THEORY OF HIGH FREQUENCY HEATING OF PLASMA

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Heating of a plasma by the high frequency field of a traveling wave with a phase velocity of the order of the mean thermal velocity of the ions is considered. An expression is obtained for the energy acquired by the ion per unit time. In the case of a strongly non-isothermal plasma, the heating of the ion gas becomes extremely effective under conditions of acoustic resonance, when the phase velocity of the wave is close to the velocity of sound.

1. In the method proposed by Berger et al [1] for the high frequency heating of a plasma, oscillations of the axial magnetic field H_z lead to the appearance of an alternating azimuthal electric field E_φ in a plasma cylinder located in a strong longitudinal magnetic field H_0. As a consequence of the drift of the particles of the plasma in the crossed fields E_φ and H_0, drift radial oscillations of the plasma develop, the phase of which changes along the axis of the cylinder (it is assumed that the frequency of the oscillations is much less than the ion cyclotron frequency $\omega_i = eH_0/Mc$). This leads to the appearance of density oscillations, i.e., an acoustic wave is produced in the plasma (see also [2]). Since the plasma oscillations are excited by the external magnetic field, this method of plasma heating has received the name of "magnetic pumping."

The flow of energy of acoustic waves in a plasma that are generated by a current carrying coil mounted on the plasma cylinder was found by Berger et al. [1] The energy of the acoustic waves leads to heating as the result of various dissipative processes (viscosity, thermal and electrical conduction), brought about by collisions. The consideration given in [1] refers, strictly speaking, only to the case of low frequencies, where $\omega\tau \ll 1$ (τ is the relaxation time of the plasma), since the propagation of ordinary sound vibrations is brought about by "near" collisions.

On the other hand, as is well known (see [3,4]), in a strongly non-isothermal plasma, where the electron gas temperature T_e is much higher than the temperature of the ion gas T_i, propagation of a collision-free ($\omega\tau \gg 1$) sound wave with the dispersion law $\omega = k_{\parallel}V_s$ becomes possible. Here, $k_{\parallel}$ is the wave vector and $V_s = \sqrt{T_e/M}$ is the sound velocity.

In the presence of a strong magnetic field, the spectrum of the acoustic vibrations has the form $\omega = k_{\parallel}V_s$, where $k_{\parallel}$ is the projection of the wave vector in the direction of the external magnetic field. [5,6] The damping decrement of these waves, due to the reverse Vavilov-Cerenkov effect (Landau mechanism), is smaller than the frequency by a factor $\sqrt{M/m}$. [5-7] Therefore, in a collision-free plasma, "magnetic pumping" is also possible, where, as has been shown by Shapiro, [2] resonance coupling can exist between the external circuit, which generates the oscillations, and the plasma cylinder, if the current in the external circuit has the form of a traveling wave with its phase velocity $V_{ph} = \omega/k_{\parallel}$ close to the velocity of "sound" V_s. The flow of energy in the plasma in the case of resonance increases by the factor M/m in comparison with the nonresonant case investigated in [1].

We note that resonance coupling between the external circuit and the plasma can evidently exist even in the case of ordinary acoustic vibrations for $V_{ph} \approx \sqrt{\kappa T/M}$ (κ is the ratio of specific heats), while the energy flow increases by the factor $(\omega/\gamma)^2$. Here γ is the damping decrement of the acoustic vibrations, brought about by the viscosity, thermal and electrical conductivity of the plasma. The problem of energy dissipation was not considered in [1,2].

In the present work, the absorption of the energy of high-frequency oscillations is studied. This absorption is produced by the external currents, as a consequence of the damping of the waves in the ion and electron gas. This damping is determined by the Landau mechanism. Ions having the velocity v_z, which is close to the phase velocity of the wave, will interact strongly with the field of the wave; as a result, the energy of the field will be pumped into the ion gas. If the phase velocity of the wave is of the order of the mean thermal velocity of the ions, then the number of resonant ions will be large, while the damping of the waves will be very strong (as is well known, free vibrations cannot generally be propagated in an unbounded plasma in this case, since the damping decrement is equal to the value of the frequency of the oscillations[8]). The heating achieved by such a method is such that the energy is transferred directly to the ion component of the plasma. The energy absorbed by the electrons of the plasma is in this case of the order of $\sqrt{m/M}$ of the energy absorbed by the ions.

