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RADIATIVE CAPTURE OF μ^- MESONS IN HYDROGEN

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Mesic-atom and mesic-molecule effects during radiative capture of μ^- mesons in hydrogen ($\mu^- + p \to n + \nu + \gamma$) are considered. Expressions for the spectrum and degree of photon circular polarization are obtained by taking into account all terms whose contribution to the amplitude of the $\mu^- + p \to n + \nu + \gamma$ process is of the order of m_μ/m and $(m_\mu/m)^2$ (m_μ and m are respectively the μ^--meson and nucleon masses).

Curves representing the spectrum and the degree of photon circular polarization for the $\mu^- + p \to n + \nu + \gamma$ process as a function of photon energy are presented for various values of the effective pseudoscalar interaction constant C_P. It is shown that the probability for radiative capture of μ^- mesons in hydrogen depends strongly on the ratio $\kappa = C_P/8C_A$ (C_A is the axial-vector weak interaction constant). Thus, for example, the probability for radiative capture of a μ^- meson from a hyperfine structure state in mesic hydrogen with a total spin $F = 0$ grows faster than κ^2 with increase of κ.

1. INTRODUCTION

In a preceding paper [1] devoted to radiative capture (RC) of a μ^- meson by a proton ($\mu^- + p \to n + \nu + \gamma$), we derived expressions for the spectrum and degree of circular polarization of the γ quanta. That paper did not consider the effects of the hyperfine structure and the mesic-molecule effects which play a rather important role in the capture of a μ^- meson in hydrogen [2-4]. Since the RC probability of μ^- mesons by nuclei with zero spin (as shown in [5]) is connected with the RC probability of a μ^- meson by a free proton, we can expect the previously obtained result [1] to serve as a model for the calculation of the corresponding quantities (circular polarization of the γ quanta and their spectrum) for RC of μ^- mesons by nuclei with zero spin.

The present paper is devoted to a calculation of the spectrum and of the degree of circular polarization of the γ quanta in RC of μ^- mesons in hydrogen, with account of the hyperfine structure of the mesic-hydrogen and mesic-molecule effects.

The RC by a proton of a μ^- meson from the mesic-atom state with $F = 0$ (F is the total spin of the proton and μ^- mesons) was considered by Dye et al [6], except that the diagrams b and f of Fig. 1 were not considered. As shown in [1], an account of diagrams b and f leads in the case of an effective pseudoscalar interaction to an increase in the probability of the RC of a μ^- meson by a proton by a factor of two, compared with the probability obtained without these diagrams. In addition, diagrams b—e of Fig. 1 were not taken into account in the amplitude of the radiative μ^- capture in [6].

In the present article, unlike in [6], the amplitude of the $\mu^- + p \to n + \nu + \gamma$ includes all terms whose contribution has an order of magnitude m_μ/m and $(m_\mu/m)^2$. In the analysis of the RC of the μ^- meson from the hyperfine structure states of the hydrogen mesic atom with $F = 0$ and $F = 1$ we used the density matrix formalism developed for mesic-atom systems in [7,8].

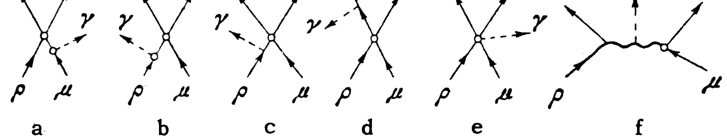

FIG. 1. Diagrams of the process $\mu^- + p \to n + \nu + \gamma$.

2. MATRIX ELEMENT. DENSITY MATRIX FOR RC FROM A DEFINITE MESIC-HYDROGEN HYPERFINE STRUCTURE STATE

Let us consider the diagrams that were taken into consideration in the calculation of the spectrum and of the circular polarization of the photons in RC of μ^- mesons in hydrogen (Fig. 1). Diagrams a and b correspond to bremsstrahlung of the μ^- meson and proton, while c and d correspond to radiation of the proton and neutron due to interaction with their anomalous magnetic moments μ_p and μ_n. It is assumed here that for diagrams c and d the nucleon form factors can be regarded with sufficient accuracy as constant in the momentum region under consideration.

