RANGE STRAGGLING OF CHARGED PARTICLES IN VARIOUS SUBSTANCES

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It is shown that straggling of heavy charged particles in any substance can be expressed by a universal function of Y = E/I, where E is the particle energy and I the mean ionization potential. Range straggling is calculated for a photographic emulsion.

1. When charged particles pass through matter they lose energy to ionization of the atoms in the matter. Owing to the random character of the ionization process, particles having identical energy have different ranges. The range scatter is characterized by a variance

$$\sigma_R^2 = \langle (R - \bar{R})^2 \rangle ,$$

where $\bar{R}$ —mean range. The ratio of the square root of the range variance to the mean, expressed in per cent, is called, following Darwin [1], range straggling. Range straggling imposes in principle a limit on the accuracy with which the particle energy can be determined from its range.

For heavy charged particles passing through sufficiently thick layers of matter, the scatter of energy loss about the mean value can be represented by a Gaussian curve with half-width $\Gamma : \sigma_E^2 = \alpha t = \Gamma^2$. According to Bohr [2], α does not depend on the particle velocity with $\alpha = 4\pi Z^2 e^4 n$, where Z is the particle charge and n is the number of electrons per unit volume.

Livingston and Bethe [3] gave a more accurate formula, in which the binding of the electrons in the atom was taken into account. Lewis [4] investigated the deviations from a Gaussian distribution. The energy-loss spread was investigated in several papers [5-7].

Taking into account the Lindhard and Scharff relativistic correction [8], range straggling is given by the formula

$$\varphi = 100 \sqrt{\sigma_R^2} / \langle R \rangle , \qquad \sigma_R^2 = \alpha \int_0^E \left(\frac{dE}{dx}\right)^{-3} \frac{1 - \beta^2/2}{1 - \beta^2} dE , \quad (1)$$

where φ is the range straggling in percent, and β is the particle velocity. Barkas [9] calculated range straggling for nuclear emulsion, while Sternheimer calculated range straggling for Be, C, Al, Cu, Pb, and air [8]. Both used the Bethe-Livingston formula for the slowing-down of the particles

$$\frac{dE}{dx} = \frac{4\pi Z^2 e^4 n}{mc^2 \beta^2} \left[\ln \left(\frac{2mc^2 \beta^2}{I}\right) - \ln (1 - \beta^2) - \beta^2 - C \right] , \quad (2)$$

where I —mean ionization energy of the atoms of the medium and C —Bethe-Fermi correction term. This formula does not yield a general expression for straggling in different substances, because it can be integrated only numerically. However, an estimate of the straggling in different substances is frequently needed in experiments. It is therefore desirable to have a simpler and more general formula.

2. Using the relation $E = Mc^2(\gamma - 1)$, let us modify (1) in the following fashion:

$$\sigma_R^2 = \alpha \int_0^E \left(\frac{dE}{dx}\right)^{-3} dE + \frac{\alpha}{Mc^2} \int_0^E \left(\frac{dE}{dx}\right)^{-3} E dE + \frac{1}{2} \frac{\alpha}{M^2 c^4} \int_0^E \left(\frac{dE}{dx}\right)^{-3} E^2 dE , \quad (3)$$

where Mc^2 is the particle rest energy. Comparison with (2) shows readily that the first term gives the nonrelativistic approximation and the remaining terms are due to the relativistic effect.

We shall calculate the range straggling for a proton (Z = 1). For the other particles we can use

$$\varphi (\beta) = (M/\mu)^{1/2} \varphi_p (\beta), \quad (4)$$

where μ is the particle mass and $\varphi_p(\beta)$ the range straggling for a proton at a velocity β.

As shown earlier [11], the range-energy ratio of heavy charged particles in any substance can be represented with good accuracy by

$$E = aR^m , \quad (5)$$

where m is a universal constant in a certain interval of y = E/I. Using this relation we find, for example that for y < 20

$$\alpha \int_0^E \left(\frac{dE}{dx}\right)^{-3} dE = \frac{\alpha}{\eta}\left(\frac{I}{a}\right)^{2/m} y^{(3-2m)/m},$$

$$\frac{\alpha}{Mc^2}\int_0^E \left(\frac{dE}{dx}\right)^{-3} EdE = \frac{1}{Mc^2}\frac{\alpha}{\xi}\left(\frac{I}{a}\right)^{2/m} y^{(3-m)/m},$$

where

$$\eta = I^2 (I/a)^{-1/m} m^2 (3-2m), \qquad \xi = I (I/a)^{-1/m} m^2 (3-m).$$

If the range is in g/cm^2, we obtain from the expression given in [10] for the universal constant k

$$\frac{\alpha}{\eta} = \frac{4\pi e^4 N_0 k}{m^2(3-2m)}, \qquad \frac{\alpha}{\xi} = \frac{4\pi e^4 N_0 k}{m^2(3-m)} I.$$

