TURBULENT DIFFUSION OF A RAREFIED PLASMA IN A STRONG MAGNETIC FIELD

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The turbulent diffusion which develops as a result of drift instability in a collisionless plasma in a strong magnetic field is considered. It is shown that in the presence of oscillations nonlinear damping, which strongly diminishes the amplitude of the stationary oscillations, occurs in addition to Landau linear damping. The oscillation spectrum and diffusion turbulence coefficient are determined approximately.

1. INTRODUCTION

IT has been shown by the author and Timofeev [1] that an inhomogeneous rarefied plasma with Maxwellian particle velocity distribution is unstable against drift waves [2] with a transverse wavelength of the order of the average ion Larmor radius, and that for a plasma of negligibly small pressure the growth increment of the small oscillations can be of the order of the frequency. As shown in [3], an increase in the plasma pressure leads to a decrease in the increment, so that when $\beta_0 \equiv 8\pi p/H^2 \gg m_e/m_i$, where p is the plasma pressure and m_i and m_e the ion and electron masses, the increment is much smaller than the oscillation frequency in the most essential region of the wave numbers. It can then be expected that the turbulent motion of the plasma developing as a result of the instability will comprise a set of relatively weak interacting oscillations, that is, the turbulence will be weak.

A general method for considering weakly turbulent states, using expansion in powers of the small ratio of the increment to the frequency, has been developed by the author and Petviashvili [4]. In the present paper we investigate by means of this method the character of the interaction of steady-state oscillations of an inhomogeneous rarefied plasma with a nonlinear Landau damping and wave decay. For simplicity we confine ourselves to the case of an isothermal current-free plasma in a homogeneous magnetic field. We thus disregard the instability due to the temperature gradient [2,3] and the longitudinal current [5]. The question of the influence of the magnetic-field inhomogeneity, or, more readily, the crossing of the force lines, will be discussed qualitatively.

2. FUNDAMENTAL RELATIONS

We assume that an inhomogeneous isothermal ($T_i = T_e = T = \text{const}$) plasma with a density gradient along the x axis: $dn/dx \equiv \kappa n$, is in a strong magnetic field H directed along the z axis. We assume that the plasma pressure is much smaller than the magnetic field pressure, namely, we assume that

$$m_e/m_i \ll \beta_0 \ll (m_e/m_i)^{1/3}. \tag{1}$$

The magnetic field can be regarded as homogeneous here.

We further assume that the magnetic field is so strong that the average Larmor radius of the ions is $\rho \ll 1/\kappa$. The wavelength of the oscillations of interest to us, which is of the order of ρ, will then be considerably smaller than the characteristic dimension κ^{-1} of the inhomogeneity, so that the oscillations can be described in the quasiclassical approximation, that is, we can assume that the dependence of the oscillating quantities on the spatial coordinate **r** is in the form $\exp(i\mathbf{k}\cdot\mathbf{r})$.

The instability of a low-pressure plasma is most affected only by perturbations of the so-called convective type, in which the longitudinal magnetic field remains constant. This means that the transverse component of the electric field can be regarded as potential, that is, $\mathbf{E}_\perp = -\nabla_\perp\varphi$. Along with φ, it is convenient to introduce a quantity ψ, defined by the relation $E_z = -\partial\psi/\partial z$. Changing to Fourier transforms, we have $\mathbf{E}_{\mathbf{k}\omega} = -i\mathbf{a}\Phi_{\mathbf{k}\omega}$ where $\Phi_{\mathbf{k}\omega}$ is the Fourier transform of the potential, and $\mathbf{a} = \mathbf{k}_\perp + \mathbf{h}k_z\alpha_0$ with $\alpha_0 = \Psi_{\mathbf{k}\omega}/\Phi_{\mathbf{k}\omega}$ and $\mathbf{h} = \mathbf{H}/H$.

Representing the distribution function for each species of particles in the form $f + f'$, where f is the average function and f' is the oscillating part, and using the averaging operation, we break up the kinetic equation for the particles of charge e and mass m into two equations:

$$\frac{\partial f}{\partial t} + \mathbf{v}\nabla f + \left(\frac{e}{m}\mathbf{E}_0 + \frac{e}{mc}[\mathbf{vH}]\right)\frac{\partial f}{\partial \mathbf{v}} = i\frac{e}{m}\frac{\partial}{\partial \mathbf{v}}\int\langle \mathbf{b}_{\mathbf{k}\omega}\Phi_{\mathbf{k}\omega}F^*_{\mathbf{k}\omega}\rangle\, d\mathbf{k}\, d\omega. \tag{2*}$$

*[vH] = v × H.

$$\left(-i\omega + i\mathbf{kv} - \Omega\frac{\partial}{\partial\vartheta}\right)F_{\mathbf{k}\omega} = i\frac{e}{m}\mathbf{b}_{\mathbf{k}\omega}\Phi_{\mathbf{k}\omega}\frac{\partial f}{\partial \mathbf{v}} + i\frac{e}{m}\int \mathbf{b}_{\mathbf{k}'\omega'}$$
$$\times\left\{\Phi_{\mathbf{k}'\omega'}\frac{\partial F_{\mathbf{k}-\mathbf{k}',\,\omega-\omega'}}{\partial \mathbf{v}} - \left\langle\Phi_{\mathbf{k}'\omega'}\frac{\partial F_{\mathbf{k}-\mathbf{k}',\,\omega-\omega'}}{\partial \mathbf{v}}\right\rangle\right\}dk'\,d\omega'. \quad (3)$$

Here $F_{\mathbf{k}\omega}$ is a Fourier transform of f', $\mathbf{E}_0$ is the average electric field, $\Omega = eH/mc$, ϑ is the azimuthal angle in the velocity space, and

$$\mathbf{b}_{\mathbf{k}\omega} = \mathbf{a}\,(1 - \mathbf{kv}/\omega) + \mathbf{k}\,(\mathbf{av})/\omega.$$

For $\kappa\rho \ll 1$ we can represent the function f in the form

$$f(\mathbf{r}, \mathbf{v}) = f_0(v_\perp^2, v_z, x) + (v_y/\Omega)\,\partial f_0/\partial x, \quad (4)$$

so that $\partial f/\partial \mathbf{v}$ in (3) can be rewritten in the form

$$\partial f/\partial \mathbf{v} = -mT^{-1}(\mathbf{v} + \mathbf{e}_y v_0)\,f_0, \quad (5)$$

where $\mathbf{e}_y$ is a unit vector along the y axis, T is the temperature, and

$$v_0 = -T\varkappa/m\Omega = -(T/m\Omega n)\,dn/dx. \quad (6)$$

To simplify the notation we shall leave out the subscripts $\mathbf{k}$ and ω where this does not cause misunderstanding, denoting with primes quantities having the indices $\mathbf{k}'$ and ω', and with double primes those having the indices $\mathbf{k}'' = \mathbf{k} - \mathbf{k}'$ and $\omega'' = \omega - \omega'$.