Diagrams e and f are so the so-called "catastrophic" diagrams. They result from the momentum-transfer dependence of the vertex parts corresponding to the effective pseudoscalar and weak magnetic interactions. Diagrams a, b, and f of Fig. 1, which correspond to the effective pseudoscalar interaction, were discussed in detail in [1].

The matrix element of the RC of a μ^- meson by a proton for the V, A, M, and P couplings, in 4-fermion vertices corresponding to the sum of diagrams of Fig. 1, assumes after simple transformations the form [1,9] 1):

$$
\begin{aligned}
M = & -\left(e\sqrt{4\pi}/m_\mu\sqrt{2k}\right) \\
& \times \{\bar{u}_n O^\mu u_p \bar{u}_\nu (1-\gamma_5)\gamma_\mu \sigma\varepsilon^* u_\mu \tfrac{1}{2}(1+\lambda) \\
& + m_\mu C_P^{(e)} \bar{u}_n \gamma_5 u_p \bar{u}_\nu (1-\gamma_5)(\sigma\varepsilon^*) u_\mu \tfrac{1}{2}(1-\lambda) \\
& - i m_\mu C_M \bar{u}_n \sigma^{\mu\nu}\varepsilon_\nu^* u_p \bar{u}_\nu (1-\gamma_5)\gamma_\mu u_\mu \\
& - (m_\mu/2m)\,[\bar{u}_n O^\mu (\sigma\varepsilon^*)(\lambda+\gamma_5) u_p \bar{u}_\nu (1-\gamma_5)\gamma_\mu u_\mu \\
& - m_\mu C_P^{(n)} \bar{u}_n (\sigma\varepsilon^*)(1+\lambda\gamma_5) u_p \bar{u}_\nu (1-\gamma_5) u_\mu] \\
& - \lambda\mu_p (m_\mu/2m)\,[\bar{u}_n O^\mu \sigma\varepsilon^* u_p \bar{u}_\nu (1-\gamma_5)\gamma_\mu u_\mu \\
& - m_\mu C_P^{(n)} \bar{u}_n \gamma_5 \sigma\varepsilon^* u_p \bar{u}_\nu (1-\gamma_5) u_\mu] \\
& + \lambda\mu_n (m_\mu/2m)\,[\bar{u}_n \sigma\varepsilon^* O^\mu u_p \bar{u}_\nu (1-\gamma_5)\gamma_\mu u_\mu \\
& - m_\mu C_P^{(n)} \bar{u}_n \sigma\varepsilon^* \gamma_5 u_p \bar{u}_\nu (1-\gamma_5) u_\mu] \\
& - 2 C_P^{(n)} \bar{u}_n \gamma_5 u_p \bar{u}_\nu (1-\gamma_5) u_\mu (\varepsilon^* \nu) P^{(e)}\},
\end{aligned}
\tag{1}
$$

where

$$O^\mu = C_V\gamma^\mu + C_A\gamma_5\gamma^\mu - iC_M\sigma^{\mu\nu}q_\nu,$$

$$C_P^{(n)} = \frac{m_\mu^2 + m_\pi^2}{\nu^2 - k^2 + m_\pi^2} C_P, \quad C_P^{(e)} = \frac{m_\mu^2 + m_\pi^2}{(\nu + k)^2 + m_\pi^2} C_P,$$

$$P^{(e)} = \frac{m_\mu^2}{(\nu + k)^2 + m_\pi^2};$$

$q_\nu = (p-n)_\nu$ — momentum transfer; C_V, C_A, C_P, C_M — vector, pseudovector, pseudoscalar, and weak-magnetism constants, respectively; m_π — mass of π^+ meson; ϵ — unit vector of photon circular polarization, and $e^2 = \alpha = 1/137$ ($\hbar = c = 1$). The parameter λ is equal to $+1$ or -1 for right-hand and left-hand polarized photons, respectively.

To calculate the probability of RC of a μ^- meson from a definite hyperfine structure state of the mesic hydrogen atom (from a state with $F = 0$ or $F = 1$) we write down the matrix element (1) in symbolic form

$$M = \bar{u}_n \bar{u}_\nu O^N O^L u_p u_\mu.$$

This notation presupposes, of course, that both the spinors and the operators O^N and O^L have corresponding matrix indices. The operator O^N acts only on the nucleon spinors while O^L acts only on the lepton spinors. Thus, the operators O^N and O^L commute, since they operate in different spinor spaces.

