# GALVANOMAGNETIC PHENOMENA IN AN ALTERNATING ELECTROMAGNETIC FIELD

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A study is made of the low-frequency electromagnetic waves in a conducting gyrotropic anisotropic medium characterized by a static specific resistivity tensor $\rho_{ik}$. If gyrotropy predominates over anisotropy, i.e., if the diagonal components of the tensor $\rho_{ik}$ are smaller than the off-diagonal, weakly attenuated elliptically polarized waves propagate in the medium. In the opposite case, when the anisotropy is large, the waves are damped. The results are applied to a metal in a strong magnetic field. Weakly attenuated waves propagate in metals with closed carrier trajectories with unequal concentrations of electrons and holes. The reflection of an electromagnetic wave from a semi-infinite space and the resonance excitation of the weakly attenuated waves in a plate are considered.

GALVANOMAGNETIC phenomena in a constant strong magnetic field have been studied in two limiting cases, the passage of a constant current through a metal,[1,2] and, the high frequency properties of metals (cyclotron resonance, etc.).

There exists a range of frequencies in which, when describing the propagation of electromagnetic waves, we can use the static electronic conductivity tensor $\sigma_{ik}$ evaluated in [1,2] for infinite space. The applicability conditions for such an analysis are the following

$$\omega\tau \ll 1, \quad \omega \ll \Omega, \quad kl \ll 1, \quad kr \ll 1. \tag{1}$$

Here $\omega$ and $\mathbf{k}$ are the frequency and wave vector of the electromagnetic wave, $\Omega$ is the cyclotron frequency, r is the radius of a Larmor orbit, and $\tau$ and $l$ are the time and mean free path of the electrons.

## 1. INFINITE SPACE

We consider the propagation of an electromagnetic wave of frequency $\omega$ in a conducting medium possessing a tensor resistivity $\rho_{ik}$ ($\rho_{ik} = \sigma_{ik}^{-1}$), where

$$\rho_{ik} = s_{ik} + a_{ik}, \qquad i, k = 1, 2, 3, \tag{2}$$

$$s_{ik} = s_{ki}, \qquad a_{ik} = -a_{ki}. \tag{3}$$

Maxwell's equations can be written in the following form when the displacement current is neglected:

$$\text{rot}\,\mathbf{H} = (4\pi/c)\,\mathbf{j}; \quad \text{rot}\,\mathbf{E} = -c^{-1}\,\partial\mathbf{H}/\partial t, \quad E_i = \rho_{ik} j_k. \tag{4*}$$

*rot = curl

Let the electromagnetic wave propagate along axis 3. It is easily seen that only components $\rho_{\alpha\alpha'}$, where $\alpha, \alpha' = 1, 2$, enter Maxwell's equations. The symmetrical part of the two-dimensional tensor $\rho_{\alpha\alpha'}$ ($s_{\alpha\alpha'}$) can be diagonalized to define the axes 1 and 2. In this system of coordinates the tensor $\rho_{\alpha\alpha'}$ has the form

$$\hat{\rho}_{\alpha\alpha'} = \begin{pmatrix} \rho_1 & \rho_{12} \\ -\rho_{12} & \rho_2 \end{pmatrix}, \tag{5}$$

and (4) can be rewritten as:

$$\partial E_\pm/\partial x_3 = -(i\omega/c\beta_\pm)\,H_\pm, \quad \partial H_\pm/\partial x_3 = (4\pi\beta_\pm/c\rho_\pm)\,E_\pm, \tag{6}$$

$$E_\pm = E_1 + \beta_\pm^{-1}E_2, \quad H_\pm = H_1 - \beta_\pm H_2, \tag{7}$$

$$\rho_\pm = \rho_1 + \beta_\pm^{-1}\rho_{21}, \tag{8}$$

$$\beta_\pm = (\rho_1 - \rho_2)/2\rho_{12} \pm [-1 + (\rho_1 - \rho_2)^2/4\rho_{12}^2]^{1/2}. \tag{9}$$

As expected, two waves with different polarizations propagate in the medium. Putting E, H $\sim \exp\{-i(\omega t - kx_3)\}$, we obtain from the system (6) the dispersion equation relating the wave vector $\mathbf{k}$ with the frequency $\omega$ for each of the waves:

$$k^2 = 4\pi i\omega/c^2\rho; \tag{10}$$

We omit the index $\pm$.

