DEPOLARIZATION OF RELATIVISTIC ELECTRONS IN A MAGNETIC FIELD

S. S. SANNIKOV

Physico-technical Institute, Academy of Sciences, Ukrainian S.S.R.

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The normal (Bohr) and anomalous (Pauli) parts of the magnetic moment of an electron moving in a magnetic field with an energy $\epsilon - m \ll m$ (m is the electron mass) have kinematically the same behavior. The expressions presented in [1] for the rotation frequency of the electron spin ω_S in the energy range $\epsilon - m \sim m$ indicate that there is a great difference in the kinematical properties of the normal and anomalous parts of the electron magnetic moment in this energy region. It is shown in the present paper that in the limiting case of high energies $\epsilon \gg m$ the normal and anomalous parts of the magnetic moment do not differ kinematically. Thus, the correspondence characteristic of the low-energy region is restored.

It is well known that an account of the radiation corrections to the magnetic moment of an electron moving in a constant, uniform magnetic field leads to additional spin rotation [1], which in its turn leads to a change in the electron polarization. It is important to note that, generally speaking, the anomalous and normal parts of the electron magnetic moment differ in their kinematic properties. The kinematic properties of both parts are the same for electron energy $\epsilon - m \ll m$ (m is the electron mass), and differ greatly for energies $\epsilon - m \gtrsim m$. This has been indicated by the expressions obtained in [1] for the electron spin rotation frequency. This difference in kinematic properties causes the additional rotation of the electron spin to depend on the orientation of the external magnetic field with respect to the particle momentum and (particularly important) on the particle energy, so that we have the ratio

$$\Delta\omega/\omega_L = (\omega_S - \omega_L)/\omega_L = \begin{cases} \alpha\varepsilon/2\pi m, & \text{when } \mathbf{H} \perp \mathbf{p} \\ \alpha/2\pi, & \text{when } \mathbf{H} \parallel \mathbf{p} \end{cases} \qquad (1')$$

(here ω_S is the spin rotation frequency, ω_L is the Larmor rotation frequency of the electron, H is the external magnetic field, p is the electron momentum, α is the fine-structure constant), i.e., it increases with increasing energy.

In this report we find an expression for ω_S which is correct in the limiting case of very high energies $\epsilon \gg m$, and show that the ratio $\Delta\omega/\omega_L$ does not depend on the orientation of the magnetic field or on the electron energy. We show thereby that, in terms of kinematic properties, the normal and anomalous parts of the magnetic moment do not differ from each other in the limiting case $\epsilon \gg m$.

For this purpose, we first write the Hamiltonian of our problem

$$H^D = \boldsymbol{\alpha}\mathbf{p} + m\beta - \mu(g-1)\beta\boldsymbol{\sigma}\mathbf{H}, \qquad (1)$$

where μ is the normal magnetic moment of the electron, equal to $e/2m$, $g \cong 1 + \alpha/2\pi$; p is the kinetic momentum of the electron (the system of units in which $\hbar = c = 1$ is used). It is convenient to change to the representation given by Cini and Touschek [2,3], which is especially adapted to the limiting case $\epsilon \gg m$ (called henceforth the E-representation). In this representation, the Hamiltonian of the problem in question takes the form (all operators in the new representation are designated by the index E)

$$H^E = e^{iT} H^D e^{-iT}, \qquad (2)$$

where

$$e^{\pm iT} \approx 1 \mp \tfrac{1}{2} m\beta\gamma_5\chi, \qquad (3)^*$$

$$\chi = e\mathbf{H}\mathbf{p}\, p^{-4} + ie\boldsymbol{\sigma}\,[\mathbf{p}\mathbf{H}]\, p^{-4} + \boldsymbol{\sigma}\mathbf{p}\, p^{-2},$$

i.e.,

$$H^E \approx \boldsymbol{\alpha}\mathbf{p} - \mu(g-1)(\beta\boldsymbol{\sigma}\mathbf{H} + m\gamma_5\mathbf{p}\mathbf{H}p^{-2}) \qquad (4)$$

(we retain only terms which are linear with respect to the field **H** and the ratio m/p). In the E-representation

$$\mathbf{p}^E \approx \mathbf{p}, \qquad \boldsymbol{\sigma}^E \approx \boldsymbol{\sigma} - im\beta\,[\boldsymbol{\alpha}\mathbf{p}]\, p^{-2} \qquad (5)$$