The matrix element for the RC of a μ^- meson from a definite hydrogen mesic-atom hyperfine-structure state can then be written in the form

$$M^F = \sum_{m,\,r} C^{Fm}_{1/2\,m-r,\ 1/2\,r} \bar{u}_n \bar{u}_\nu O^N O^L (u_p)_{1/2\,m-r} (u_\mu)_{1/2\,r}.$$

In this expression $(u_p)_{\frac{1}{2}m-r}$ and $(u_\mu)_{\frac{1}{2}r}$ are the spin wave functions of the proton and negative muon and $C^{a\alpha}_{b\beta,d\delta}$ Clebsch-Gordan coefficients. The sum is taken over the projections of the proton and muon spin on the quantization axis.

It must be noted that

$$\Psi^F_m = \sum_r C^{Fm}_{1/2\,m-r,\ 1/2\,r} (u_p)_{1/2\,m-r} (u_\mu)_{1/2\,r}$$

is the wave function of the mesic hydrogen atom with definite values of spin F and spin projection m.

The normalized density matrix describing the spin states of the mesic hydrogen atom can be introduced in standard fashion:

$$\rho^F = \frac{1}{2F+1} \sum_m \Psi^F_m \bar{\Psi}^F_m.$$

By direct calculation we can show if the μ^- meson is not polarized, then the density matrix ρ^F for the states of the mesic hydrogen with $F = 0$ and $F = 1$ is of the form

$$
\begin{aligned}
\rho^0 &= \tfrac{1}{4}(1 - \sigma_p\sigma_\mu)\Lambda_\mu\Lambda_p, \\
\rho^1 &= \tfrac{1}{4}\left(1 + \tfrac{1}{3}\sigma_p\sigma_\mu\right)\Lambda_\mu\Lambda_p.
\end{aligned}
\tag{2}
$$

Here σ_p and σ_μ are 4×4 matrices containing 2×2 Pauli σ-matrices on the principal diagonal. The matrices σ_p and σ_μ, as already noted, act in the

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1)The following γ matrices were used in (1):
$\gamma_i^+ = -\gamma_i (i = 1, 2, 3)$; $\gamma_0^+ = \gamma_0 = \beta$; $\gamma_5^+ = \gamma_5$, $\sigma^{\mu\nu} = [\gamma^\mu, \gamma^\nu]/2i$.

nucleon and lepton spaces. The operators Λ_p and Λ_μ are the ordinary energy projection operators pertaining to the muon and the proton. If we neglect the proton and muon momenta, then

$$\Lambda_\mu = (1+\beta^L)/2, \qquad \Lambda_p = (1+\beta^N)/2.$$

Expressions (2) for the density matrix ρ^0 and ρ^1 coincide with those obtained in [7,8] by a different method.

3. SPECTRUM AND CIRCULAR POLARIZATION OF PHOTONS FOR RC OF A μ^- MESON FROM A DEFINITE MESIC-HYDROGEN HYPERFINE-STRUCTURE STATE

The differential probability of the RC of a μ^- meson from a definite hyperfine structure state of the mesic hydrogen atom is of the form

$$W^F(k)\,dk = \frac{\alpha}{2\pi^3 a_\mu^3}(m_\mu - k)^2 k\,dk \int\limits_0^\pi \sin\theta\, d\theta \times \left\{1 - \frac{k^2 + 2(m_\mu - k)^2}{m(m_\mu - k)} - 3\frac{k}{m}\cos\theta\right\} D^F,$$

$$D^F = {}^1\!/_4\, \mathrm{Sp}\,\{\bar{O}\Lambda_\nu\Lambda_n O\rho^F\}, \quad O = O^N O^L,$$
$$\bar{O} = \beta^L\beta^N (O^N)^+ (O^L)^+ \beta^N\beta^L, \tag{3}$$

where Λ_ν and Λ_n — energy projection operators of the neutrino and the neutron, θ — angle between the photon and neutrino momenta, a_μ — Bohr K-orbit radius of the mesic hydrogen atom.