To begin with we consider the case when $|\rho_1 - \rho_2|/2|\rho_{12}| < 1$. Here $\beta = a \pm bi$ (a, b are real numbers; $a^2 + b^2 = 1$). From equation (7) it is seen that each of the eigen waves is elliptically polarized. The equation of the corresponding ellipse is:

$$E_\mu^2/b_\mu^2 + E_\nu^2/b_\nu^2 = 1, \tag{11}$$

$$b_{\mu,\nu}^2 = E_0^2/2\,(1 \pm a), \qquad a = (\rho_1 - \rho_2)/2\rho_{12}, \tag{12}$$

where the amplitude $E_0$ is related to the energy density of the electromagnetic field W by the usual relation

$$W = E_0^2/4\pi. \tag{13}$$

The axes $\mu, \nu$ of the ellipse (11) are rotated with respect to the coordinate axes 1, 2, by an angle of $\pi/4$. The natural waves of different polarization differ only in the direction of rotation of the field vector.

Equations (8)—(10) show that the waves under consideration are in general strongly attenuated, and only when the strong inequality

$$|\rho_1 + \rho_2| / 2 |\rho_{12}| \ll 1 \tag{14}$$

is satisfied is one of the waves almost unattenuated.[1)]

For a real wave vector **k**, the real and imaginary parts of the frequency $\omega = \omega' + i\omega''$ are determined by the equalities

$$\omega'_{\pm} = \mp (c^2k^2/4\pi) [\rho_{12}^2 - (\rho_1 - \rho_2)^2/4]^{1/2},$$
$$\omega''_{\pm} = -c^2k^2 (\rho_1 + \rho_2)/16\pi. \tag{15}$$

The two signs for $\omega'$ in (15) correspond to the two directions of propagation of the waves. The waves then differ in the direction of rotation of the polarization vector. It is clear from (15) that when $|\rho_1 + \rho_2| \ll |\rho_{12}|$ the attenuation is small ($|\omega''| \ll |\omega'|$).

We give the dependence of the wave vector **k** on frequency only when condition (14) is satisfied:

$$k_{\pm}^2 \approx \mp \frac{4\pi\omega}{c^2\rho_{12}} \left(1 \mp i \frac{\rho_1 + \rho_2}{2\rho_{12}}\right), \quad \left|\frac{\rho_1 + \rho_2}{2\rho_{12}}\right| \ll 1. \tag{16}$$

The attenuation of one of the waves is associated with the dissipative terms of the tensor $\rho_{\alpha\beta}$ and is small over one wavelength, but the attenuation of the other is not associated with dissipation.

When $|\rho_1 + \rho_2| \ll \rho_{12}$ the gyrotropic-anisotropic medium can be considered as a medium with a real dielectric constant [$\epsilon = \pm 4\pi/\omega\rho_{12}$; see (8)—(10)]. For the unattenuated wave ($\epsilon > 0$) the phase velocity is $v_\phi = c(\omega\rho_{12}/4\pi)^{1/2}$, and the group velocity is $v_g = 2v_\phi$.

Now let $|\rho_1 - \rho_2|/2|\rho_{12}| > 1$. Then $\beta = a \pm |b|$ is a real quantity ($a^2 - b^2 = 1$), and, consequently, the natural waves [the solutions of (6)] are linearly polarized. The angle $\psi$ between the directions of polarization is given by the quantity

$$\mathrm{tg}\,\psi = b, \quad |b| = \sqrt{(\rho_1 - \rho_2)^2/4\rho_{12}^2 - 1}. \tag{17}*$$

We note that when $(\rho_1 - \rho_2)^2/4\rho_{12}^2 \gg 1$ (strong anisotropy) the angle $\psi$ is close to $\pi/2$; when $(\rho_1 - \rho_2)^2/4\rho_{12}^2 \ll 1$ the angle $\psi$ is close to zero.