Since we are interested in oscillations having frequencies much smaller than the cyclotron frequency Ω, Eq. (3) can be simplified by the expansion in inverse powers of the cyclotron frequency. In the zeroth approximation we have

$$ik_\perp v_\perp\cos(\vartheta - \alpha)\,F - \Omega\partial F/\partial\vartheta$$
$$= -ieT^{-1}k_\perp v_\perp\cos(\vartheta - \alpha)\,f_0\Phi,$$

where α is the angle between $k_\perp$ and the x axis. From this we get

$$F = -e\Phi T^{-1}f_0 + F_1(v_\perp^2, v_z)\exp(i\,[\mathbf{kv}]\mathbf{h}/\Omega). \quad (7)$$

We substitute this expression in (3), then multiply (3) by $\exp(-i[\mathbf{k}\times\mathbf{v}]\cdot\mathbf{h}/\Omega)$, and average over ϑ; this is equivalent to imposing the orthogonality condition on the solution of the adjoint zeroth-approximation equation. Neglecting in the quadratic terms the terms linear in k, which is fully justified when $k_\perp \gg \kappa$, we obtain

$$-i(\omega - k_z v_z)F_1$$
$$= -i\frac{e}{T}\frac{\omega + \omega_*}{\omega}(\omega - k_z v_z + k_z v_z\alpha_0)\,f_0 J_0\left(\frac{k_\perp v_\perp}{\Omega}\right)\Phi$$
$$-\frac{e}{m}\int\frac{[\mathbf{kk}']\,\mathbf{h}}{\Omega}\left\{1 + \frac{k_z' v_z(\alpha_0' - 1)}{\omega'}\right\}J_0\left(\frac{k_\perp' v_\perp}{\Omega}\right)$$
$$\times\{\Phi'F'' - \langle\Phi'F''\rangle\}\,dk'd\omega', \quad (8)$$

where

$$\omega_* = k_y v_0,\ \Phi' = \Phi_{\mathbf{k}'\omega'},\ F'' = F_{\mathbf{k}-\mathbf{k}',\omega-\omega'},\ \alpha_0' = \alpha_0(\mathbf{k}'\omega'),$$

J_0 is the Bessel function of zero order.

In the case of weak turbulence, the quadratic term must be regarded as small, and consequently the dependence of F_1 on $v_\perp$ will be determined by the first term in the right half of (8), from which we see that F_1 can be represented in the form

$$F_1(v_z)\,J_0(k_\perp v_\perp/\Omega)\,f_m(v_\perp)\,\beta^{-1}(s).$$

Here

$$f_m(v_\perp) = (m/2\pi T)\exp(-mv_\perp^2/2T)$$

is the Maxwell function, $F_1(v_z)$ the longitudinal-velocity distribution function, $\beta(s) = E^{-s}I_0(s)$ a factor added for normalization, $s = k_\perp^2\rho^2 = k_\perp^2 T/m\Omega^2$, and I_0 is the Bessel function of imaginary argument.

Multiplying (8) by $J_0(k_\perp v_\perp/\Omega)2\pi v_\perp dv_\perp$ and integrating with respect to $v_\perp$, we obtain for $F_1(v_z)$:

$$(\omega - k_z v_z)\,F_1(v_z) = eT^{-1}(\omega + \omega_*)\,Af_0\beta\Phi$$
$$-i\frac{c}{H}\int\frac{\chi(\mathbf{k}, \mathbf{k}')}{\beta''}A'\,\{\Phi'F_1''(v_z) - \langle\Phi'F_1''(v_z)\rangle\}\,dk'\,d\omega', \quad (9)$$

where f_0 stands for the Maxwellian function of v_z, $\beta'' = \beta_{\mathbf{k}-\mathbf{k}'}$,

$$A = 1 - k_z v_z(1 - \alpha_0)/\omega,\quad A' = 1 - k_z' v_z(1 - \alpha_0')/\omega',$$

and the function $\chi(\mathbf{k}, \mathbf{k}')$ is equal to

$$\chi(\mathbf{k}, \mathbf{k}') = [\mathbf{k}, \mathbf{k}']\,\mathbf{h}\xi(\mathbf{k}, \mathbf{k}')$$
$$= [\mathbf{kk}']\,\mathbf{h}\int J_0\left(\frac{k_\perp v_\perp}{\Omega}\right)J_0\left(\frac{k_\perp' v_\perp}{\Omega}\right)J_0\left(\frac{k_\perp'' v_\perp}{\Omega}\right)f_m(v_\perp)dv_\perp. \quad (10)$$

Equation (9) pertains both to the ion $F_1^i(v_z)$ and to the electron distribution functions, and we can assume for the electron $\chi^e(\mathbf{k}, \mathbf{k}') = [\mathbf{k}\times\mathbf{k}']\cdot\mathbf{h}$ and $\beta^e = 1$.

These equations must be supplemented by the equations for the electric field. For this purpose we can use the quasineutrality condition which, taking (7) into account, we represent in the form

$$\frac{T}{en}\int[F_1^i(v_z) - F_1^e(v_z)]\,dv_z - 2\Phi = 0, \quad (11)$$

and the equation*

$$\partial j_z/\partial t = -(c^2/4\pi)\,(\text{rot rot }\mathbf{E})_z,$$

which with account of (11) is best rewritten in the form

$$\frac{T}{en}\int(\omega - k_z v_z)\,[F_1^i(v_z) - F_1^e(v_z)]\,dv_z$$
$$-2\omega\Phi = \frac{sk_z^2 v_A^2}{\omega}(\Psi - \Phi), \quad (12)$$

where $v_A = H/\sqrt{4\pi n m_i}$ is the Alfven velocity, $s = k_\perp^2 T/m_i\Omega_i^2$, and $\Psi = \alpha_0\Phi$. In a weakly turbulent

*rot = curl.

