COMPARISON OF ELASTIC πp AND pp SCATTERING ON THE BASIS OF A MODEL WITH THREE REGGE POLES

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An attempt is made to correlate the data on $\pi^{\pm}p$ scattering [1] with the pp and $\overline{p}p$ scattering data [2,3] by invoking the three pole model suggested by Rarita et al [5]. It is shown that the model in which $\pi^{\pm}p$ scattering is described by P, P′ and ρ poles and pp and $\overline{p}p$ scattering by P, P′ and ω poles leads to qualitative disagreement with the experiments, which apparently do not yield any shrinkage of the diffraction cone in the π^-p system for $3.4 \le s/M^2 \le 30$ and $|t| \le 0.4$ (BeV/c)^2. In order to interpret the results of Ting et al [1] on the basis of the Regge pole theory, some additional poles must be introduced for which the trajectories have a negative slope for $t < 0$.

BY comparing data on elastic π^-p scattering with $|t| \le 0.4$ and $3.6 \le s \le 30$ (t—square of 4-momentum transfer, s—square of energy in c.m.s., expressed in units of the square of the nucleon mass M^2), Ting et al [1] have concluded that there is no narrowing down of the diffraction cone. The same authors have shown that the invariant cross section of elastic scattering for $|t| \le 0.4$ is well approximated by the formula

$$\frac{d\sigma}{d(-t)} \Big/ \left(\frac{d\sigma}{d(-t)}\right)_{t=0} = \exp\,(tA_{\pi^-}(s)), \qquad (1)$$

where $A_{\pi^-}(s) \approx 8$ and changes little in the region $1 \le \ln s \le 3.5$.

We have made a similar analysis of the literature data [2,3] on pp scattering with $1.5 \le \ln(s/2) \le 3.5$. All these data are also well approximated by (1) when $|t| \le 0.4$. In Fig. 1 are compared the values of $A_{\pi^-}(s)$ given in [1] and the values of $A_p(s)$ [2,3]. It follows from Fig. 1 that the slopes of both curves indeed cannot be compatible with each other.

If we confine ourselves in the description of πp and pp scattering to the principal vacuum Regge pole only (P pole), then the elastic scattering cross section for small $|t|$ should be approximated by the formula

$$d\sigma/d(-t) = F_{\pi,p}(t)\exp(2t\alpha_P'\ln S) \approx \exp\{t\,[\gamma_{\pi,p} + 2\alpha_P'\ln S]\}, \qquad (2)$$

where F_π and F_p are the form factors of the πp and pp scatterings, respectively, and $\alpha_P' \approx 1$ is a universal constant, characterizing the trajectory

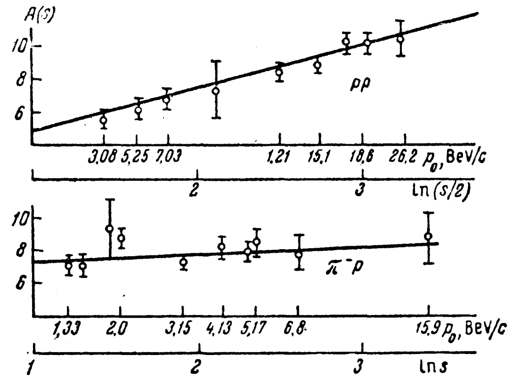

FIG. 1. Plot of the coefficient [see (1)] $A_{\pi^-}(s)$ (lower curve) and $A_p(s)$ (upper curve) against ln s (s is in BeV^2).

of the P pole. In this case the slope of the $A_p(s)$ curve is equal to $\partial A(s)/\partial \ln s = 2$, and the result of [1], from which it follows that $A_{\pi^-}(s) \approx$ const. contradicts the theory.

However, the description of scattering by means of a single pole is incorrect, for in this region of s the total cross sections of the $\pi^{\pm}p$ and pp scattering vary appreciably, and the differences in the cross sections of the particles and antiparticles still constitute a noticeable fraction of the total cross section. If we assume that the course of the cross sections is monotonic for E > 20—30 BeV, then it is necessary for the description of experiments with E < 20—30 BeV to use in addition to the P pole at least three other poles [4]: the second vacuum pole P, which determines the manner in which the quantities $\sigma_t(pp) + \sigma_t(p\overline{p})$ and $\sigma_t(\pi^+p) + \sigma_t(\pi^-p)$ approach their limiting values, and also the poles ω and ρ, which determine the decrease of the quantities[1)] $\sigma_t(\bar{p}p) - \sigma_t(pp)$ and $\sigma_t(\pi^- p) - \sigma_t(\pi^+ p)$, respectively.

