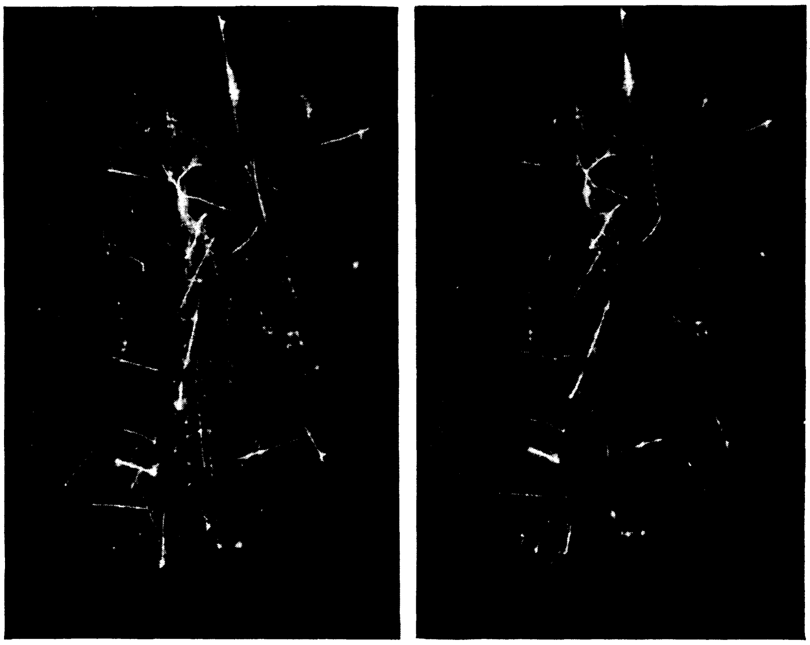

FIG. 3

uring the parallax showed that in our geometry (stereoscopic base equal 10.6 cm, distance of the camera to the object equal 120 cm, and magnification 10x), the z coordinates of the luminous centers can be determined accurate to several millimeters.

The new method of spark chamber operation enables us therefore to reconstruct the charged particle tracks in space, and also to detect showers (Fig. 3), making the spark chamber a true track detector comparable with such classical instrumoments as the cloud chamber.

In conclusion the authors express their deep gratitude to É. L. Andronikashvili for stimulating the present work and his constant interest in it.

Note added in proof (August 20, 1963). During the course of further work we were able to make the single and shower tracks identical by specially shaping the high-voltage supply pulse.

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ANOMALOUS BEHAVIOR OF SPECIFIC HEAT OF CRYSTALS WITH HEAVY IMPURITY ATOMS

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THE authors have previously[1] called attention to the fact that the oscillation spectrum (i.e., the spectral density of the mean square of the displacement) of an isolated heavy impurity atom with $m'/m \gg 1$ (m' and m are the masses of the impurity atoms and the ideal-lattice atom) has a sharply pronounced resonance character, being localized near the frequency

$$x_0 \equiv \omega_0^2/\omega_{max}^2 = 1/|\varepsilon|\langle x^{-1}\rangle. \tag{1}$$

Here $x = \omega_{\mathrm{max}}^2$, $\epsilon = 1 - m'/m$, $\langle\ \rangle$ denotes averaging over the phonon spectrum of the initial ideal lattice (matrix), and ω_{max} is the upper limit of this spectrum. It is precisely this circumstance that has made it possible to obtain a limiting expression for the probability of the Mössbauer effect on a heavy impurity nucleus (see also [2]). An analogous result was obtained later in [3,4].

It turns out that such localization of the spectral density of the oscillations brings about a noticeable increase (compared with the ideal lattice) in the specific heat at low temperatures, even at relatively low concentration of the heavy impurity atoms. Physically this is connected with the fact that for frequencies close to (1) the transition to the classical limit occurs much earlier than for the main part of the phonon spectrum of the matrix.