The trace of the product of the matrices in (3) contains summation over two indices [see the expression for the matrix element (1)]. For a simpler calculation of this trace it is convenient to express all the operators in the matrix elements (1) in terms of the matrices γ_5, σ, and β with the aid of the quality $\gamma = \gamma_5\beta\sigma$.

The matrix element (1) is expressed in rather complicated form in terms of the matrices σ. We note that the contribution of the first term of the density matrix ρ^F (i.e., of unity) to the probability is equal, apart from the statistical factor $1/4$, to the RC probability of the μ^- meson by the proton, summed over the hyperfine-structure states with F = 0 and F = 1 [1]. The contributions from the second term of ρ^F for the states with F = 0 and F = 1 differ only in a coefficient (-1 or $1/3$).

As a result of the calculations we have obtained the following expressions for D^F, which were integrated immediately over the angle θ

$$D^0 = D_1 - D, \tag{4}$$

$$D^1 = D_1 + \tfrac{1}{3}D, \tag{5}$$

$$D_1 = D_1^L + D_1^R, \tag{6}$$

$$D = D^L + D^R. \tag{7}$$

Here D_1^L, D^L and D_1^R, D^R are those parts of the expressions for D_1 and D which represent the contributions from the left-hand and right-hand circularly-polarized photons,

$$\begin{aligned} D_1^L = {} & 2C_M\,(C_M + m^{-1}(C_V + C_A)) + \tfrac{1}{2}m_\mu m^{-2}C_P^{(n)}[C_V + C_A \\ & + (\mu_p - \mu_n)\,C_V + \tfrac{1}{2}m_\mu C_P^{(n)} + \tfrac{1}{3}\nu C_P^{(e)} \\ & - (2\nu^2/3m_\mu)\,C_P^{(n)}P^{(e)}] + \tfrac{1}{4}(\mu_p - \mu_n)^2 m^{-2}\,(C_V^2 + C_A^2) \\ & + \tfrac{1}{4}m_\mu m^{-3}\,(1 + \mu_p - \mu_n)\,C_P^{(n)} \\ & \times [kC_P^{(n)} + (\mu_p - \mu_n)\,(kC_V - \tfrac{1}{3}\nu C_A)] \\ & + m^{-1}C_P^{(e)}\,[(\tfrac{2}{3}\nu - k)\,C_M + \tfrac{1}{2}m^{-1}(\tfrac{1}{3}\nu - k)\,(C_V + C_A) \\ & - \tfrac{1}{6}\nu\,(\mu_p - \mu_n)\,C_V - \tfrac{1}{3}\nu\,(\nu^2 + k^2)\,C_P^{(n)}P^{(e)}/mm_\mu] \\ & + \tfrac{1}{4}m^{-2}\,(m_\mu^2 - \tfrac{8}{3}\nu k)\,(C_P^{(e)})^2, \end{aligned} \tag{8}$$

$$\begin{aligned} D_1^R = {} & m_\mu^{-2}[C_V^2 + 3C_A^2 + 8\,(k - m_\mu)\,C_A C_M \\ & + \tfrac{2}{3}\nu\,(7m_\mu - 10\,k)\,C_M^2] + (mm_\mu)^{-1}[C_V^2 + 3C_A^2 - 4C_V C_A \\ & + (5m_\mu - 4k)\,C_V C_M + 2\,(4k - 3m_\mu)\,C_A C_M \\ & + m_\mu C_A C_P^{(n)} + m_\mu\,(k - \tfrac{2}{3}\nu)\,C_M C_P^{(n)}] \\ & - (\mu_p - \mu_n)\,(m_\mu m)^{-1}\,C_V\,[2C_A - (\tfrac{4}{3}\nu - k)\,C_M] \\ & - (2\nu/3m_\mu m)\,C_A C_P^{(n)}P^{(e)} + \tfrac{1}{4}m_\mu m^{-2}C_P^{(n)}\,[2\,(C_A - C_V) \\ & + m_\mu C_P^{(n)} - \tfrac{1}{2}m_\mu m^{-2}\,(\mu_p - \mu_n)\,C_V - \tfrac{1}{3}\nu^2 m^{-2}C_P^{(n)}P^{(e)}] \\ & + \tfrac{1}{4}(\mu_p - \mu_n)^2 m^{-2}\,(C_V^2 + C_A^2) + m^{-1}m_\mu^{-2}\,[k\,(C_V + C_A)^2 \\ & + \nu\,(C_V - C_A)^2] \\ & + \tfrac{1}{2}m^{-2}\,(2 + \mu_p - \mu_n)\,C_P^{(n)}\,[\tfrac{1}{3}\nu C_A + (k - 2\nu/3)\,C_V] \\ & + \tfrac{1}{4}m_\mu m^{-3}\,(\mu_p - \mu_n)\,(1 + \mu_p - \mu_n)\,C_P^{(n)}\,(\tfrac{1}{3}\nu C_A + kC_V), \end{aligned} \tag{9}$$