This raises the question: can the data on π^-p scattering[1] be reconciled with the known data on pp scattering on the basis of a theory with three Regge poles? Leaving out the complications connected with the spin, we define the scattering amplitude T:

$$|T|^2 = d\sigma/d(-t), \qquad \mathrm{Im}\, T(0, s) = \sigma_t/4\sqrt{\pi}. \tag{3}$$

The amplitudes of $\pi^\pm p$, pp, and $\bar{p}p$ scattering can be represented in the form

$$T\binom{\pi^+p}{\pi^-p} = T_{\pi P} + T_{\pi P'} \pm T_{\pi\rho} + \Sigma^\pm_\pi,$$

$$T\binom{pp}{\bar{p}p} = T_{pP} + T_{pP'} \pm T_{p\omega} + \Sigma^\pm_p, \tag{4}$$

where T_{ik} corresponds to the contribution of the poles P, P′, ρ, and ω, while $\Sigma^\pm_{\pi,p}$ denotes the contribution of the minor poles. Then

$$T_{ik} = F_{ik}(t)\frac{1 \pm \exp(-i\pi\alpha_k)}{\sin \pi\alpha_k} z^{\alpha_k - 1}, \tag{5}$$

where the plus sign corresponds to the poles P and P′, and the minus sign to the poles ω and ρ; $F_{ik}(t)$—real function; $z = E/M = (S - N^2 - M'^2)/2M^2$ (E—total laboratory-system energy of the incoming particle, $M' = M$, M_π, respectively for the pp and πp system)[2)]. For $|t| \lesssim 0.4$ we can put

$$\alpha_k = \alpha_k(0) + \alpha'_k t. \tag{6}$$

Dardel et al[9] have established that the total cross sections of $\pi^\pm p$ scattering in the region E ~ 3—20 BeV are approximated by the formula

$$\sigma_t = \sigma_\infty + b^\pm E^{-\beta}, \tag{7}$$

with the values of σ_∞, $b^\pm$, and β given in Table 2 of[9]. Comparing (7) with (4) and neglecting the contribution of $\Sigma^\pm$, we obtain the following values for the πp scattering parameters at t = 0:

$\alpha_P(0)$	$\alpha_{P'}(0)$	$\alpha_\rho(0)$	$F_{\pi P}(0)$	$F_{\pi P'}(0)$	$F_{\pi\rho}(0)$
1	0.5	0.5	—2.96 mb	—3.25 mb	—0.5 mb

(8)

An analogous analysis of the total cross sections of pp and $\bar{p}p$ scattering, made by Rarita et al[5], gives the following parameters

$\alpha_P(0)$	$\alpha_{P'}(0)$	$\alpha_\omega(0)$	$F_{pP}(0)$	$F_{pP'}(0)$	$F_{p\omega}(0)$
1	0.5	0.5	—5.66 mb	—3.75 mb	—3.75 mb

(9)

We note the following two circumstances:

1. The close values of $\alpha_{P'}(0)$ in $\pi^\pm p$, pp, and $\bar{p}p$ systems are evidence that the three-pole model is internally consistent at t = 0.

2. A sensitive check on the applicability of the extrapolation formula (7), or the analogous formula for pp and $\bar{p}p$ scattering, in the energy region above the experimental boundary is the measurement of Re T (s, 0) at large s. The skimpy data available for this purpose apparently do not contradict (8) and (9).