plasma, the interaction between the oscillations is small, and therefore the oscillation frequency is essentially determined by the linear approximation. Discarding in (9) the quadratic term and expression F_1 in terms of Φ and Ψ, we reduce the linearized equations (11) and (12) to the form

$$B_1\Phi + B_2\Psi = 0, \quad (11a)$$

$$[(\omega + \omega_*)\omega(1-\beta)/sk_z^2v_A^2 - 1]\Phi + \Psi = 0, \quad (12a)$$

where

$$\omega_* \equiv \omega_*^i = -\omega_*^e, \ \beta \equiv \beta^i, \quad \beta^e = 1,$$

$$v_i = \sqrt{2T/m_i} \quad v_e = \sqrt{2T/m_e},$$

$$B_1 = -(1-\beta)\frac{\omega+\omega_*}{\omega};$$

$$B_2 = -\beta\frac{\omega+\omega_*}{\omega}\left[1 - \frac{\omega}{k_zv_i}Y\left(\frac{\omega}{k_zv_i}\right)\right] - \frac{\omega-\omega_*}{\omega}\left[1-\frac{\omega}{k_zv_e}Y\left(\frac{\omega}{k_zv_e}\right)\right], \quad (13)$$

$$Y\left(\frac{\omega}{k_zv_i}\right) = \frac{k_zv_i}{\sqrt{\pi}}\int\frac{\exp(-v^2/v_i^2)}{\omega - k_zv + i\nu}dv/v_i = 2\int_0^{\omega/k_zv_i}\exp\left(-\frac{\omega^2}{k_z^2v_i^2}+x^2\right)dx - \frac{i\sqrt{\pi}k_z}{|k_z|}\exp\left(-\frac{\omega^2}{k_z^2v_i^2}\right). \quad (14)$$

From (12a) we obtain

$$\alpha_0 = \Psi/\Phi = 1 - (\omega+\omega_*)\omega(1-\beta)/sk_z^2v_A^2, \quad (15)$$

and from (11a) with an account of (15) we obtain a dispersion relation for the frequency

$$D \equiv B_1 + \alpha_0B_2 = 0.$$

In the case of greatest interest to us, $v_i \ll \omega/k_z \ll v_e$, the dispersion equation assumes the form

$$D = -2 + \beta\frac{\omega+\omega_*}{\omega} + \frac{\omega^2-\omega_*^2}{sk_z^2v_A^2}(1-\beta) - i\sqrt{\pi}\frac{\omega-\omega_*}{|k_z|v_e}\left[1-\frac{\omega(\omega+\omega_*)(1-\beta)}{sk_z^2v_A^2}\right] = 0. \quad (16)$$

From this we can easily obtain the frequency and the buildup increments of the small oscillations.

A corresponding investigation was carried out in [3], in which all the drift-wave branches were considered. We are primarily interested in oscillations built up in time, since these are the ones that will have the maximum amplitude. At small s, the frequency of these oscillations is close to ω_*, and for sufficiently large s, when

$$\beta = e^{-s}I_0(s) \cong 1/\sqrt{2\pi s} \ll 1,$$

we have

$$\omega_k \cong \omega_*\zeta^2/2\sqrt{2\pi s}\ (\zeta^2 + \sin^2\alpha) = v_0\zeta^2\sin\alpha/2\sqrt{2\pi}\ \rho(\zeta^2+\sin^2\alpha), \quad (17)$$

where

$$\rho = \sqrt{T/m_i\Omega_i^2}, \ \zeta = \sqrt{2}\rho k_zv_A/v_0 = 2\rho k_z/\varkappa\sqrt{\beta_0}.$$

The growth increment γ of the small oscillations for $s > 1$ and for not too small ζ is approximately equal to

$$\gamma \cong \sqrt{\pi}\omega_k^2/|k_z|v_e\beta. \quad (18)$$

From a comparison of (17) and (18) we see that the increment reaches a maximum $\sim\beta^{-1}\omega_k(m_e/m_i\beta_0)^{1/2}$ when $\zeta \sim 1$, that is when the phase velocity ω/k_z along the z axis reaches a maximum value $\sim v_A$.

With the aid of the real part of (16), we can easily show that for the natural oscillations

$$\alpha_0 = (1-\beta)(\omega_* + \omega_k)/(\omega_* - \omega_k). \quad (19)$$

Thus, in the region $s > 1$ we have $\alpha_0 \approx 1$, that is, the oscillations are almost longitudinal.

3. KINETIC EQUATION FOR WAVES

Inasmuch as $\gamma/\omega \ll 1$, Eq. (9) can be solved by expansion in the oscillation amplitude, assuming that in the zeroth approximation we deal with free non-interacting waves. For simplicity we confine ourselves to only those oscillations which are growing in the linear approximation, since the amplitude of damped waves should be noticeably smaller.

Following the work by the author and Petviashvili [4], we set up a chain of linked equations for the correlation functions. Multiplying (9) by Φ^* and averaging it over the statistical ensemble, we obtain

$$(\omega - k_zv_z)P_{k\omega} = \frac{e}{T}(\omega+\omega_*)Af_0\beta I_{k\omega} - i\frac{c}{H}\int\frac{\chi(k,k')}{\beta''}A'Q_{k'\omega',k\omega}dk'd\omega',$$
$$\delta(k-k')\delta(\omega-\omega')I_{k\omega} = \langle\Phi^*_{k'\omega'}\Phi_{k\omega}\rangle,$$
$$\delta(k-k')\delta(\omega-\omega')P_{k\omega}(v_z) = \langle\Phi^*_{k'\omega'}F_{1k\omega}(v_z)\rangle,$$
$$\delta(k-k_1)\delta(\omega-\omega_1)Q_{k'\omega',k\omega} = \langle\Phi_{k'\omega'}\Phi^*_{k\omega}F_{1k_1-k',\omega_1-\omega'}\rangle. \quad (20)$$

According to (20), the pair correlation function is coupled with the triple function. Analogously, by multiplying (9) by the product of the two Φ's we can obtain an equation for Q, which will contain the quadrupole correlation function. In the weak-coupling approximation we can neglect the correlation between oscillations having four different k and ω, that is, we can express the quadruple correlation function in terms of the product of the pair correlation function. In this approximation we obtain for Q

$$(\omega'' - k_z''v_z)\, Q_{k'\omega',\, k\omega} = eT^{-1}(\omega'' + \omega_*'')A''f_0\beta''q_{k'\omega',\, k\omega}$$
$$- i\,(c/H)\,\{\chi(k'', k)\, A I_{k\omega} P_{-k',-\omega'}/\beta'$$
$$+ \chi(k'', -k')\, A' I_{k'\omega'} P_{k\omega}/\beta\},$$
$$\delta(k - k_1)\,\delta(\omega - \omega_1)\, q_{k'\omega',\, k\omega} = \langle \Phi^*_{k\omega}\Phi_{k'\omega'}\Phi_{k_1-k',\, \omega_1-\omega'}\rangle. \quad (21)$$