$$\begin{aligned} D^L = {} & -C_M\,[6C_M + 2m^{-1}\,(C_V + C_A + m_\mu C_P^{(n)} \\ & + (\mu_p - \mu_n)\,(C_V - C_A))] + \tfrac{1}{2}m_\mu m^{-2}C_P^{(n)}\,[-C_A - C_V \\ & + C_M\,(\tfrac{2}{3}\nu - k) + (\mu_p - \mu_n)\,(C_A - \tfrac{2}{3}\nu C_M) \\ & - (2\nu k/3m_\mu)\,C_P^{(n)}P^{(e)} + C_P^{(e)}\,(\tfrac{2}{3}\nu - k)] \\ & + \tfrac{1}{2}m^{-2}\,[(\mu_p - \mu_n)^2\,C_V C_A + C_P^{(e)}\,(C_V + C_A)\,(\tfrac{1}{3}\nu - k) \\ & + C_P^{(e)}C_M\nu\,(\tfrac{7}{3}\nu - k)] + \tfrac{1}{4}m_\mu^2 m^{-3}\,(1 + \mu_p - \mu_n)\,C_P^{(n)} \\ & \times [\tfrac{1}{3}\nu m_\mu C_P^{(n)} + \tfrac{1}{3}(\mu_p - \mu_n)\,\nu C_V + (\mu_p - \mu_n) \\ & \times (k - 2\nu/3)\,C_A - C_P^{(e)}\,(m_\mu^2 - 8\nu k/3)] - kC_M C_P^{(e)}/m \\ & - \tfrac{1}{2}(\mu_p - \mu_n)\,m^{-2}C_P^{(e)}\,[C_V\,(k - 2\nu/3) \\ & - C_A k + \tfrac{2}{3}C_M\nu\,(k - \nu)], \end{aligned} \tag{10}$$

$$D^R = 2m_\mu^{-2}[C_A^2 - C_AC_V + 2C_VC_M(m_\mu + \nu/3) + 2kC_AC_M - m_\mu(k+3m_\mu)C_M^2] + (mm_\mu)^{-1}[C_V^2 + 3C_A^2 - 4C_VC_A + m_\mu C_P^{(n)}(C_V + 2C_A - C_M(7\nu/3 + m_\mu)) + C_AC_M(5\nu/3 - m_\mu) + C_VC_M(2m_\mu + \nu) - \tfrac{14}{3}\nu m_\mu C_M^2] - (\mu_p - \mu_n)(m_\mu m)^{-1}[C_V^2 - C_A^2 + 2C_VC_A - C_VC_M(7\nu/3 + m_\mu) + 2C_AC_M(m_\mu + 2\nu/3) - \tfrac{4}{3}\nu m_\mu C_M^2] - (2/3\, mm_\mu^2)C_P^{(n)}P^{(e)}[C_V\nu(\nu - k) + 2\nu^2C_A + C_M\nu(km_\mu - 2\nu(m_\mu + \nu))] + \tfrac{1}{2}m_\mu m^{-2}C_P^{(n)}[C_A - C_V - C_M(m_\mu + \nu) - (\mu_p - \mu_n)(C_A - \tfrac{2}{3}\nu C_M) + (2\nu k/3m_\mu)C_P^{(n)}P^{(e)}] + \tfrac{1}{2}(\mu_p - \mu_n)^2 m^{-2}C_VC_A - (2/mm_\mu)[\tfrac{4}{3}\nu C_AC_V - C_V^2(2k + \nu/3) - \tfrac{1}{3}\nu C_A^2 - \tfrac{1}{3}\nu(\nu - 2k)C_AC_M + C_VC_M(\nu k - 2\nu^2 - 3k^2) + \tfrac{2}{3}\nu C_M^2(\nu^2 + k^2)] - \tfrac{1}{2}m^{-2}C_P^{(n)}[C_V(\nu - 2k) - C_A(k + 2\nu/3) + C_M(m_\mu^2 - 4\nu k/3)] - \tfrac{1}{2}(1 + \mu_p - \mu_n) \times m^{-2}C_P^{(n)}[C_A(\nu - 2k) - C_V(k + 2\nu/3) + C_M(5\nu k/3 - k^2 - 4\nu^2/3) + (\nu m_\mu^2/6m)C_P^{(n)}] + \tfrac{1}{4}(\mu_p - \mu_n)m^{-3}(1 + \mu_p - \mu_n) \times m_\mu C_P^{(n)}[\tfrac{1}{3}\nu C_V - C_A(k + 2\nu/3) + C_M\nu(\nu - k/3)]. \quad (11)$$