## 2. SEMI-INFINITE SPACE

To consider the reflection of an electromagnetic wave from a semi-infinite space, it is convenient to use the concept of surface impedance, which in this case is conveniently defined as follows: $\zeta = \beta_\pm (E/H)_0$ (the index zero signifies that the quantities E and H are evaluated when $x_3 = 0$). According to equation (6) we obtain the usual expression for $\zeta_\pm$:

$$\zeta_\pm = \sqrt{\omega\rho_\pm i/4\pi}, \quad \mathrm{Re}\,\zeta_\pm < 0. \tag{18}$$

Using the definition of impedance and also the first equation of system (6), we obtain an expression for the complex reflection coefficient R:

$$R_\pm = -(1 + \zeta_\pm)/(1 - \zeta_\pm). \tag{19}$$

The choice of sign of the real part of $\zeta_\pm$ can be made if we require that the modulus of the reflection coefficient R be smaller than unity ($|R_\pm| < 1$). Under these conditions, when an unattenuated wave is propagating in the medium [i.e., when condition (14) is satisfied] the surface impedance is

$$\zeta_\pm \approx \sqrt{\pm\omega\rho_{12}/4\pi}, \quad \mathrm{Re}\,\zeta_\pm < 0, \tag{20}$$

i.e., $\zeta_+$ is real and $\zeta_-$ imaginary. Therefore

$$|R_+| \approx |\sqrt{4\pi} - \sqrt{\omega\rho_{12}}| / |\sqrt{4\pi} + \sqrt{\omega\rho_{12}}| < 1, \ |R_-| \approx 1. \tag{21}$$

In the case of strong anisotropy ($|\rho_1 - \rho_2| > 2\rho_{12}$), when an arbitrary plane polarized wave falls on the surface of the metal, the reflected wave is in general elliptically polarized. If the incident wave is polarized in one of the principal directions of which we spoke in the preceding section, the polarization direction of the reflected wave is unchanged. We recall that the angle between these directions is determined by formula (17).

## 3. THE EXCITATION OF ELECTROMAGNETIC WAVES IN A PLATE

Maxwell's equations in a plate of thickness 2d ($-d < z < d$) admits of two types of solution: 1) symmetrical electric field, antisymmetrical magnetic field; 2) symmetrical magnetic field,

[1)]A general analysis (taking into account temporal and spatial dispersion) of the unattenuated waves in metals in a strong magnetic field has been given by Kaner and Skobov.[3] The work of Konstantinov and Perel',[4] in which the propagation of an electromagnetic wave along a magnetic field is treated with spatial dispersion taken into account, should be noted.

*tg = tan

antisymmetrical electric field. We denote a solution of the first type by the index s and of the second type by a. If it is assumed that the electric and magnetic field outside the plate are zero, then the plate is a resonator, and, consequently, the wave vector **k** assumes discrete values.

Thus, the proper solutions in the cases considered have the following form:

1) symmetrical solution ($k_n = (n + 1/2)\pi/d$)

$$E_n^{(s)} = E_n^{(s)}(0)\cos k_n z,\quad H_n^{(s)} = E_n^{(s)}(0)\,(\beta c k_n/i\omega)\sin k_n z; \tag{22}$$

2) antisymmetrical solution ($k_n = n\pi/d$)

$$E_n^{(a)} = E_n^{(a)}(0)\sin k_n z,\quad H_n^{(a)} = E_n^{(a)}(0)\,(i\beta\, c k_n/\omega)\cos k_n z. \tag{23}$$

The choice of such solutions is associated with the assumption concerning the existence of surface currents (in the first case antisymmetrical, in the second case symmetrical).

