(u)$, which determines the position of the pole.

5. ASYMPTOTIC VALUES OF THE CROSS SECTION AND POLARIZATION

A detailed summary of the expressions for the cross section and all possible polarization effects is contained in the paper of Moravcsik [11]. By determining the connection between the functions

*ch = cosh, sh = sinh, tg = tan.

A_i in (3) and the Moravcsik amplitudes A, B, C, and D, we obtain the following expression for the latter in terms of the α_i and β_i of (12):

$$A = \sqrt{-u}\,(\beta_2 + \alpha_1 - m\alpha_2),$$

$$B \sin\theta = \sqrt{-u}\,(\beta_2 - \alpha_1 + m\alpha_2),$$

$$C = m\beta_2 - \beta_1 - u\alpha_2,\quad D \sin\theta = m\beta_2 - \beta_1 + u\alpha_2, \tag{16}$$

where θ —scattering angle in the s channel.

With the aid of (16) and (15) as well as of the table given by Moravcsik, it is possible to obtain the differential cross section and all the polarization effects of the photoproduction process. We present some of them:

$$I_0 = -u\,(\rho_1^{\mp})^2\,(1 + \alpha_{\pm}^2)\,\{[1 + (m^2 - u)\,(\rho_2^{\mp})^2 - 2m\rho_2^{\mp}\cos\varphi_2] - 2\sqrt{-u}\rho_2^{\mp}\sin\varphi_2\cos 2\Phi\}\,s^{2l'},$$

$$I_0 P_m = -2u\,(\rho_1^{\mp})^2\,\alpha_{\pm}\sin(\pm\beta)\,\{2\sqrt{-u}\rho_2^{\pm}\sin\varphi_2 - [1 + (m^2 + u)\,(\rho_2^{\mp})^2 - 2m\rho_2^{\mp}\cos\varphi_2]\cos 2\Phi\}\,s^{2l},$$

$$I_0 T_{mn} = 4u\,(\rho_1^{\mp})^2\,\{\cos\Psi_1 + \alpha_{\pm}^2\cos\Psi_2 - 2m\rho_2^{\mp} \times [\cos(\Psi_1 + \varphi_2) + \alpha_{\pm}^2\cos(\Psi_2 + \varphi_2)] + (\rho_2^{\mp})^2 \times [\cos(\Psi_1 + 2\varphi_2) + \alpha_{\pm}^2\cos(\Psi_2 + 2\varphi_2)]\}\sin 2\Phi,$$

$$\cos^2\Phi = \frac{(\varepsilon q)^2}{\sin^2\theta},\ \Psi_1 = 2\,(l''\xi + \varphi_1),\ \Psi_2 = 2\,(l''\xi + \varphi_1 \mp \beta). \tag{17}$$

The notation is from Moravcsik's paper. We see that in spite of the oscillatory character of the energy dependence of the amplitude, the differential cross section and some polarization effects are monotonic functions of the energy. However, as can be seen from the last formula, some of the polarization effects oscillate with the energy. An identical picture is obtained for other processes in [4,12]. We can therefore assume that such a behavior of the cross sections and polarizations is a characteristic feature of Regge-ization.

6. CONNECTION BETWEEN CROSS SECTIONS

Relation (1) between the differential cross sections can be readily obtained by using the unitarity condition for the partial amplitudes of the processes $\gamma + N \to \gamma + N$, $\gamma + N \to N + \pi$, $\pi + N \to \pi + N$ in the crossed u-channel. Let us consider to this end the partial helicity amplitudes for photon-nucleon scattering $g^j_{\lambda\lambda'}$, for photoproduction of pions on a nucleon $f^j_{\lambda\lambda'}$, and for pion-nucleon scattering $h^j_{\lambda\lambda'}$. The helicity partial amplitudes are related to the partial amplitudes with definite parity. For photoproduction this relation is given in (6) and (7); for γN scattering we have the following relations:

$$g^j_{1/2,\,1/2} = A^j_+ + A^j_-,\quad g^j_{1/2,\,-3/2} = B^j_+ + B^j_-,\quad g^j_{-3/2,\,3/2} = C^j_+ + C^j_-,$$

$$g^j_{1/2,\,-1/2} = A^j_+ - A^j_-,\quad g^j_{-1/2,\,-3/2} = B^j_+ - B^j_-,\quad g^j_{-3/2,\,3/2} = C^j_+ - C^j_-. \tag{18}$$

Finally, for πN scattering

$$h_{1/2,\,1/2} = h^j_+ + h^j_-,\quad h_{-1/2,\,1/2} = h^j_+ - h^j_-, \tag{19}$$

where the quantities with index (+) denote partial amplitudes with parity $(-1)^{j+1/2}$, and the quantities with index (−) denote partial amplitudes with parity $(-1)^{j-1/2}$.

Assuming that the extreme right-hand singularity of all the amplitudes with definite parity is a pole, we obtain asymptotically at high energies ($u < 0$) the following differential cross section for γN scattering:

$$d\sigma/d\Omega = (1 + \alpha^2)\,[(\omega_1)^2 + 2\,(\omega_2)^2 + (\omega_3)^2]\,s^{2j'-1}, \tag{20}$$

where ω_1, ω_2, and ω_3 are the residues of the amplitudes A^j_+, B^j_+, and C^j_+.

Analogously we obtain the differential cross section of the πN scattering[4]:

$$d\sigma/d\Omega = (1 + \alpha^2)\,|\,\Delta\,|^2 s^{2j'-1}, \tag{21}$$

where Δ —residue of the amplitude h^j_+.

The photoproduction differential cross section is given in (17). It follows from unitarity that, first, the residues ω_1, ω_2, and ω_3 are connected by (see [12])

$$\omega_2^2 = \omega_1\omega_3, \tag{22}$$

and, second, that the following connection holds

$$r_1^2 = \Delta\omega_1,\quad r_2^2 = \Delta\omega_3. \tag{23}$$

From (22) and (23) in conjunction with (17), (14), (11), (20), and (21) we get (1).

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