We also write down formulas for C_1 and C:

$$C_1 = C_1^R = m_\mu^{-2}(C_V^2 - C_A^2 - m_\mu^2 C_A C_P^{(n)}/m), \quad (12)$$

$$C = C^R + C^L = \{-2m_\mu^{-2}(C_V + C_A)C_A + C_VC_P^{(n)}/m - \tfrac{1}{2}m_\mu^2 m^{-2}(C_P^{(n)})^2\} + \tfrac{1}{2}m_\mu^2 m^{-2}(C_P^{(n)})^2, \quad (13)$$

which are those parts of the expressions for D_1 and D which, unlike the remaining parts, are multiplied in formula (3) only by $-k/m$. The factor $3\cos\theta$ vanishes in this case as a result of integration with respect to θ.

We thus obtain the following expression for the spectrum of the photons emitted in the RC of a μ^- meson from a definite state of the mesic hydrogen atom hyperfine structure:

$$W^F(k)\,dk = \frac{\alpha}{\pi^3 a_\mu^3}(m_\mu - k)^2 k\,dk\left\{1 - \frac{k^2 + 2(m_\mu - k)^2}{m(m_\mu - k)} - \frac{k}{m}\right\}D^F, \quad (14)$$

where D^F is given by (4)—(13) with $\nu = m_\mu - k$.

The spectrum of the photons emitted in the RC of a μ^- meson by a proton, summed over the hyperfine-structure states, is determined (by virtue of the foregoing remarks concerning this structure of the density matrix ρ^F) by expression (14), in which we replace D^F by D_1[1].

The expression for the degree of circular polarization of the photons in RC from a definite hyperfine structure state of the mesic hydrogen atom has the form

$$\beta(k) = 1 - 2W_L^F(k)/W^F(k), \quad (15)$$

where $W_L^F(k)$ is the part of $W^F(k)$ due to the contribution of the left-circularly polarized photons [see formulas (4)—(14)]. The expression for the degree of circular polarization of the photons emitted in the RC of a μ^- meson by a proton, summed over the states of the hyperfine structure, is obtained from (15) in fashion similar to that with which the spectrum of these photons is obtained from (14).

4. SPECTRUM AND CIRCULAR POLARIZATION OF PHOTONS FOR THE RC OF A μ^- MESON FROM THE MESIC-MOLECULE STATE

Since thermal motion and collisions with the free protons and molecules of the hydrogen cause the hydrogen mesic atoms to go over completely to the hyperfine structure ground state with F = 0 during the lifetime of the μ^- meson[10,11], the RC in mesic-hydrogen atoms is only from the state with F = 0. The RC from the hyperfine structure state of the hydrogen mesic atom with F = 0 can be experimentally studied by using deuterium-free hydrogen, the density of which is 20—30 times smaller than the density of liquid hydrogen at T = 20°K[2].

The μ^- mesons produce in the liquid hydrogen the mesic-molecule ions $(pp\mu)_+$ by electric dipole transition with energy given up to the conversion electron[2,3]:

$$(\mu p) + (ep) \to (pp\mu)_+ + e.$$

The probability of this process is[2,12] $1/\tau_{pp\mu} \approx 1.5 \times 10^6\ \mathrm{sec}^{-1}$.