(to the accuracy required below). The Heisenberg equations for the momentum and spin operators take the form

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* $[\mathbf{pH}] = \mathbf{p} \times \mathbf{H}$.

$$\dot{\mathbf{p}}^E = -i[\mathbf{p}^E, H^E] = [\boldsymbol{\alpha}\mathbf{H}],$$

$$\dot{\sigma}^E = -i[\boldsymbol{\sigma}^E, H^E] = -2[\boldsymbol{\alpha}\mathbf{p}] + 2\mu(g-1)\left\{\beta[\boldsymbol{\sigma}\mathbf{H}] + \frac{m}{p}\left[\gamma_5\frac{\mathbf{p}}{p}\mathbf{H}\right]\right\}. \quad (6)$$

Averaging these equations over to the positive-energy state $|E\rangle_+ = \Lambda_+^E|E\rangle$, where $|E\rangle$ is the solution of the Dirac equation in the E-representation and Λ_+^E is the projection operator for the positive-energy state,

$$\Lambda_+^E \approx \frac{1}{2}\left(1 + \boldsymbol{\alpha}\mathbf{p}\frac{1}{p} + \frac{e}{2p^2}\gamma_5\frac{\mathbf{pH}}{p}\right), \qquad (\Lambda_+^E)^2 \approx \Lambda_+^E, \quad (7)$$

we obtain

$$\dot{\mathbf{p}} = \frac{e}{p}[\mathbf{pH}], \qquad \dot{\boldsymbol{\xi}} = \frac{eg}{p}[\boldsymbol{\xi}\mathbf{H}], \quad (8)$$

in which we designated the quantity $\langle E|\Lambda_+^E \mathbf{p}^E \Lambda_+^E|E\rangle$ by $\mathbf{p}$, and

$$\boldsymbol{\xi} = \langle E|\Lambda_+^E\boldsymbol{\sigma}^E\Lambda_+^E|E\rangle = \langle E|\gamma_5 p^E p^{-1}\Lambda_+^E|E\rangle. \quad (2')$$

Here

$$\langle E|\Lambda_+^E\boldsymbol{\alpha}\Lambda_+^E|E\rangle \approx \frac{\mathbf{p}}{p} - \frac{e}{2p^2}\gamma_5\mathbf{H}. \quad (9)$$

It follows from (8) that when $\epsilon \gg m$ the electron and spin rotation frequencies in the magnetic field are equal respectively to

$$\omega_L = eH/p, \qquad \omega_s = egH/p, \quad (10)$$

and consequently $\Delta\omega/\omega_L$ equals $\alpha/2\pi$, just as in the limiting case $\epsilon - m \ll m$. It can thus be concluded that in the limiting case $\epsilon \gg m$ the kinematic properties of the normal and anomalous parts of the electron magnetic moment are the same.

It should be noted that, owing to the constant change of the electron polarization in the magnetic field, many difficulties are encountered in carrying out experiments on colliding beams of polarized particles by means of accumulators.

In conclusion, the author would like to express his deep appreciation to Professor A. I. Akhiezer for his advice and consideration.

Note added in proof (August 14, 1963). We give here a simple proof of the statement made in the abstract regarding the nature of electron spin motion in the nonrelativistic limit. In the limit $\epsilon - m \ll m$, the Hamiltonian of the problem in question takes the form

$$H = \frac{p^2}{2m} - \frac{p^4}{8m^3} - \mu g\,\boldsymbol{\sigma}\mathbf{H}. \quad (3')$$

The Heisenberg equations for the momentum operator and polarization matrix of the density $\rho = \frac{1}{2}(1 + \xi\sigma)$ ($\frac{1}{2}\xi$ is the mean electron spin) now yield

$$\dot{\mathbf{p}} = \frac{e}{m}[\mathbf{pH}], \qquad \dot{\boldsymbol{\xi}} = \frac{eg}{m}[\boldsymbol{\xi}\,\mathbf{H}]. \quad (4')$$

Thus, in this limiting case $\Delta\omega/\omega_L = \alpha/2\pi$, i.e., it does not depend on the field or on the particle energy.

[1] H. Mendlowith and K. M. Case, Phys. Rev. **97**, 33 (1955).

[2] M. Cini and B. Touschek, Nuovo cimento 7, 422 (1958).

[3] Bose, Gamba, and Sudarshan, Phys. Rev. **113**, 1661 (1959).

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