Determining Q from this and substituting in (20), we get

$$(\omega - k_z v_z + \eta_{k\omega})\, P_{k\omega} = eT^{-1}(\omega + \omega_*)\, A f_0 \beta I_{k\omega}$$
$$- \frac{c^2}{H^2} I_{k\omega}\int \frac{\chi(k, k')\,\chi(k'', k)}{\beta''\beta'} AA' \frac{P_{-k',-\omega'}}{\omega'' - k_z''v_z + i\nu}\, dk'd\omega'$$
$$- i\frac{c}{H}\frac{e}{T} f_0\int \chi(k, k')\, \frac{(\omega'' + \omega_*'')\, A'A''}{\omega'' - k_z''v_z + i\nu}\, q_{k'\omega',\, k\omega}dk'd\omega', \quad (22)$$

where

$$\eta_{k\omega}(v_z) = \frac{c^2}{H^2}\int \frac{\chi(k, k')\,\chi(k'', -k')}{\beta''\beta}\, \frac{A'^2 I_{k'\omega'}}{\omega'' - k_z''v_z + i\nu}\, dk'd\omega'. \quad (23)$$

The positive quantity $\nu \to 0$ has been added in order to circuit the poles correctly.

It is easy to see that Im $\eta > 0$. Thus, an account of the interaction between the oscillations leads automatically to an additional damping in $P_{k\omega}$, and consequently, to an elimination of the pole in $P_{k\omega}$ as a function of v_z. Setting up an equation for the quadruple function, we can easily see that an analogous addition should appear in Equation (21) for Q and in general in all equations for the correlation functions. Therefore in place of $i\nu$ in (22) it would be necessary to write $\eta_{k\omega}$, but since $\eta_{k\omega}$ is itself small, this circumstance is quite insignificant, except for the one case which will be noted below.

To find $q_{k'\omega',k\omega}$, we represent $\Phi_{k\omega}$ (and accordingly $\Psi_{k\omega}$) in the form $\Phi_{k\omega} = \Phi^{(0)}_{k\omega} + \Phi^{(1)}_{k\omega}$, where $\Phi^{(0)}_{k\omega}$ pertains to the free oscillations and satisfies Eqs. (11a) and (12a), while $\Phi^{(1)}_{k\omega}$ describes the forced oscillations under the influence of a "force" which is given by the quadratic term in (9). Substituting in this term the approximate values $\Phi^{(0)}$ and $\Psi^{(0)}$ in lieu of Φ and Ψ, we obtain with the aid of (11) and (12)

$$B_1\Phi^{(1)}_{k\omega} + B_2\Psi^{(1)}_{k\omega} = i\frac{c}{H}\int M_{k\omega,\, k'\omega'}\,\Phi^{(0)}_{k'\omega'}\Phi^{(0)}_{k-k',\, \omega-\omega'}\, dk'd\omega', \quad (24)$$

$$-\alpha_0\Phi^{(1)}_{k\omega} + \Psi^{(1)}_{k\omega}$$
$$= -i\frac{\omega}{sk_z^2v_A^2}\frac{c}{H}\int N_{k\omega,\, k'\omega'}\,\Phi^{(0)}_{k'\omega'}\Phi^{(0)}_{k-k',\, \omega-\omega'}\, dk'd\omega', \quad (25)$$

from which we get

$$D\Phi^{(1)}_{k\omega} = i\frac{c}{H}\int L_{k\omega,\, k'\omega'}\Phi^{(0)}_{k'\omega'}\Phi^{(0)}_{k-k',\, \omega-\omega'}\, dk'd\omega', \quad (26)$$

where D is the determinant of the system of equations (24) and (25), and

$$L_{k\omega,\, k'\omega'} = M_{k\omega,\, k'\omega'}$$
$$+ \frac{B_2\omega}{sk_z^2v_A^2} N_{k\omega,\, k'\omega'} = \frac{1}{n}\int\left\{\frac{1}{\omega - k_zv_z + i\nu} + \frac{B_2\omega}{sk_z^2v_A^2}\right\}$$
$$\times \frac{\chi^i(k, k')(\omega'' + \omega_*'')f_0^i + \chi^e(k, k')(\omega'' - \omega_*'')f_0^e}{\omega'' - k_z''v_z + i\nu} A'A''dv_z. \quad (27)$$

As regards $\Psi^{(1)}_{k\omega}$, we can put approximately $\Psi^{(1)} = \alpha_0\Phi^{(1)}$ inasmuch as $\Psi^{(1)}$ and $\Phi^{(1)}$ have a sharp maximum at a frequency that coincides with the natural frequency.

Since $\Phi^{(1)}$ is assumed to be a small quantity, it would be necessary to use for D in (26) the value Re D. However, since the natural oscillations $\Phi^{(1)}$ should damp out if higher corrections are taken into account, it is necessary to replace D in the left side of (26) by $D_+ = D_1 = \nu\partial D_1/\partial\omega$, where D_1 = Re D, and the small quantity $\nu > 0$ takes into account the damping of $\Phi^{(1)}$.

Taking this remark into account, as well as the fact that the triple correlation function of the free oscillations is equal to zero, we obtain

$$q_{k'\omega',\, k\omega} = i\frac{c}{H}\left\{\frac{L_{k''\omega'',k\omega} + L_{k''\omega'',\,-k'-\omega'}}{D_+(k''\omega'')} I_{k\omega}I_{k'\omega'}\right.$$
$$+ \frac{L_{k'\omega',\, k\omega} + L_{k'\omega',\, -k''-\omega''}}{D_+(k'\omega')} I_{k\omega}I_{k''\omega''}$$
$$- \frac{L^*_{k\omega,\, k'\omega'} + L^*_{k\omega,\, k''\omega''}}{D^*_+(k\omega)} I_{k'\omega'}I_{k''\omega''}. \quad (28)$$