Now, the free parameters left for the reconstruction of the experimental dependence of $d\sigma/d(-t)$ on t and s are the slopes of the trajectories α'_k and $F_{ik}(t)/F_{ik}(0)$. Since the shrinkage of the diffraction cone with increasing E is essentially determined by α'_k, these quantities are varied, with the exception of α'_P, which is assumed equal to unity[6]. We chose $F_{ik}(t)/F_{ik}(0)$ by means of the following procedure: $F_{ik}(t)/F_{ik}(0)$ was represented in the form $r_k(t)\exp(\gamma_i t)$, where $r_k(t)$ was determined from the condition[5,10]

$$r_k^2 \left|\frac{1 \pm \exp(-i\pi\alpha_k)}{\sin \pi\alpha_k}\right|^2 = 1 \tag{10}$$

for t < 0. The exponent $\gamma_{\pi,p}$ was assumed the same for the residues of all the poles respectively in the systems $\pi^\pm p$ and pp, $\bar{p}p$ and was chosen such that for a specified set of α_k the calculated value of $d\sigma/d(-t)$ coincided with the experimental one at an energy corresponding to the middle of the working interval of ln s.

Figure 2 shows plots of $\ln[(d\sigma/d(-t))/(d\sigma/d(-t))_{t=0}]$ against $|t|$ in the π^-p and the pp systems for different sets of α'_k. To each set of α'_k there corresponds a family of three curves representing the values ln s = 1, 2, 3. Thus, the slope of the curves of Fig. 2 determines A(s), and its increase as ln s varies from 1 to 3 corresponds to the shrinkage of the diffraction cone. Figure 2 shows schematically also the results of the experiment (in this case the values of A(s) are taken in accordance with Fig. 1). If we assume $\alpha'_{P'} = \alpha'_\omega = \alpha'_\rho = 1$[5] (Fig. 2, pp_2 and πp_2), we obtain approximately the same values of A(s) for π^-p and pp scattering, which agree with the case pp_1 and

[1)]The ω-pole makes no contribution to the πp amplitude, being forbidden by G parity ($\omega \to 3\pi$). The ρ-pole determines the small difference $\sigma_t(np) - \sigma_t(pp)$, and therefore gives only a small contribution to pp scattering.[6]

[2)]In the theory of Gribov[7] and Chew and Frautschi[8] $z = \cos\theta_t$, where θ_t is the angle of the reaction in the crossed channel:

$$\cos\theta_t = (4EM + t)\,[(t - 4M^2)(t - 4M'^2)]^{-1/2}.$$

For small t it is possible to split off in the expression for $(\cos\theta_t)^{\alpha_k - 1}$ the factor $[4M^2/((t - 4M^2)(t - 4M'^2))^{1/2}]^{\alpha_k - 1}$ by suitably redefining the function $F_{ik}(t)$. This is indeed done in (5). The condition for the applicability of the representation (5) consists in the requirement $|\cos\theta_t| \gg 1$. This condition can be regarded as satisfied when $|t| \lesssim 0.4$, $E_\pi \gtrsim 1.2$ BeV, and $E_p \gtrsim 3.5$ BeV.