If the plasma is strongly non-isothermal ($T_e \gg T_i$), then the energy flow in the plasma can be sharply increased in the resonant case ($V_{ph} \approx V_s$); here, the energy of the waves is absorbed primarily by the electrons of the plasma (the ion damping is exponentially small). However, if the ratio T_e/T_i is not too large ($T_e < 10T_i$), the ion damping at resonance still exceeds the electron damping, although resonance between the external circuit and the plasma may have already set in. In this case, the energy flow in the plasma can increase by an order of magnitude, or even by two orders of magnitude, "heating" both ions and electrons.

We note that collision-free heating of the plasma by the solenoidal electric field E_φ, which is created by a coil of finite dimensions wound around the plasma cylinder, has already been studied in [1] (the so-called heating due to the finite time of flight. Here the charged particle, passing along H_0 through a region with a slowly changing alternating magnetic field, was considered in [1] as a magnetic dipole, on which the force $F_z = -\mu\,\partial H/\partial z$ was acting ($\mu = Mv_\perp^2/2H_0$ is the magnetic moment of the particle). The energy flow carried by the particles is maximum for $\omega L \sim v_i$ (L is the dimension of the coil). However, in such a single-particle analysis, the quasineutrality of the plasma and the possibility of the generation of acoustic waves were not considered.

2. We consider the vibrations of a plasma excited by external azimuthal currents which flow over the cylindrical surface with radius a:

$$j_\varphi = j_0 \cos(k_\parallel z - \omega t)\,\delta(r - a). \tag{1}$$

The electric field in the plasma is determined from the equation

$$\text{rot rot } \mathbf{E} = \omega^2 c^{-2}(\mathbf{E} + 4\pi i\omega \mathbf{j}). \qquad (2)^*$$

*rot = curl.

If the magnetic pressure in the plasma, $H_0^2/8\pi$, is significantly larger than the gas-kinetic pressure of the plasma, $p = n_0(T_e + T_i)$, then the wavelength excited in the plasma, $\lambda \sim v_i/\omega$, will, for $\omega \ll \omega_i$, be much larger than the Larmor radius of the particles, having a velocity of the order of the mean thermal velocity $v_i = \sqrt{T_i/M}$. In this case, the electrical current density j in the plasma is not difficult to find, by using the well-known approximate expressions for the dielectric constant tensor $\epsilon_{ik}(\mathbf{k}, \omega)$. We have for the Fourier component of the current density

$$4\pi i\omega^{-1} j_i(\mathbf{k}, \omega) = [\varepsilon_{ik}(\mathbf{k}, \omega) - \delta_{ik}] E_k(\mathbf{k}, \omega);$$
$$\varepsilon_{ik} = a_1\delta_{ik} + a_2 h_i h_k + a_3(\varkappa_i [\varkappa\mathbf{h}]_k - \varkappa_k [\varkappa\mathbf{h}]_i) + a_4 [\varkappa\mathbf{h}]_i [\varkappa\mathbf{h}]_k \qquad (3)^*$$

The coefficients $a_1, a_2, \ldots$ have the form

$$a_1 = \frac{c^2}{V_A^2},\ a_2 = -a_1 + \frac{\Omega_i^2}{k_\parallel^2 v_i^2}[1 + \frac{T_i}{T_e} + i\sqrt{\pi} z_i w(z_i) + \frac{T_i}{T_e} i\sqrt{\pi} z_e w(z_e)],$$
$$a_3 = k^2\tilde{a}_3 = -\frac{\sqrt{\pi}\,\Omega_i^2 k^2}{\omega\omega_i k_\parallel^2}[z_i w(z_i) - z_e w(z_e)],$$
$$a_4 = k^2\tilde{a}_4 = \frac{2v_i^2 k^2 c^2}{V_A^2\omega^2}[i\sqrt{\pi} z_i w(z_i) + \frac{T_e}{T_i} i\sqrt{\pi} z_e w(z_e)]. \tag{4}$$