Repeating the arguments of (24)—(26), we can eliminate from (11), (12), and (22) the functions $P^e_{k\omega}$ and $P^i_{k\omega}$ and obtain the equation

$$D_\eta I_{k\omega} = i\frac{c}{H}\int L_{k\omega,\, k'\omega'}\, q_{k'\omega',\, k\omega}\, dk'd\omega' - (\Gamma^i_{k\omega} + \Gamma^e_{k\omega})\, I_{k\omega}, \quad (29)$$

where

$$\Gamma^i_{k\omega} = -\frac{c^2}{H^2n}\int\left\{\frac{1}{\omega - k_zv_z + \eta_{k\omega}} + \frac{B_2\omega}{sk_z^2v_A^2}\right\}$$
$$\times \frac{\chi^i(k, -k')\;\chi^i(k + k', k)\,(\omega' + \omega_*')\,AA'^2 f_0^i I_{k'\omega'}}{\beta_{k+k'}(\omega' - k_z'v_z + i\nu')\,(\omega + \omega' - (k_z + k_z')\, v_z + i\nu)}\, dk'd\omega'dv, \quad (30)$$

and the expression for $\Gamma^e_{k\omega}$ differs from (30) only in that the index i is replaced by e and ω_* is replaced by $-\omega_*$. Substituting (28) in (29) we get

$$\widetilde{D}I_{k\omega} \equiv (D_\eta + \Gamma^i + \Gamma^e + \Gamma^0)\, I_{k\omega}$$
$$= \frac{1}{D^*_+(k\omega)}\frac{c^2}{2H^2}\int |\,l_{k\omega,\, k'\omega'}|^2\, I_{k'\omega'}I_{k-k',\, \omega-\omega'}\, dk'd\omega', \quad (31)$$

where

$$l_{\mathbf{k}\omega,\,\mathbf{k}'\omega'} = L_{\mathbf{k}\omega,\,\mathbf{k}'\omega'} + L_{\mathbf{k}\omega,\,\mathbf{k}-\mathbf{k}',\,\omega-\omega'},$$

and Γ_0 is given by the relation

$$\Gamma^0_{\mathbf{k}\omega} = \frac{c^2}{H^2}\int \frac{l_{\mathbf{k}\omega,\,\mathbf{k}'\omega'}\,l_{\mathbf{k}'\omega',\,\mathbf{k}\omega}}{D_+(\mathbf{k}'\omega')} I_{\mathbf{k}-\mathbf{k}',\,\omega-\omega'}\,d\mathbf{k}'d\omega'. \quad (32)$$

In (29)—(31) the quantities Γ^e and Γ^i are the result of the second term in (22), in which we have substituted in place of $P_{-\mathbf{k}',-\omega'}$ the approximate value obtained from (22) by discarding the second and third terms of the right half. The quantity D differs from the determinant D introduced earlier only in the fact that in expression (14) for Y the infinitesimal quantity $i\nu$ is replaced by $\eta_{\mathbf{k}\omega}(v_z)$. The quantity Γ^0 on the left side of (31) as well as the right half of (31) are the result of $q_{\mathbf{k}'\omega',\mathbf{k}\omega}$.

Equation (31) describes the forced oscillations of a weakly turbulent plasma with a dispersion relation determined from the condition $\widetilde{D} = 0$, under the influence of a noise source whose intensity is given by the right half of (31). It is obvious that under equilibrium the equation $\widetilde{D} = 0$ should describe damped oscillations, that is, $\widetilde{D}_2\,\partial\widetilde{D}_1/\partial\omega > 0$ where $\widetilde{D}_1 = \mathrm{Re}\,\widetilde{D}$ and $\widetilde{D}_2 = \mathrm{Im}\,\widetilde{D}$. Since $\widetilde{D}$ differs from D by a small quantity of the order of the ratio of the increment to the frequency, we shall assume approximately $\widetilde{D}_1 = D_1$, that is, we neglect the shift in the natural frequencies due to the oscillations.

Inasmuch as $\widetilde{D}_2 \ll D_1$ and the sign of $\widetilde{D}_2$ coincides with the sign of Im D_+, we can replace D_+^* by D^* in the right half of (31), which is small. Taking into account the fact that $\widetilde{D}_2 \ll \widetilde{D}_1 \cong D_1$, we obtain

$$I_{\mathbf{k}\omega} = K\,[(\partial D_1/\partial\omega)^2(\omega-\omega_{\mathbf{k}})^2 + \widetilde{D}_2^2]^{-1}$$
$$\cong (\widetilde{D}_2\partial D_1/\partial\omega)^{-1}\pi K\delta(\omega-\omega_{\mathbf{k}}), \quad (33)$$

where K stands for the coefficient of $1/D_+^*(\mathbf{k}\omega)$ in the right half of (31). Putting $I_{\mathbf{k}\omega} = I_{\mathbf{k}}\delta(\omega-\omega_{\mathbf{k}})$, we reduce (33) to the form

$$\mathrm{Im}\,(D_\eta + \Gamma^i + \Gamma^e + \Gamma^0)\,I_{\mathbf{k}} = \left(\frac{\partial D_1}{\partial\omega}\right)^{-1}\frac{\pi c^2}{2H^2}\int |\,l_{\mathbf{k}\omega,\,\mathbf{k}'\omega'}|^2$$
$$\times\,\delta(\omega_{\mathbf{k}}-\omega_{\mathbf{k}'}-\omega_{\mathbf{k}-\mathbf{k}'})\,I_{\mathbf{k}'}I_{\mathbf{k}-\mathbf{k}'}\,d\mathbf{k}'. \quad (34)$$

Equation (34) is the sought stationary kinetic equation for the waves. It can be greatly simplified by using the fact that we are essentially interested in waves having phase velocities $\sim v_A$, that is, $v_i \ll \omega/k_z \ll v_e$. It follows therefore that it is possible to neglect in Im D_η the added term connected with η^e, inasmuch as Im $\eta^e \sim$ Im η^i, but in the series expansion of an integral of the type (14) a small factor $\sim \omega^2/k_z^2 v_e^2$ appears in front of Im η^e. On the other hand, the same condition makes it possible to replace in an expression of the type (14) the term f_0^i by $\delta(v_z)$ for Y_η^i, from which we get

$$\mathrm{Im}\,D_\eta - \mathrm{Im}\,D \cong -\beta\,\frac{\omega_*}{\omega}\,\mathrm{Im}\,\eta^i_{v_z=0}$$
$$= -\frac{\omega_*}{\omega_{\mathbf{k}}}\frac{\pi c^2}{H^2}\int\frac{\chi_i^2(\mathbf{k},\mathbf{k}')}{\beta''}\delta(\omega_{\mathbf{k}}-\omega_{\mathbf{k}'})\,I_{\mathbf{k}'}\,d\mathbf{k}'. \quad (35)$$