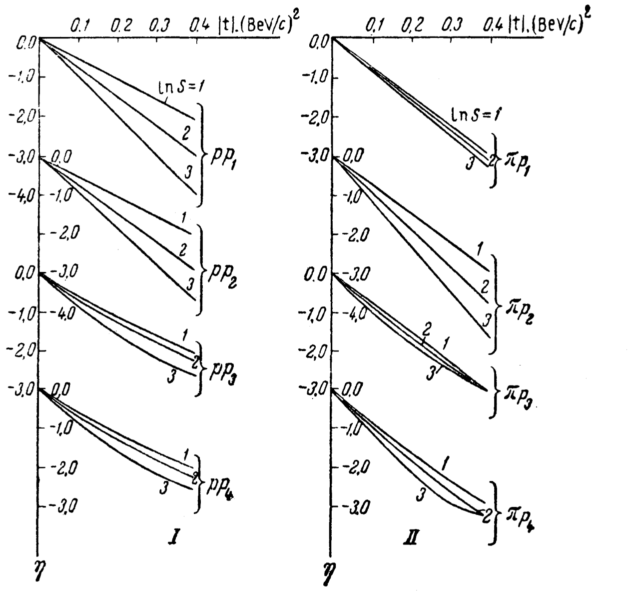

FIG. 2. Plot of $\eta \equiv \ln[(d\sigma/d(-t))/(d\sigma/d(-t))_{t=0}]$ against $|t|$ for three values of ln s, calculated under different assumptions concerning the values of α'_P, $\alpha'_{P'}$, α'_ρ, and α'_ω for elastic pp scattering (I) and π^-p scattering (II); the cases pp_1 and πp_1 – experiments; case $pp_2 - \alpha_P(t) = 1 + t$, $\alpha_{P'}(t) = 0.5 + t$, $\alpha_\omega(t) = 0.5 + t$; case $\pi p_2 - \alpha_P(t) = 1 + t$, $\alpha_{P'}(t) = 0.5 + t$, $\alpha_\rho(t) = 0.5 + t$; case $pp_3 - \alpha_P(t) = 1 + t$, $\alpha_{P'}(t) = 0.5 - t$, $\alpha_\omega(t) = 0.5 + t$; case $\pi p_3 - \alpha_P(t) = 1 + t$, $\alpha_{P'}(t) = 0.5 - t$, $\alpha_\rho(t) = 0.5 + 0.5t$; case $pp_4 - \alpha_P(t) = 1 + t$, $\alpha_{P'}(t) = 0.5 - t$, $\alpha_\omega(t) = 0.5 + 2t$; case $\pi p_4 - \alpha_P(t) = 1 + t$, $\alpha_{P'}(t) = 0.5 + t$, $\alpha_\rho(t) = 0.5 - 2t$.

contradict the case πp_1 of Fig. 2.

If we attempt to reconcile A(s) for π^-p scattering with experiment by putting $\alpha'_{P'} = -1$ and $\alpha'_\rho = +0.5$ (Fig. 2, πp_3) and retaining $\alpha'_\omega = +1$, a contradiction with experiment arises in the case of pp scattering (see Fig. 2, cases pp_3 and pp_1). This contradiction cannot be eliminated by strongly increasing the positive slope of the ω-trajectory ($\alpha'_\omega = +2$) (see Fig. 2, pp_4). It is possible to reduce somewhat the shrinkage of the cone in the π^-p system by making the slope of the ρ-trajectory highly negative, $\alpha'_\rho = -2$ with $\alpha'_{P'} = +1$ (see Fig. 2, πp_4). This, however, greatly disturbs the exponential dependence (1) for large s, and when $t < -0.25$ ($\alpha'_\rho = -2$) the model becomes altogether meaningless, since it leads to a power-law growth of $d\sigma/d(-t)$ as $s \to \infty$ (violation of unitarity [11]).

We have not yet used the available data on $d\sigma/d(-t)$ for $p\bar{p}$ (E = 3 BeV [12]) and π^+p (E = 3.15 BeV [1]) scattering. Rarita et al [5] have shown that the parameters (9) together with $\alpha'_P = \alpha'_{P'} = \alpha'_\omega = 1$ are in qualitative agreement with the fact that $p\bar{p}$ scattering gives a much narrower diffraction cone than pp scattering.

In the case of $\pi^\pm$p scattering we have from the data of Ting et al [1]

$$A_{\pi^-}(s) - A_{\pi^+}(s) \approx 0.5 \quad \text{for } \ln s = 2. \tag{11}$$

If we put $\alpha'_P = \alpha'_{P'} = 1$, then there follows from condition (11) the restriction $1 \geq \alpha'_\rho > -0.5$. But such a set of parameters leads to a shrinkage of the diffraction cone of π^-p scattering, almost equal to that for pp scattering. Thus, the aggregate of data on $\pi^\pm$p scattering from [1,9] cannot be reconciled with the known data for pp and $\bar{p}$p scattering within the framework of the three-pole model.

We doubt that the contradictions will be eliminated by suitable variation of the function $F_{\pi k}$ or by taking into account the spin dependence of the scattering. It also seems to us that the apparent lack of shrinkage of the diffraction cone in π^-p scattering cannot be reconciled with the shrinkage of the cone in pp scattering by taking into account the nonlinear terms in (6).

The absence of shrinkage of the diffraction cone for $\pi^\pm$p scattering in a limited region of s could be reconciled with the data on pp scattering by introducing additional poles. Our analysis shows, however, that it is necessary to make use for this purpose of lower-order poles, in which the trajectories have a negative slope in the region $t < 0$.[3)] The presence of such trajectories makes doubtful, in our view, the concept of Chew-Frautschi-Udgaonkar [14,4] concerning the identification of various trajectories that determine the asymptotic behavior of the amplitude in the s channel for $t < 0$ with the known resonances in the region $t > 0$ in the t-channel.

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