INVESTIGATION OF THE FERMI SURFACE IN LEAD BY THE CYCLOTRON RESONANCE METHOD

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Experiments are described for the measurement of the effective masses of the current carriers in lead, as a result of which exact values are obtained for the effective masses and detailed information is secured on their anisotropy in the two principal crystallographic planes. The experimental results are discussed in detail and compared with the model of the lead Fermi surface set up in the approximation of almost free electrons. The characteristic features of the model of surfaces of the second and third zones are found to be in complete qualitative agreement with the experimental results. The principal dimensions of the surface of the third zone are determined and minor differences between the zone and the model are noted. Some facts are considered which indicate that the fourth zone is vacant. The results obtained are compared with published data, some of which are used for confirming the conclusions made in the present paper.

THE first results of a detailed investigation of cyclotron resonance in lead were given in [1]. The principal conclusions given in the research concern the shape of the multiply-connected Fermi electron surface of lead of the third Brillouin zone and emphasize the correctness of the model constructed in the approximation of nearly free electrons.[2,3] The paper contains the description of new, more detailed investigations, which supplement appreciably and make more precise the results of previous research, in particular in relation to the hole Fermi surface of lead of the second zone. The conclusions given in [1] relative to the surface of the third zone are supported by new experimental facts. At the same time, some secondary conclusions and assumptions made in [1] needed to be made more exact or be reconsidered; these items are noted in the discussion of results.

MEASUREMENT OF THE EFFECTIVE MASSES OF THE CURRENT CARRIERS IN LEAD

Single crystal samples were prepared in the shape of rectangular plates by the method described in [1]. The material used was lead, the purity of which was characterized by the resistance ratio $\rho(20°C)/\rho(4.2°K) = (0.6-1) \times 10^4$.

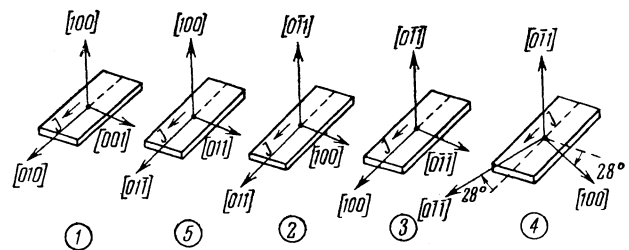The distribution of the crystallographic axes in the five samples studied is shown in Fig. 1.

FIG. 1. Orientation of the crystallographic axes in five investigated lead samples: 1 and 2—13 × 6 × 1 mm³; 3,4,5—13 × 6 × 0.5 mm³, J—direction of the high-frequency currents.

The departure of the flat surfaces of the samples actually investigated from coincidence with the orientations shown in this drawing amounted to the following: for samples 3 and 5, not more than 30′, for samples 1 and 2, about 1°, and for sample 4, about 2°.

Use of samples 2—4, which have the same orientation of the plane surface (0$\bar{1}$1), but different distribution of the [100] and [011] axes in the plane of the surface relative to the direction of the high frequency currents J, had as its purpose the possibility of experimentally distinguishing cyclotron resonance in the current carriers belonging to the central section of the Fermi surface from resonance on the noncentral sections. For a magnetic field lying in the (100) plane, that is, in experiments with specimens 1 and 5, the difference in

$\frac{1}{\chi}\frac{\partial\chi}{\partial H}$

H^{-1}, kOe^{-1}

FIG. 2. Record of the spectrum of cyclotron resonances χ obtained with the sample 2 (Fig. 1) when the high-frequency current **J** and the magnetic field **H** are parallel to the [011] axis.
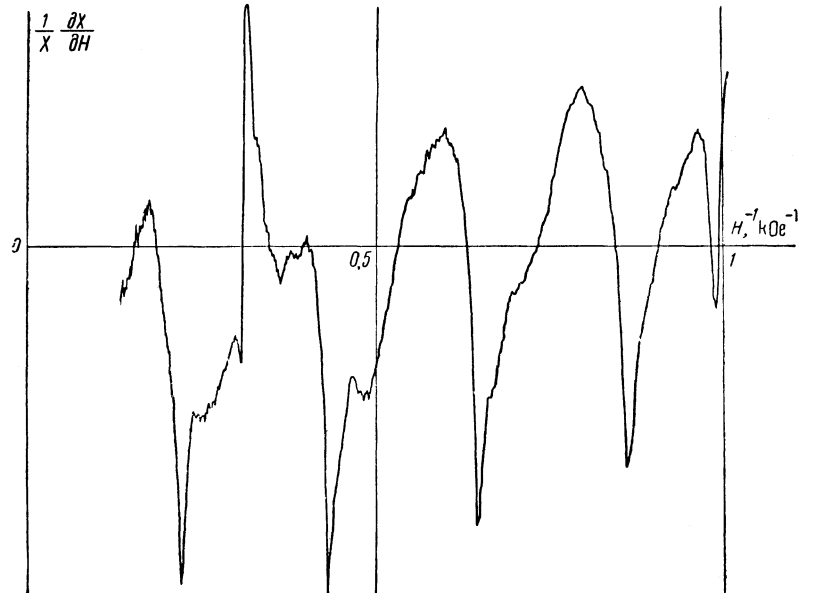
the excitation of these two types of resonances can be observed in each specimen, by using the equivalence of directions which make an angle of 90° with one another.