An analogous simplification yields

$$\mathrm{Im}\,\Gamma^i_{\mathbf{k}\omega} = \frac{\pi c^2}{H^2}\frac{C_{\mathbf{k}}}{\omega_{\mathbf{k}}}\int\frac{\chi_i^2(\mathbf{k},\mathbf{k}')}{\beta''}\left(1+\frac{\omega_*'}{\omega_{\mathbf{k}'}}\right)\delta(\omega_{\mathbf{k}}-\omega_{\mathbf{k}'})\,I_{\mathbf{k}'}\,d\mathbf{k}', \quad (36)$$

where

$$C_{\mathbf{k}} = (\omega_* - \omega_{\mathbf{k}})/(\omega_* + \omega_{\mathbf{k}})\,(1-\beta) = 1/\alpha_0.$$

As regards Im $\Gamma^{(e)}$, it is easy to show that when $\zeta > 1$ it is considerably smaller than Im Γ^i, and when $\zeta < 1$ it is outwardly of the same order of magnitude, but contributions are made to it only by oscillations with $\zeta < 1$. As will be seen below, the intensity of the oscillations decreases very rapidly with ζ, so that Im Γ^e can be neglected. An analogous situation arises for $l_{\mathbf{k}\omega,\mathbf{k}'\omega'}$. It is easy to verify that the electrons give a contribution to $l_{\mathbf{k}\omega,\mathbf{k}'\omega'}$ which is comparable with that of the ions only in the case when $\zeta < 1$ and $\zeta' < 1$ simultaneously, that is, we can neglect again the electron terms, so that

$$l_{\mathbf{k}\omega,\,\mathbf{k}'\omega'} \cong l^i_{\mathbf{k}\omega,\,\mathbf{k}'\omega'} = C_{\mathbf{k}}\omega_{\mathbf{k}}^{-1}\chi^i(\mathbf{k},\mathbf{k}')\,(\omega_*''/\omega_{\mathbf{k}-\mathbf{k}'} - \omega_*'/\omega_{\mathbf{k}'}). \quad (37)$$

It must be noted that in calculating the right half of (34), a difficulty arises connected with the fact that in $|\,l_{\mathbf{k}\omega,\mathbf{k}'\omega'}|^2$ there is a contribution from both the real and imaginary parts of $l^e_{\mathbf{k}\omega,\mathbf{k}'\omega'}$, and as ν tends to zero $|\,l^e_{\mathbf{k}\omega,\mathbf{k}'\omega'}|$ tends to infinity. But if we take account of the fact that $i\nu$ should in fact be replaced everywhere by η, we can show that the contribution from $l^e_{\mathbf{k}\omega,\mathbf{k}'\omega'}$ can be neglected. Substituting (35)—(37) in (34), replacing in G^0 the integration variable, and recognizing that approximately $\partial D_1/\partial\omega \cong -\beta\omega_*/\omega^2$, we ultimately obtain

$$-\frac{H^2}{c^2\sqrt{\pi}}\frac{\omega_*(\omega_*+\omega_{\mathbf{k}})}{v_e|k_z|}(1-\beta)$$
$$+\frac{H^2}{c^2\sqrt{\pi}}\frac{\omega_*(\omega_*+\omega_{\mathbf{k}})}{v_i|k_z|}\beta\exp\left(-\frac{\omega^2}{k_z v_i^2}\right)$$
$$+\frac{\omega_*^2}{\omega_{\mathbf{k}}^2}\int\frac{\chi_i^2(\mathbf{k},\mathbf{k}')}{\beta''}\delta(\omega_{\mathbf{k}}-\omega_{\mathbf{k}'})\,I_{\mathbf{k}'}\,d\mathbf{k}'$$
$$-\frac{C_{\mathbf{k}}}{\omega_{\mathbf{k}}^2}\int\frac{\chi_i^2(\mathbf{k},\mathbf{k}')}{\beta''}\omega_*(\omega_*'+\omega_{\mathbf{k}'})\,\delta(\omega_{\mathbf{k}}-\omega_{\mathbf{k}'})\,I_{\mathbf{k}'}\,d\mathbf{k}'$$

$$+\frac{C_{\mathbf{k}}}{\omega_{\mathbf{k}}^2}\int C_{\mathbf{k}''}\frac{\chi_i^2(\mathbf{k},\mathbf{k}')}{\beta''}\,\frac{\omega_*(\omega_*\omega_{\mathbf{k}'}-\omega_*'\omega_{\mathbf{k}})^2}{\omega_*''\omega_{\mathbf{k}'}^2}$$

$$\times\,\delta(\omega_{\mathbf{k}}-\omega_{\mathbf{k}'}-\omega_{\mathbf{k}-\mathbf{k}'})\,I_{\mathbf{k}'}\,d\mathbf{k}' =$$

$$=\frac{C_{\mathbf{k}}^2}{2I_{\mathbf{k}}\beta}\int\frac{(\omega_*\omega_{\mathbf{k}'}-\omega_*'\omega_{\mathbf{k}})^2}{\omega_{\mathbf{k}'}^2\omega_{\mathbf{k}''}^2}\,\chi_i^2(\mathbf{k},\mathbf{k}')\,\delta(\omega_{\mathbf{k}}-\omega_{\mathbf{k}'}-\omega_{\mathbf{k}-\mathbf{k}'})$$

$$\times\,I_{\mathbf{k}'}I_{\mathbf{k}-\mathbf{k}'}\,d\mathbf{k}'. \tag{38}$$

The first two terms are the result of linear Landau damping, the next two terms are naturally defined as nonlinear damping and, finally, the last term on the left, together with the expression on the right, describes the breakup of the waves into two parts and the merging of two waves into one. With the exception of linear damping on ions, which is negligibly small when $\beta_0 \gg (m_e/m_i)^{1/3}$, all the terms in (38) are of the same order of magnitude, and consequently all are essential in the analysis of the interaction between oscillations.

4. QUALITATIVE INVESTIGATION OF THE KINETIC EQUATION FOR WAVES

It follows from (38) that I_k, as a function of k_z, should have a maximum at that value of k_z corresponding to $\zeta = 1$. Indeed, with increasing k_z the first term, which describes the buildup of the oscillations by the electrons, decreases and therefore the amplitude of the oscillations should decrease with k_z. On the other hand, when $\zeta < 1$ decreases, ω_k in front of the integral terms in the left half of (38) begins to decrease rapidly, and therefore the function I_k for $\zeta < 1$ should decrease like $\omega_k^2 \sim \zeta^4$.