Thus, the mesic-molecule ions $(pp\mu)_+$ are produced in hydrogen within a time much longer than the lifetime of the μ^- meson relative to the hyperfine mesic atom transition[10] ($1/\tau_c \approx 2 \times 10^9\ \mathrm{sec}^{-1}$), but smaller than the lifetime of the μ^- meson ($1/\tau_\mu = 0.5 \times 10^6\ \mathrm{sec}^{-1}$). The mesic molecules are produced here principally in the ortho state. The ratio of the probabilities of the production of the mesic molecule in the para and ortho states[2,3] is 2×10^{-4}.

Thus, the μ^- mesons are captured in liquid hydrogen from the state of the mesic molecule $(pp\mu)_+$. The probability of capture (both radiative and non-radiative) from the $(pp\mu)_+$ mesic molecule state is[3]:

$$W_{pp\mu} = \xi W\,(^1/_2) + (1-\xi)\,W\,(^3/_2),$$

where $W(^1/_2)$ and $W(^3/_2)$ are respectively the probabilities of capture from the mesic molecule states with total spin $^1/_2$ and $^3/_2$, and ξ is the statistical weight of the state with total spin $^1/_2$.

The probability $W_{pp\mu}$ can be expressed in terms of the probability W^0 of μ^- capture from the mesic atom state with $F = 0$ and the probability $\widetilde{W}$ of μ^- capture by a proton, averaged over the hyperfine structure state[3]:

$$W_{pp\mu} = 2\gamma_0\,\{(^4/_3 - \xi)\,\widetilde{W} + (\xi - {}^1/_3)\,W^0\}, \tag{16}$$

where γ_0 is the ratio of the square of the modulus of the μ^--meson wave function of one of the protons in the $(pp\mu)_+$-ortho molecule to the square of the modulus of the μ^--meson wave function in the $(p\mu)$ atom[13]. Weinberg[3] obtained $2\gamma_0 = 1.165$. Kroll and Halpern[14-16] found by calculation that $\xi = 1$, with an uncertainty of several percent. Then expression (16) for $W_{pp\mu}$ goes over into

$$W_{pp\mu} = 2\gamma_0\left\{\frac{1}{3}\,\widetilde{W} + \frac{2}{3}\,W^0\right\}. \tag{16a}$$

An experimental investigation of the production of $pp\mu$ mesic molecules has established[15] that approximately $^3/_4$ of all the muons captured on the mesic-atom K orbit produce $(pp\mu)_+$ molecules, while the remainder ($^1/_4$) are in the mesic-atom hyperfine-structure state with $F = 0$. Then the RC probability of a μ^- meson in hydrogen is given by the sum of the probabilities for capture from the mesic-molecule and mesic-atom (with $F = 0$) states:

$$W = \frac{3}{4}\,W_{pp\mu} + \frac{1}{4}\,W^0. \tag{17}$$

5. DISCUSSION OF RESULTS

It follows from expressions (4)—(14) that μ^- capture from a mesic-atom hyperfine-structure state with $F = 0$ is forbidden if $C_M = C_P = 0$ and $C_A = -C_V$. This result coincides with the deductions of [17], where the angular momentum conservation law was used to demonstrate that RC from the state with $F = 0$ is forbidden if a V–A weak-interaction Hamiltonian is used.

Thus, RC occurs in the state with $F = 0$ only as a result of weak-interaction renormalization effects caused by strong interactions, i.e., is the result of anomalous magnetic moments of the nucleons and the induced effective pseudoscalar interaction in the "anomalous" part of the axial-vector interaction. An experimental investigation of the RC from a state with $F = 0$ is therefore of great interest, for it can yield additional information on renormalization effects in weak interactions.

Expressions (14) and (17) for the probabilities $\widetilde{W}$, W^0, and W of radiative capture of a μ^- meson in hydrogen were calculated for different values of the effective pseudoscalar interaction constant C_A at fixed values of the remaining constants, chosen the same as in [18,19]. It is seen from the accompanying table that for all values of the ratio $C_P/8C_A$ the probability W^0 of RC from a mesic-hydrogen state with $F = 0$ is small compared with the probability $\widetilde{W}$ averaged over the hyperfine structure states. $\widetilde{W}$ and W increase more slowly than κ^2 with increasing κ, while W^0 increases more rapidly (see Figs. 2 and 4).