The frequency appearing in (22) and (23) is determined by the dispersion equation (10), from which it is clear that when condition (14) is satisfied the vibration with $\beta \approx +i$ [see formula (9)] is weakly attenuated, and

$$\mathrm{Re}\,\omega \approx c^2k^2\rho_{12}/4\pi,\quad \mathrm{Im}\,\omega \approx -c^2k^2(\rho_1+\rho_2)/16\pi. \tag{24}$$

We now consider the question of the excitation of the weakly decaying vibrations in the plate by a field of frequency $\omega$. If the plate is placed in a symmetrical electric field, the field in the plate is described by:

$$E_s(z) = E_s(d)\cos kz/\cos kd,$$
$$H_s(z) = E_s(d)\,(\beta ck/i\omega)\sin kz/\cos kd \tag{25}$$

[$k^2$ from (10)]. In the case of an antisymmetrical field

$$E_a(z) = E_a(d)\sin kz/\sin kd,$$
$$H(z) = E_a(d)\,(i\beta ck/\omega)\cos kz/\sin kd. \tag{26}$$

By placing the plate at an antinode of the electric field in a resonator we establish the symmetrical case, and by placing at an antinode of the magnetic field, the antisymmetrical one. Usually the quantities measured in an experiment can be expressed in terms of the surface impedance of the plate.

According to (25) and (26) we have

$$\zeta_s = (i\omega/ck)\,\mathrm{ctg}\,kd,\qquad \zeta_a = -(i\omega/ck)\,\mathrm{tg}\,kd. \tag{27*}$$

The formulae obtained show that the surface impedance has a resonant character: neglecting attenuation, $\zeta_s$ and $\zeta_a$ tend to infinity when

$$k = \left(n+\tfrac{1}{2}\right)\pi/d\quad(\zeta_s)\quad\text{and}\quad k = n\pi/d\quad(\zeta_a).$$

*ctg = cot

To describe the resonance of the excitation we introduce an expression for the ratio $\Gamma$ of the power absorbed in the plate to the energy density of the electromagnetic wave:

$$\Gamma = \frac{2c}{d}\,\frac{\Delta\omega_n}{(\omega-\omega_n)^2+\Delta\omega_n^2}, \tag{28}$$

where, for the symmetrical electric field,

$$\omega_n = (c^2\rho_{12}/4\pi)\left[\left(n+\tfrac{1}{2}\right)\pi/d\right]^2,$$
$$\Delta\omega_n = (c^2/4\pi)(\rho_1+\rho_2)\left[\left(n+\tfrac{1}{2}\right)\pi/d\right]^2, \tag{29}$$

and for the antisymmetrical

$$\omega_n = (c^2\rho_{12}/4\pi)(n\pi/d)^2,\quad \Delta\omega_n = (c^2/4\pi)(\rho_1+\rho_2)(n\pi/d)^2. \tag{30}$$

It is obvious from the formulae obtained that the height of the resonance peak $\Gamma_n = \Gamma(\omega=\omega_n)$ decreases with increasing harmonic number as $n^2$. We note that a measurement of the resonance frequency, the width of the resonance curve, and the polarization of the resonating wave, makes it possible to determine in a single experiment all the components of the tensor $\rho_{\alpha\beta}$ [see (28)–(30) and also (12)].

## 4. METAL IN A STRONG MAGNETIC FIELD

The results obtained in the preceding sections refer most naturally to a metal placed in a strong magnetic field. In the present section we consider the conditions placed on the magnetic field and the propagation direction for various metals.