Experiments on the study of cyclotron resonance have been carried out at a temperature of ~ 1.8°K in magnetic fields ~ 0.8—10 kOe. The lower bound of the magnetic field intensity is determined by the destruction of the superconductivity of lead. The impossibility of observations in weak fields leads to the result that these experiments can give no information whatever on small effective masses $\mu < 0.3$, including information on their very existence.

The methods and accuracy of adjustment of the magnetic field, and also all other practical means of carrying out the experiments, were identical with the methods of investigation described in [4].
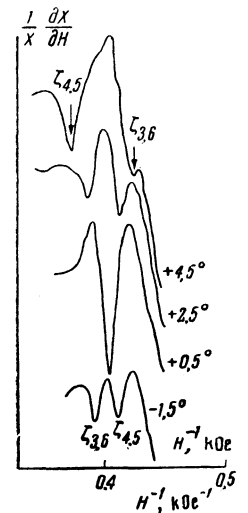
A sample record of the spectrum of cyclotron resonances is shown in Fig. 2. Each of the series of such spectra, which were recorded in the course of the experiments, and which differ by rotation of the magnetic field in the plane of the surface of the specimen by an angle of 1—3°, was used to calculate the relative effective masses of the current carriers:

$$\mu = H_{epr}^{-1} / (H_{n+1}^{-1} - H_n^{-1}), \tag{1}$$

where H_{epr} is the intensity of the magnetic field for electron paramagnetic resonance, and H_n is the field for cyclotron resonance of order n; here we have used the fact that H_{epr} corresponds to the value $\mu = 1$ with sufficient accuracy.

The interpretation of cases similar to that shown in Fig. 3 requires special attention: two

FIG. 3. Record of the case of "intersection" of cyclotron resonances. To the right of the curves is shown the angle between the field **H** and the [001] axis in the plane of the (01̄1) surface of the specimen.

$\frac{1}{\chi}\frac{\partial\chi}{\partial H}$

$\zeta_{4,5}$

$\zeta_{3,6}$

$+4,5°$

$+2,5°$

$+0,5°$

$-1,5°$

$\zeta_{3,6}$ $\zeta_{4,5}$ H^{-1} kOe

H^{-1}, kOe^{-1}

resonances, similar in amplitude and shape, coincide for a certain direction of the magnetic field (+0.5°). In order to determine whether the two resonances come together monotonically upon rotation of the field or whether they first come to within a small distance of each other and then diverge, it is necessary to carry out an additional experiment. This experiment consists in the observation of the same resonances for different polarizations of the high frequency currents relative to the axis of the crystal, that is, using another specimen (when working with rectangular single crystals). The direction of polarization has been so chosen that one of the resonances was greatly weakened or disappeared entirely.