Figure 3 shows the degree of photon circular polarization $\beta(x)$, averaged over the hyperfine structure states of the hydrogen mesic atom, vs. the effective pseudoscalar interaction constant C_P.

Since 75% of all the orbital μ^- mesons are captured from the $(pp\mu)_+$ mesic-molecule state, while 25% are captured from the mesic-atom state with $F = 0$, the spectrum and the circular polarization

$\varkappa = C_P/8C_A$	$\widetilde{W}$, 10^{-2}sec^{-1}	W^0, 10^{-2}sec^{-1}	W, 10^{-2}sec^{-1}
1	11.4	0.3	3.6
2	14.8	1.6	5.6
3	19.3	3.7	8.7

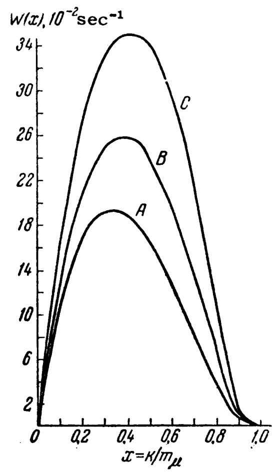

FIG. 2. Photon spectrum averaged over the states of the hyperfine structure of the hydrogen mesic atom (curves A, B, and C correspond to κ = 1, 2, 3).

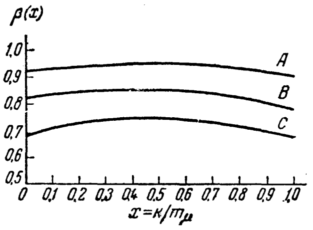

FIG. 3. Degree of photon circular polarization averaged over the states of the hyperfine structure of the mesic atom of hydrogen (curves A, B, C correspond to κ = 1, 2, 3).

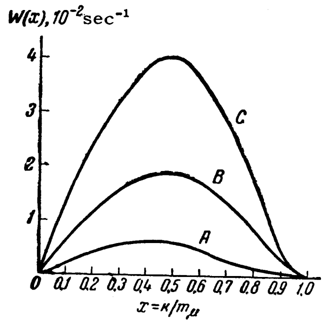

FIG. 4. Photon spectrum for radiative capture of μ^- meson from the mesic hydrogen hyperfine structure state with F = 0 (curves A, B, C correspond to κ = 1, 2, 3).

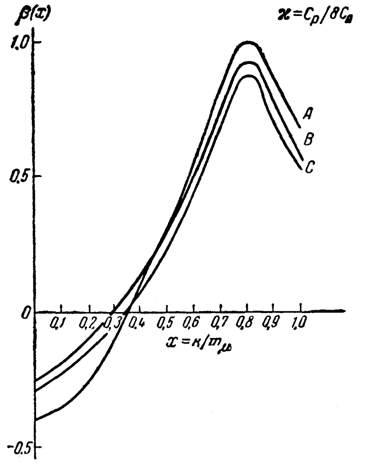

FIG. 5. Degree of photon circular polarization for radiative μ^- capture from the mesic-atom state with F = 0 (curves A, B, C correspond to κ = 1, 2, 3).

of the photons can be readily obtained for this case from the curves of Figs. 2 and 4 and from (16a) and (17).

It is seen from Fig. 5 that the photons emitted in the RC of a μ^- meson in hydrogen have left-hand polarization in the energy region $0 < k/m_\mu \lesssim 0.4$ in the case of capture from a mesic-atom state with F = 0. In all other cases the emitted photons have right-hand polarization.

At the present time there are experimental data on the total probability and spectrum of the photons emitted in RC of μ^- mesons by Fe^{58} [20] and Cu^{64} [21], but not accurate enough to conclude definitely that theory and experiment agree. The experimental study of the energy spectrum, the degree of photon circular polarization, and the total probability of the RC of μ^- mesons in hydrogen entails no specific nuclear difficulties in the interpretation of the experimental results, and is therefore of great interest.

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