If the electron trajectories on the Fermi surface are closed and the number of electrons $n_1$ is not equal to the number of holes $n_2$, then, in the system of coordinates associated with the magnetic field, the tensor $\rho_{ik}$ (i, k = x, y, z) has the following form (the z axis is directed along the magnetic field):

$$\hat{\rho}_{ik} = \begin{pmatrix} b_{xx} & \rho_{xy} & b_{xz} \\ \rho_{yx} & b_{yy} & b_{yz} \\ b_{zx} & b_{zy} & b_{zz} \end{pmatrix}, \tag{31}$$

$$\rho_{xy} = H/(n_1-n_2)ec + b_{xy},$$
$$\rho_{yx} = -H/(n_1-n_2)ec + b_{yx}, \tag{32}$$

and the elements of the matrix $b_{ik}$ are of order of the resistance in the absence of a magnetic field, where $|b_{ik}| \ll H/(n_1-n_2)ec|$ (this is a consequence of the relation $r \ll l$).

Choosing the y axis in the k, z plane and diagonalizing the "plane" tensor $\rho_{\alpha\beta}$ ($\alpha, \beta = 1, 2$), we obtain

$$\rho_{\alpha\beta} = \begin{pmatrix} c_{11} & \dfrac{H}{(n_1-n_2)ec}\cos\vartheta \\ -\dfrac{H}{(n_1-n_2)ec}\cos\vartheta & c_{22} \end{pmatrix}, \tag{33}$$

where $\vartheta$ is the angle between the wave vector and the z axis and, respectively,

$$c_{11}, c_{22} = \frac{1}{2}\rho_d \pm \frac{1}{2}[(b_{xy} + b_{yx})\cos\vartheta + (b_{xz} + b_{zx})\sin\vartheta],$$

$$\rho_d = \rho_{xx} + \rho_{yy}\cos^2\vartheta + \rho_{zz}\sin^2\vartheta + (\rho_{yz} + \rho_{zy})\sin\vartheta\cos\vartheta.$$

In the components $\rho_{21}$, $\rho_{12}$ subsequent terms of order $c_{11}$ and $c_{22}$ are neglected.

From this and from (10), (8), and (9), it is seen that in the case considered the dispersion equation for the unattenuated solutions is

$$\mathrm{Re}\,\omega \approx (ck^2/4\pi e)\,[H/(n_1 - n_2)]\cos\theta, \quad (34)$$

and the small attenuation is determined by the following relation:

$$\mathrm{Im}\,\omega \approx -c^2k^2\rho_d/16\pi. \quad (35)$$

We note that when the angle $\vartheta$ is close to $\pi/2$ all components of the tensor $\rho_{\alpha\beta}$ become of the same order of magnitude, and, consequently, the vibration is strongly attenuated. The unattenuated vibration can propagate if the angle between the wave vector and the magnetic field satisfies the following condition:

$$\mathrm{tg}\,\vartheta < r/l. \quad (36)$$

Since the resonance frequency is inversely proportional to the square of the thickness of the plate [see (29), (30)], to satisfy condition (1) it is necessary that d be not too small. The strongest of the inequalities, as estimates indicate, has the form

$$d \gg \sqrt{\Omega\tau}\cdot 10^{-5}\,\mathrm{cm}.$$

Recently several experimental papers have appeared in which unattenuated waves in metals in a strong magnetic field were observed.[5] These waves were called helicoidal. When open trajectories play an important part, or there is volume compensation ($n_1 = n_2$) the diagonal elements of the matrix are always significantly greater than the off-diagonal.[1,2] Therefore the propagation of unattenuated vibrations is impossible when conditions (1) are satisfied. In metals of such a type effects occur associated with the predominance of anisotropy over gyrotropy [see formula (17) and the end of Sec. 2]. This circumstance will be displayed with especial clarity in metals with open surfaces, in which for certain directions of the magnetic field the elements $\rho_1$ and $\rho_2$ have different asymptotic dependences on the magnetic field (for example, $\rho_1 \sim \rho_0(H/H_0)^2$, $\rho_2 \sim \rho_0$; $H_0$ is the magnetic field for which $r = l$; $\rho_0$ is the resistance when $H = 0$).

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