Here

$$\varkappa = \mathbf{k}/k,\ \mathbf{h} = \mathbf{H}_0/H_0,\quad V_A = H_0/\sqrt{4\pi n_0 M},$$
$$\Omega_i = \sqrt{4\pi n_0 e^2/M},$$
$$w(z) = e^{-z^2}\left(1 + \frac{2i}{\sqrt{\pi}}\int_0^z e^{t^2}dt\right),\ z_i = \frac{\omega}{\sqrt{2}\,k v_i},$$
$$z_e = \frac{\omega}{\sqrt{2}\,k v_e},\quad v_e = \sqrt{\frac{T_e}{m}}.$$

From (3) we find that

$$4\pi i\omega^{-1}\mathbf{j} = a_1\mathbf{E} + a_2\mathbf{E}_\parallel + ik_\parallel\tilde{a}_3(\text{rot } \mathbf{E}_\parallel + \mathbf{h}(\mathbf{h}\text{ rot } \mathbf{E})) - \tilde{a}_4[\mathbf{h}, \nabla(\mathbf{h}\text{ rot } \mathbf{E})],$$

where $\mathbf{E}_\parallel = \mathbf{h}(\mathbf{h}\cdot\mathbf{E})$.

Substituting (5) in (2), and taking it into account that $|a_2| \gg a_1, |a_3|, |a_4|$, we obtain the result that

$$E_z = -\frac{ik_\parallel\tilde{a}_3}{a_1 + a_2}\left(E_\varphi' + \frac{1}{r}E_\varphi\right),$$
$$E_r = -\frac{k_\parallel^2\tilde{a}_3}{a_1 + a_2}E_\varphi + \frac{i\omega^2\tilde{a}_4}{k_\parallel c^2}\left(E_\varphi' + \frac{1}{r}E_\varphi\right).$$
$$E_\varphi = AI_1(k_\perp r)\qquad (r < a),$$
$$E_\varphi = A\frac{I_1(k_\perp a)}{K_1(k_\perp a)}K_1(k_\perp r)\qquad (r > a);$$
$$A = \frac{4\pi i j_0\omega a}{c^2(1 - \zeta)}K_1(k_\perp a), \tag{6}$$

*$[\kappa\mathbf{h}] = \kappa \times \mathbf{h}$.

where I_n is the modified Bessel function and K_n is the Macdonald function. The intensity of the alternating magnetic field is determined from the equation $\mathbf{H} = (c/i\omega)$ curl $\mathbf{E}$. The value of $k_\perp$ is

$$k_\perp = k_\parallel\,(1-\zeta)^{-1/2},\ \ \zeta = \frac{\omega^2}{c^2}\tilde{a}_4 + k_\parallel^2\,\omega^2\tilde{a}_3^2\,/\,c^2\,(a_1+a_2). \quad (7)$$

The time average of the energy flow in the plasma per unit length, Q, is

$$Q = -2\pi\int_0^\infty \overline{E_\varphi j_\varphi}\,r\,dr = \frac{4\pi^2 j_0^2\omega}{k_\parallel^2 c^2}\,\mathrm{Im}\,[z^2 K_1(z)\,I_1(z)], \quad (8)$$

$z = k_\perp a$. Since the electromagnetic field penetrates a distance $\sim k_\parallel^{-1}$ in the plasma, the mean energy acquired by a single particle located in a cylinder with radius $\sim k_\parallel^{-1}$ will be equal in order of magnitude to $dW/dt = Qk_\parallel^2/\pi n_0$.