A similar picture appears with respect to the dependence on $k_\perp$. Recognizing that for small values of $k_\perp\rho$ we have $C_k \approx 1$ for $k_z > \omega_*/v_A$ and $C_k \approx 1/s$ for $k_z < \omega_*/v_A$, we can easily verify that as $k_\perp \to 0$ the function I_k decreases linearly with $k_\perp$ in the region $k_z > \omega_*/v_A$ and $I_k \sim 1/k_\perp$ for $k_z < \omega_*/v_A$. And since the region of integration $k_z < \omega_*/v_A$ contracts to a point as $k_\perp \to 0$, the oscillations in this region will not play any role at all, and I_k can be regarded as a decreasing function of $k_\perp$ for $k_\perp\rho$.

In order to clarify the character of the behavior of I_k for large $k_\perp\rho$, it is necessary to know the specific form of $\chi_i^2(k, k')$. It is seen from (10) that the function $\chi_i^2(k, k')$ is completely symmetrical with respect to k, k′ and k″. For large $k_\perp\rho$ and $k'_\perp\rho$ the integral in (10) can be calculated by the saddle-point method. For the case $k'/k \ll 1$, for example, such a calculation yields

$$\chi_i^2(\mathbf{k},\mathbf{k}') \cong (s'/2\pi\rho^4)\sin^2(\alpha-\alpha')\,e^{-s'}I_0^2(\tfrac{1}{2}s'\cos(\alpha-\alpha')). \tag{39}$$

For $s' \gg 1$, this expression assumes the form

$$\chi_i^2(\mathbf{k},\mathbf{k}') \cong (\alpha-\alpha')^2\exp[-\tfrac{1}{2}s'(\alpha-\alpha')^2]/2\pi^2\rho^4.$$

Integration of this function with respect to the angles leads to the relation $\langle\chi_i^2\rangle \sim (k'\rho)^{-3}$, that is, to a very rapid decrease with increasing k′. Thus, when $k_\perp\rho \gg 1$ the main contribution to the integral terms of (38) should be made by those regions of integration, where either $k'\rho$ or $k''\rho$ is of the order of unity. In this case the contribution from the region $k''\rho \sim 1$ in the integrals of the left half of (38) is much smaller than the corresponding contribution from the region $k'\rho \sim 1$. Indeed, the first two integrals in the left half of (38) cancel each other in the region $k''\rho \sim 1$, that is, when $k \approx k'$, while the last integral, taken over the region $k''\rho \sim 1$ is small, for then $\beta'' \sim 1$ and is not a small quantity as in the case when $k'\rho \sim 1$.

Further, it is seen from (38) that for large $k\rho$ the decay processes play no role. Indeed, for $k'/k \ll 1$ we can assume that in the integral on the left side $\omega_*/\omega_*'' \approx 1$, while in the integral on the right we can put $I_{k-k'} \approx I_k$ and take I_k outside the integral sign, doubling the result to account for the possibility of $k''\rho \sim 1$. In this approximation the integrals cancel each other.

Thus, in the region of large $k_\perp\rho$ the interaction between waves having wave numbers of the same order of magnitude is negligibly small, leaving only the interaction with the long-wave oscillations, which is described by the first two integral terms in the left half of (38). For large $k_\perp\rho$, the first integral exceeds the second and increases like k^3, inasmuch as $\beta'' \sim 1/k$. For sufficiently large $k\rho$ it exceeds the first term, which describes the buildup of the waves by the electrons. In other words, the presence of oscillations in the region $k_\perp\rho \sim 1$ causes all the short-wave oscillations to become damped. And since the decay diffusion of the waves has low efficiency in the region of large wave numbers, all the short-wave oscillations will be suppressed.

Thus, the plot of the spectral function I_k against $k_\perp$ should have a maximum for $k_\perp\rho \sim 1$, and decreases quite rapidly as $k_\perp \to \infty$.

Inasmuch as for $k_\perp\rho \sim 1$ all the functions in (38) have a complicated character, the determination of I_k entails considerable difficulties. Nonetheless, Eq. (38) enables us to estimate the integral of the spectral functions, in terms of which the turbulent-diffusion coefficient is expressed. To this end we multiply (38) by $I_k dk$ and integrate it with respect to k. It is easy to see that in such an integration, the decay terms drop out. Neglecting the ionic damping and the small quantities of β and ω_k/ω_*, and also symmetrizing the integral terms, we obtain the following integral equation

$$\frac{H^2}{c^2\sqrt{\pi}\,v_e}\int\frac{\omega_*^2}{|k_z|}I_k\,dk = \frac{1}{2}\int\frac{(\omega_*-\omega_*')^2}{\omega_k\omega_{k'}}\frac{\chi_i^2(k,k')}{\beta''}\delta(\omega_k-\omega_{k'})I_kI_{k'}\,dk\,dk'. \quad (40)$$

In the left half of this equation we find just the integral which we need [compare with (43)]. Equation (40) can be interpreted as the equality of the "friction" forces between the oscillations and the electrons or ions, respectively. Inasmuch as the decay terms have dropped out, this means that they make no contribution to the friction force for the ions.

Since I_k has a clearly pronounced maximum at $\zeta = 1$, that is, when $k_z = \kappa\sqrt{\beta_0}/2\sqrt{2}$ we can put approximately $\zeta = 1$ in (40). In the integral on the right side, to which the main contribution is made by the integration region near α, $\alpha' = \pi/2, 3\pi/2$, we can put approximately $\omega_k = v_0 \sin\alpha/4\sqrt{2\pi}\,\rho$. We shall assume I_k to be isotropic in the transverse direction. Then, substituting in (40) the approximate value for $\chi_i^2 \cong \sin^2(\alpha-\alpha')\,2\pi^2\rho^4$ we can readily integrate over the angles. Integration of the δ-function with respect to α' yields $\sin\alpha = \sin\alpha'$, hence $\alpha' = \pi - \alpha$, and then integration with respect to α is elementary. Putting $\beta'' \cong \frac{1}{2}\sqrt{2\pi}\,\rho k''$, we obtain in the right half of (40) an integral of the form

$$\int (k-k')^2 k'' I_k I_{k'}\,dk\,dk',$$

which can be approximately replaced by

$$\frac{A}{\rho}\left\{\int k^2 I_k\,dk\right\}^2,$$

where A is a numerical factor of the order of 10^{-1}.

Substituting this expression in (40), we obtain the approximate value of the integral which we need

$$\int k^2 I_k\,dk = \frac{3\pi^2}{2^7A}\sqrt{\frac{m_e}{m_i\beta_0}\frac{T^2\varkappa^2}{e^2}} \approx \sqrt{\frac{m_e}{m_i\beta_0}\frac{T^2\varkappa^2}{e^2}}. \quad (41)$$

Recognizing that the fluctuation of the density n' is connected with the fluctuation of the potential by the relation $n' = e\varphi'/T$, we obtain from this

$$\langle (n'/n)^2\rangle \approx \sqrt{m_e/m_i\beta_0}\,\varkappa^2\rho^2, \quad (42)$$

which is to be expected, since the density perturbation is produced by the displacement of the plasma through a distance $\sim\rho$.