Let us consider a monatomic cubic matrix (a lattice of arbitrary symmetry can be considered analogously), in which part of the atoms is replaced by the relatively heavy impurity atoms ($m'/m > 1$), the concentration η of the latter being small ($\eta \ll 1$). Neglecting, as usual, the change in the force constants, we obtain for the specific heat of such a crystal (per particle) (see, for example, [5])

$$c(T) = c_0(T) + \Delta c(T),$$

$$\Delta c(T) = -3\eta \int_0^1 \frac{\partial z(x)}{\partial x} \varphi(x, T)\, dx, \qquad (2)$$

where

$$\varphi(x, T) = kx \left(\frac{\hbar\omega_{max}}{kT}\right)^2 \exp\left(\frac{\hbar\omega_{max}\sqrt{x}}{kT}\right)\left[\exp\left(\frac{\hbar\omega_{max}\sqrt{x}}{kT}\right) - 1\right]^{-2},$$

$$z(x) = -\frac{1}{\pi}\cot^{-1}\left[\frac{1}{\pi|\varepsilon| x g(x)}\left(1 + |\varepsilon| x \int_0^1 \frac{g(x')\,dx'}{x - x'}\right)\right],$$

$$-1 \leqslant z \leqslant 0. \qquad (3)$$

The remaining symbols are: $g(x)$ —distribution function of the squares of the matrix frequencies, normalized to unity, and $c_0(T)$ —specific heat of the matrix per atom.

Let us analyze the behavior of the functions $z(x)$ for $|\epsilon| \gg 1$. We consider a region $x \gg 1$ containing the point x_0 for which, in particular, the representation $g(x) = d\sqrt{x}$ (d = const) is valid. It is easy to see that $z(x)$ falls sharply in a very narrow interval $\sim 3x_0$ from the value $z(0) = 0$ to

$$z(3x_0) \approx -1 + \frac{d}{2|\varepsilon|^{1/2}}\left(\frac{3}{\langle x^{-1}\rangle}\right)^{3/2}.$$

The derivative $-\partial z/\partial x$ has in this interval a sharply pronounced δ-function form:

$$\frac{\partial z}{\partial x} \approx -\frac{1}{\pi}\,\frac{\pi d x^{3/2}/\langle x^{-1}\rangle}{(x - x_0)^2 + \pi^2 d^2 x^3 \langle x^{-1}\rangle^{-2}}\,\frac{1}{2}\left(3\,\frac{x_0}{x} - 1\right). \qquad (4)$$

$\partial z/\partial x$ vanishes when $x \approx 3x_0$ and reverses sign for larger x. Recognizing that

$$\int_0^1 \frac{\partial z(x)}{\partial x}\, dx = z(1) - z(0) = 0, \qquad \int_0^{3\lambda_0} \frac{\partial z(x)}{\partial x}\, dx \approx -1, \qquad (5)$$

we arrive at the relation

$$\int_{3x_0}^1 \frac{\partial z(x)}{\partial x}\, dx \approx 1.$$

It follows immediately that if T does not exceed several times $T_0 = \sqrt{x_0}\,\hbar\omega_{max}/k$ ($x_0 \ll 1$), then we can use (4) for $\partial z/\partial x$ in the calculation of the change in the specific heat (2). We present the results of the direct analysis.

1. Let $T \ll T_0$. Then only the frequencies $x \ll x_0$ are significant in (2), and we get

$$\Delta c(T)/c_0(T) = {}^3/_2\eta|\varepsilon|.$$

Thus, the relative change in the specific heat turns out to be $3/2\,|\epsilon|$ larger than the concentration of the impurity atoms.

2. Let $T \sim T_0$. Then, obviously

$$\Delta c(T) \approx 3\eta\varphi(x_0, T).$$

The function $\varphi(x_0, T)$, which is the specific heat of an oscillator of frequency $\sqrt{x_0}\,\hbar\omega_{max}$, is close to its classical value when $T > T_0$, so that

$$\Delta c(T) \approx 3\eta k, \qquad T \gg T_0.$$

If we use the standard expression for the low-temperature limit of the specific heat $c_0(T)$, then

$$\frac{\Delta c(T)}{c_0(T)} \approx \eta \frac{5}{4\pi^4}\left(\frac{\Theta}{T}\right)^3, \qquad T \gg T_0.$$

Then, in particular, putting approximately $\hbar\omega_{max} = k\Theta$, we get

$$\frac{\Delta c(T_0)}{c_0(T_0)} \approx \eta \frac{5}{4\pi^4}\langle x^{-1}\rangle^{3/2}|\varepsilon|^{3/2}.$$

(In the Debye model $\langle x^{-1}\rangle = 3$).

By way of an example let us consider a solid solution of tungsten in beryllium ($m' = 184$, $m = 9$, $\Theta = 1000$°K). We readily obtain

$$\frac{\Delta c(T)}{c_0(T)} \sim 30\eta, \quad T \ll T_0; \qquad \frac{\Delta c(T_0)}{c_0(T_0)} \sim 6\eta.$$

With further increase in T, we have $\Delta c(T) \to 0$. This can be readily verified by recognizing that $\varphi(x, t) \to k$ as the temperature increases and taking also the first equation of (5) into account.