3. One can simplify the expression (8) by taking into account the fact that in order of magnitude,

$$|\zeta| \sim 8\pi n_0 T_i/H_0^2 \ll 1.$$

In this case,

$$Q = 4\pi^2 j_0^2\omega a^2 c^{-2}\,8\pi n_0 T_i H_0^{-2}\,\psi(k_\parallel a)\,f(z_i); \quad (9)$$

$$\psi(x) = K_1(x)\,I_1(x) + {}^1/_2\,x\,[K_1(x)\,I_1'(x) + K_1'(x)\,I_1(x)], \quad (10)$$

$$f(z_i) = \sqrt{\pi}\,z_i\left\{e^{-z_i^2} + \xi + \frac{2v\,(e^{-z_i^2} - \theta\xi)(1+\theta-v) + (e^{-z_i^2} + \theta^2\xi)\,[v^2 - \pi z_i^2\,(e^{-z_i^2} - \theta\xi)^2]}{2\,[(1+\theta-v)^2 + \pi z_i^2\,(e^{-z_i^2} + \theta^2\xi)^2]}\right\},$$

$$\theta = T_i/T_e;\ \ \xi = \sqrt{mT_e/MT_i},\ \ v = 2z_i e^{-z_i^2}\int_0^{z_i} e^{t^2}dt. \quad (11)$$

If $T_e \lesssim T_i$, then $f(z_i)$ increases in proportion to z_i for $z_i \ll 1$, reaches a maximum at $z_i \sim 1$, and then falls off. In the region $z_i \gg 1$, the ion damping falls off as $\exp(-z_i^2)$ and one can neglect the value of $\exp(-z_i^2)$ as soon as $z_i \sim 3$, in comparison with the value of ξ, which is determined by the electron damping. The expression (11) for $f(z_i)$ is obtained for $z_e \ll 1$. If $z_i \gg 1$, then $f(z_i) = (\sqrt{\pi}\,T_e/2T_i)z_e e^{-z_e^2}$, i.e., for $z_e \ll 1, \ll z_i$, the value of f increases as z_e, for $z_e = 1/\sqrt{2}$, it approaches a maximum, and decreases thereafter.

A graph of the function $f(z_i)$ is shown in Fig. 1 for $T_e = T_i$. For $0.1T_i < T_e \le T_i$ and $z_i < 2$, the graph of $f(z_i)$ differs but little from the graph shown in Fig. 1.

Thus, when the number of resonant ions is large ($z_i \sim V_{ph}/v \sim 1$), then the order of magnitude is

$$dW/dt \sim (\tilde{H}^2/H_0^2)\,\omega T_i,$$

where $\tilde{H} = 4\pi j_0/c$ is the amplitude of the alternating magnetic field.

It is interesting to compare this expression with the energy acquired by one particle under optimal conditions of gyro-relaxation heating: $dW/dt \sim \tilde{H}^2 T_i/H_0^2\tau$. It is then seen that the intensity of the absorption resulting from the Landau mechanism is $\omega\tau \gg 1$ times larger than from the gyro-relaxation heating.

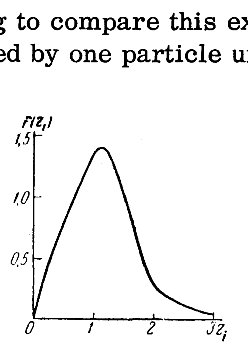

FIG. 1

The use of strong fields for heating the plasma leads to the formation of a "plateau" in the distribution function of the ions in the region $v_z = V_{ph}$ and, consequently, to a decrease in the absorption coefficient.[9] However, this does not generally mean that the absolute value of the absorbed power will decrease in strong fields. As the damping of Langmuir oscillations, studied without the quasi-linear approximation, shows, the increase of damping of waves with large amplitude E_0 is proportional to $E_0^{-3/2}$.[10,11] Since the energy density $\sim E_0^2$, the absorption of plasma energy of waves with large amplitude is proportional to $\sqrt{E_0}$, i.e., the absolute value of the absorption nevertheless increases upon an increase in the amplitude of the wave, although much more slowly than in the linear approximation. Collisions will make the distribution Maxwellian.