The range straggling is then given, in accordance with (1) and (3), by

$$\varphi = Cy^{(1-2m)/2m} + IAy^{1/2m},$$

where

$$C = 100\left[\frac{4\pi e^4 N_0 k}{m^2(3-2m)}\right]^{1/2}, \qquad A = \frac{C}{2Mc^2}\frac{(3-2m)}{(3-m)}.$$

In the general case we obtain

$$\varphi = \varphi_0 + \varphi_I, \quad \varphi_0 = C_i y^{(1-2m_i)/2m_i} + B_i y^{(2m_i-5)/2m_i},$$

$$\varphi_I = IA_i y^{1/2m_i}, \tag{6}$$

where

$$A_i = \frac{C_i}{2Mc^2}\frac{(3-2m_i)}{(3-m_i)}, \qquad C_i = 100\left[\frac{4\pi e^4 N_0 k_i}{m_i^2(3-2m_i)}\right]^{1/2},$$

$$B_i = \frac{C_i}{2}\left[-y_i^{(3-2m_i)/m_i} + y_i^{(3-2m_{i-1})/m_{i-1}}\frac{k_i^{-3} m_i^2 (3-2m_i)}{k_{i-1}^{-3} m_{i-1}^2 (3-2m_{i-1})}\right].$$

Table I

y	m_i	$k_i \cdot 10^{-2}$	y_i
$0 < y < 20$	0.635	0.382	0
$20 \leq y < 600$	0.574	0.233	20
$600 < y < 2700$	0.656	1.006	600

The values of m_i, k_i, and y_i are given in Table I.

Equations (6) and (7) lead to the following conclusions. Since m_i and k_i are constants that do not depend on the medium, the constants C_i, B_i, and A_i in (6) are likewise independent of the medium. In addition, as shown below, the third term in (6) is smaller than 0.1% percent up to y ~ 1500. Therefore the range straggling in the interval 0 < y < 1500 is given by a universal function

$$\varphi = F(E/I), \tag{8}$$

i.e., the range straggling of particles of energy E in a substance having an average ionization potential I is equal to the straggling possessed by a particle with energy $E_0 = (I_0/I)E$ in a substance with average ionization potential I_0.

3. Owing to the universality of the function F(E/I), range straggling can be calculated for any substance. We have calculated F(E/I) for a nuclear emulsion, using the values of m and k given in Table I. The result is shown in Table II. For comparison, this table lists also the results obtained by Barkas. We see that the numerical agreement is sufficiently good.

Table II

E, MeV	φ_0, %	φ_I, %	φ, %	ϕ (Barkas), %
1	2,32	0.00	2,32	2,11
2	2,00	0,00	2.00	1,94
5	1.64	0,00	1,64	1,66
10	1.55	0.00	1.55	1.53
20	1,45	0.01	1,46	1,42
50	1,28	0,03	1.31	1.29
100	1.17	0.05	1.22	1.21
200	1.07	0.09	1.16	1.13
500	1.00	0,08	1.08	1.02

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[1] Darwin, Phil. Mag. **23**, 901 (1912).
[2] N. Bohr, Phil. Mag. **25**, 10 (1913), Phil. Mag. **30**, 581 (1915).
[3] M. S. Livingston and H. A. Bethe, Revs. Modern Phys. **9**, 261 (1937).
[4] H. W. Lewis, Phys. Rev. **85**, 20 (1952).
[5] L. Landau, J. Phys. USSR **4**, 201 (1944).
[6] P. V. Vavilov, JETP **32**, 920 (1957), Soviet Phys. JETP **5**, 749 (1957).
[7] O. Blunck and K. Westphalk, Z. Physik **130**, 641 (1951).
[8] J. Lindhard and M. Scharff, Dan. Mat. Fys. Medd. **27**, 15 (1953).
[9] Barkas, Smith, and Birnbaum, Phys. Rev. **98**, 605 (1955).
[10] R. M. Sternheimer, Phys. Rev. **117**, 485 (1960).
[11] Do In Seb, JETP **43**, 121 (1962), Soviet Phys. JETP **17**, 87 (1963).

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131