5. COEFFICIENT OF TURBULENT DIFFUSION

With the aid of Eq. (2) for the average distribution function we can readily obtain an expression for the diffusion current. It is possible to use here either the ion or the electron equation—both give precisely the same result. In practice it is more convenient to use the electron equation. Indeed, in the derivation of (38) it was established that the electron contribution to the nonlinear terms can be neglected. This means that in the averaged equation (2) for the electrons we can neglect all the nonlinear terms, with the exception of the quadratic term. Thus, in the present problem the quasilinear approximation turns out to be correct for the electrons [6].

Multiplying Eq. (2) for the electrons by v_y and integrating it then with respect to $\mathbf{v}$, we obtain the electron current along the density gradient, and consequently also the coefficient of turbulent diffusion

$$D_\perp = \frac{\sqrt{\pi}c^2}{H^2v_e}\int\frac{\alpha_0k_y^2}{|k_z|}\left(1+\frac{\omega_k}{\omega_*}\right)(1-\beta)\,I_k\,dk. \quad (43)$$

If we neglect here the small quantities β and ω_k/ω_* and substitute the expression obtained above for the integral of the spectral function, we obtain approximately

$$D_\perp \cong m_e\rho^2 v_i\varkappa/m_i\beta_0. \quad (44)$$

Thus, in the case considered here $m_e/m_i \ll \beta_0 \ll (m_e/m_i)^{1/3}$, the turbulent diffusion coefficient does not depend on the magnetic field.

6. DISCUSSION OF RESULTS

Thus, in the present work we have investigated the turbulent diffusion of a collisionless plasma in a strong magnetic field. We have shown that the character of the transfer of the vibrational energy differs appreciably from the transfer of energy in an ordinary turbulent liquid. Namely, whereas in ordinary hydrodynamics the vortices break up and are transformed into pulsations of smaller scale, with conservation of the total energy, so that the energy flux is constant over the spectrum, in a turbulent collisionless plasma the additional damping of the waves, which can naturally be called the nonlinear Landau damping, is a very important factor, along with this type of energy diffusion in the wave-number space, described with the aid of the decays.

As can be seen from (22) and (23), this damping is the result of the interaction between the waves $\mathbf{k}$, ω and $\mathbf{k}'$, ω' through the resonant particles whose longitudinal velocity coincides with the "phase velocity" of the beats at these wavelengths, that is, $v_z = (\omega - \omega')/(k_z - k_z')$. Nonlinear damping of waves by waves causes the spectral function to decrease rather rapidly in regions far from the maximum increment, where the maximum supply to the waves comes from the ground state.

For simplicity we confine ourselves to the case of a homogeneous magnetic field. It is easy to determine qualitatively the result of an inhomogeneity. The greatest influence on the drift waves will be produced by the crossing of the force lines. This effect, analyzed in detail by Mikhaĭlovskiĭ and Galeev, can be described qualitatively in the following manner. We shall characterize the crossing of the lines by means of a parameter $\theta = 1/\kappa L$, where L is the length of a plasma pinch such that the angle of rotational transformation referred to it varies along the pinch by unity. Then the projection $k_{\parallel}$ of the wave vector by the magnetic-field vector will vary like $k_{\parallel} \approx \theta k_{\perp} \kappa x$ at a distance x from the point where the spacing of the perturbation coincides with the spacing of the force lines. However, as soon as $k_{\parallel}$ becomes of the order of $\kappa/10$, the drift wave begins to be absorbed by damping on the ions[1].

Thus, the amplitude of an arbitrary wave train increases until the train becomes localized on a length $\Delta x \sim 1/10 k_{\perp}\theta$, that is, within a time interval $\Delta t \sim \Delta x/u$, where $u \sim \omega/k_{\perp}$ is the group velocity. The wave train then enters a region of strong damping and disappears. As a result, in the linear approximation with $\theta > \rho\kappa$ the instability occurs only in the presence of turning points, when the wave packet is blocked by "potential" barriers from the damping region.

We now consider the effect of the crossing of the force lines on the development of the oscillation. The presence of the absorption region leads in this case to wave diffusion in the x direction, and this effect can be taken into account in the kinetic equation for the waves by means of a term of the type $u\,\partial I_k/\partial x$[5]. The diffusion of the waves in the absorption region leads to a decrease in the effective increment by an amount $\sim (1-R)u/\Delta x \sim 10(1-R)\omega\theta$, where R is the coefficient of reflection from the "potential" barriers, if such exist. And since the increment in our case is $\gamma \sim \omega\sqrt{m_e/m_i}\,\kappa/k_{\parallel}$, the crossing effect can be taken into account in the oscillation amplitude and in the diffusion coefficient by means of the additional factor $\sim [1-(1-R)\theta\sqrt{m_i/m_e}\,]$. Thus, the anomalous diffusion should disappear for $\theta \gtrsim (1-R)^{-1} \times \sqrt{m_e/m_i}$. At smaller values of θ, the presence of crossing leads only to a decrease in $D_{\perp}$, and in this case it is possible to distinguish between instability with respect to infinitesimally small and finite perturbations. If the finite perturbation is specified in the form of a set of wave trains, then in the presence of an interaction between the waves, each individual train, before entering the damping region, can give rise, through scattering, to a wave that travels in the opposite direction, and this process is perfectly analogous to reflection from a potential barrier.

Thus, a plasma which is stable in the linear approximation in an inhomogeneous magnetic field, may turn out to be unstable with respect to perturbations of finite amplitude. On going over to such finite perturbations, the stability problem itself changes, for in view of the presence of interaction between the waves, the finding of the eigenfunctions corresponding to a definite asymptotic behavior as $t \to \infty$ recedes to the background, and it is necessary to trace instead the behavior of wave trains during finite time intervals $\Delta t \sim 1/\gamma$. This is precisely why it is perfectly justified to use when $\kappa\rho \ll 1$ the expansion which we employed not in the eigenfunctions of the linear problem, but in simple Fourier integrals.

In conclusion we make one stipulation. We have assumed everywhere that the particles have a Maxwellian distribution. In fact, the oscillations distort f(v) but estimates show that this distortion can be neglected when $D_{\perp}\kappa^2 < \nu_i$, where ν_i is the ion collision frequency.

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