We note in conclusion that anomalies will occur in the analogous case in the kinetic phenomena, too.

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NEW METHOD FOR THE DETERMINATION OF ANISOTROPY RELAXATION TIME AND MODULATION OF LIGHT IN A KERR CELL

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A light beam transmitted through an anisotropic material located in a Kerr cell (condenser, segment of wave guide, resonator) is modulated in frequency and amplitude [1] if a variable field is applied to the Kerr cell.

The use of the Kerr cell as a light modulator has been considered frequently, [1-5] especially in recent years, as a method of modulating coherent optical radiation. [3,6] In the present note we propose a new method for determining the relaxation time of the anisotropy based on measurements of the intensity of the components of the amplitude-modulation spectrum of the light transmitted through a Kerr cell.

Optimum amplitude modulation of the light is realized when the principal directions of the polarization device form an angle of 90° with each other and an angle of 45° with the electric field (another case of optimum amplitude modulation corresponds to parallel orientation of the principal directions of the polarizers and an angle of 45° with the field direction). The field of a light wave transmitted through the polarizer, Kerr cell, and analyzer is expressed as follows:

$$Y_a = Y_p \sin\left\{\frac{k}{2}\int_{-L/2}^{+L/2}(n_p - n_s)\,dx\right\}$$

$$\times \exp\left\{i\left[\omega t - \frac{k}{2}\int_{-L/2}^{+L/2}(n_p + n_s)\,dx \pm \frac{\pi}{2}\right]\right\}, \qquad (1)$$

$$n_p = n + \tfrac{2}{3}\lambda B E^2(t), \qquad n_s = n - \tfrac{1}{3}\lambda B E^2(t), \qquad (2)$$

where Y_p is the amplitude of the field leaving the polarizer; k and ω are the wave number and frequency of the light; L is the path length of the light in the electric field; n_p and n_s are the refractive indices for light with electric vector parallel and perpendicular to the electric field E; n is the refractive index in the absence of field; B is the Kerr constant and λ is the wavelength of the light.

If the frequency of the electric field $\Omega \gg 1/\tau$ (τ is the relaxation time of the anisotropy) the molecules cannot follow the field variations and Eq. (1) will not contain frequency-dependent components, but will only exhibit a constant (dc) component; the spectrum of the transmitted light will only contain the frequency of the incident light. However, if $\Omega\tau \ll 1$ the transmitted light will contain all the components of the modulation spectrum with maximum intensity. In the intermediate case $\Omega\tau \sim 1$ the strengths of the amplitude-modulation components will depend on Ω or, for fixed Ω, on τ.

In order to describe the effects quantitatively we assume that $y_i = (n_i - n)$ [where $(n_i - n)$ is either $n_p - n$ or $n_s - n$ at a definite point in the Kerr cell] and is given by the following equation:

$$\frac{dy_i}{dt} + \frac{1}{\tau_i} y_i = \frac{1}{\tau_i} b_i \lambda B E^2(t). \qquad (3)$$

Here, the subscript i = p or s, with

$$b_p = {}^2/_3, \qquad b_s = -{}^1/_3.$$

If $dy_i/dt = 0$, then $y_i = b_i B E^2(t)$ and we obtain Eq. (2); however, if the field E is switched on and off instantaneously, i.e., if E = 0 in Eq. (3) then $y_i = y_i^0 \exp(-t/\tau)$ and the artificial anisotropy produced by the field decays exponentially.

We assume for simplicity that E(t) does not contain a dc term and is expressed by the harmonic function $E(t) = E_0 \cos \Omega t$. Solving Eq. (3) for this case we have

$$y_i = \tfrac{1}{2} b_i B \lambda E_0^2 \{1 + [1 + (2\Omega t)^2 \tau_i^2]^{-1/2} \cos(2\Omega t + \varphi)\}. \qquad (4)$$

Here, φ is the phase shift between the electric field and the double refractor with $\tan\varphi = 2\Omega\tau_i$. The usual methods for determining τ_i are essentially different ways of determining φ.

To determine the quantity $\psi = \int_{-L/2}^{L/2} (n_i - n)\,dx$ we must integrate Eq. (4) in the direction of the light beam within the limits − L/2 and L/2 with the origin of coordinates taken at the center of the element. Account should also be taken of the fact that $t = nx/c$ in Eq. (4) while the phase of the