Let us estimate the critical field $\tilde{H} = H_{cr}$, above which the distortion of the distribution function in the region $v_z = V_{ph}$ will be appreciable. The number of resonant particles which extract energy from the field is smaller than the total number of particles by the factor $v_i/\Delta v \sim [k_\parallel T_i/\mu\,|\partial H_z/\partial z|]^{1/2}$. Under optimal conditions ($z_i \sim 1$, $k_\parallel a \sim 1$), $v_i/\Delta v \sim \sqrt{H_0/\tilde{H}}$. The energy which the resonant particles acquire in the relaxation time τ for $\tilde{H} \sim H_{cr}$ is of the order of T_i: $\tau\,dW/dt \sim T_i\Delta v/v_i$. We then find that

$$H_{cr} = H_0(\omega\tau)^{-2/3}.$$

This energy is distributed among the remaining particles of the plasma in a time $\sim\tau$. Therefore, for $\tilde{H} = H_{cr}$, the mean energy acquired by a single particle of the plasma will be equal, in order of magnitude, to

$$dW/dt \sim T_i/\tau\,(\omega\tau)^{1/3}.$$

For example, for $n_0 \sim 10^{14}\ cm^{-3}$, $T_i = 100$ eV, $H_0 = 10^4$ Oe ($\omega_i \sim 10^8$/sec) and $\omega = 10^7$/sec, we have $\tau \sim T_i^{3/2}/n_0 \sim 10^{-5}$ sec, $H_{cr} \sim 100$ Oe, $E_{\varphi\,cr} \sim 10$ V/cm and $dW/dt \sim 1$ MeV/sec.

Inasmuch as $\tau \sim T_i^{3/2}$, $\omega \sim v_i \sim \sqrt{T_i}$, $W \sim T_i$, the intensity of the heating, under optimal conditions, falls off as $T_i^{-7/6}$ $dT_i/dt \sim (T_0/T_i)^{7/6}T_0/\tau_0 \times (\omega_0\tau_0)^{1/3}$, i.e.,

$$T_i = T_0\,[1 + t\alpha/\tau_0\,(\omega\tau_0)^{1/3}]^{6/13},$$

where T_0 and τ_0 are the values of T_i and τ as the start of heating, for $t = 0$ and $\alpha \sim 1$.

It is obvious that, to maintain the condition $z_i \sim 1$, it is necessary either to modulate the frequency or to transfer to an operating condition with a different frequency, dependent on the increase in the temperature T_i (as is seen from Fig. 1, the damping is sufficiently great for $0.6 \le z_i \le 1.8$, so that it is not necessary to maintain the condition $z_i = 1.2$, when the absorption is maximum).

The heating of the plasma can be intensified by using certain waves with phase velocities differing by (2—3)Δv, where $\Delta v \sim v_i\sqrt{\tilde{H}/H_0}$. Obviously, the maximum number of such waves $\sim \sqrt{H_0/\tilde{H}}$. In this case, under the best conditions,

$$dW/dt \sim T_i/\tau,$$

i.e.,

$$T_i = T_0\,(1 + \alpha' t/\tau_0)^{2/3},$$

where $\alpha' \sim 1$.

4. In the case of a non-isothermal plasma $T_e = 10\,T_i$, the graph of the function $f(z_i)$ is shown in Fig. 2. The presence of the maximum at $z_i = 2.63$ is connected with the resonance $V_{ph} \sim V_S$. For $z_i \le 2.7$, the damping of the wave in the ion gas is larger than the electron damping; for $z_i \approx 2.7$, the electron and ion damping are comparable; for $z_i \ge 2.7$, the electron damping predominates. As is seen from Fig. 2, the energy flow in the plasma at $T_e = 10\,T_i$ increases by more than an order of magnitude in comparison with the case $T_e \le T_i$.

For still more non-isothermal conditions, the

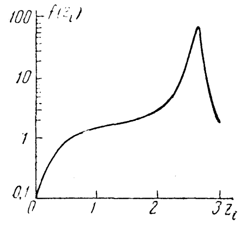

FIG. 2

resonance $V_{ph} \approx V_S$ takes place for $z_i \gg 1$. Taking it into account that $v \approx 1 - \frac{1}{2}z_i^2$ for $z_i \gg 1$, we get the following expression for $f(z_i)$:

$$f = \frac{T_e}{T_i}\frac{\sqrt{\pi}z_e}{2\,[(1 - k_{\parallel}^2V_s^2/\omega^2)^2 + \pi z_e^2]}, \quad z_e \ll 1. \qquad (12)$$

It then follows that the maximum absorption occurs for $\omega \approx k_{\parallel}V_S$. However, this case is less suitable for plasma heating, since only the electrons are heated.

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196