The polar plot of Fig. 4a shows the results of the study of the anisotropy of the effective masses of the current carriers in lead in two crystallo-
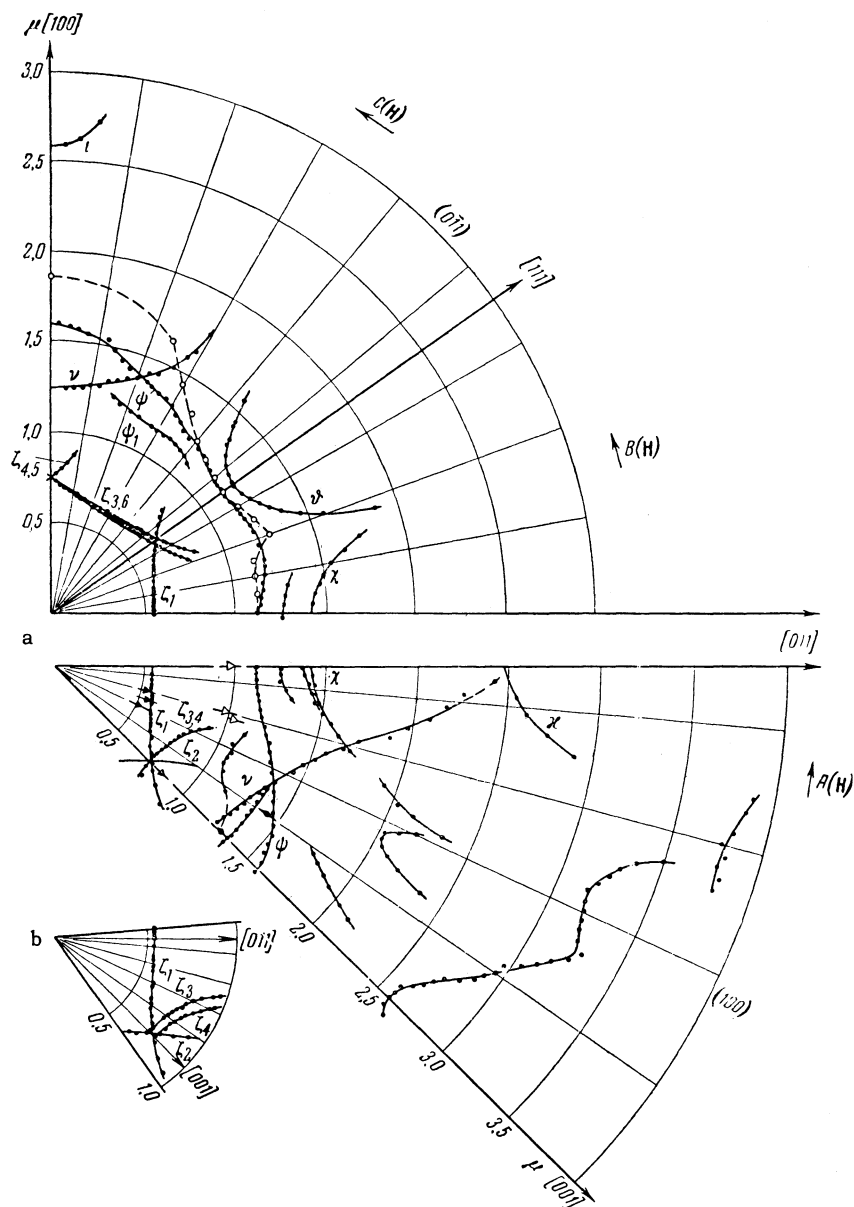
FIG. 4. Anisotropies of the effective masses of current carriers in lead: a—data obtained on sample 4 — the (0$\bar{1}$1) plane and on 5 — the (100) plane, with the exception of the line ζ_2 drawn from the results of the study of sample 1; b—data obtained on sample 1. The masses, which were measured by Gold,[2] are indicated by the sumbols: $\triangle$ — from α oscillations, $\diamond$ from β oscillations, $\blacktriangle$ from y oscillations.

graphic planes. Data on the (0$\bar{1}$1) plane were obtained in experiments with sample 4, while that of the (100) plane with sample 5. The results of experiments, carried out with different specimens, with account of corrections for difference in orientation, agree with one another within the accuracy of the experiment. This is confirmed by comparison of the positions of the lines ζ_1 in Fig. 4a and Fig. 4b, where part of the results of investigating specimen 1 is shown.

Note should be taken of the splitting of the resonances corresponding to the masses of ζ_3 and ζ_4 which is observed in specimen 1 (Fig. 4b), a splitting that was not resolved in sample 5 (Fig. 4a). This splitting is associated with a deviation of $\sim 1°$ of the surface of specimen 1 relative to the (100) plane, which disrupts the symmetry of the position of the field vector H, which lies in the

plane of the surface of the specimen relative to the crystallographic axes. A similar splitting of the resonances ζ_3 and ζ_6 is observed in sample 4 (Fig. 4a); their gradual fusion in proportion to the approach of the direction of the field H to the [100] axis permits us to comment on the resolving power of these measurements. It is evident that the discovery of the splitting of the cyclotron resonance has a great value for the correct interpretation of the results of the experiments.

CLOSED HOLE FERMI SURFACE OF THE SECOND ZONE OF LEAD

The ψ line in the diagram of Fig. 4a illustrates the anisotropy of the effective mass of the group of current carriers, cyclotron resonance in which is observed for any direction of the magnetic field

vector **H** in the (01̄1) and (100) planes, but under the condition that the high frequency current **J** is not parallel to the field **H**. It follows from what has been said that this cyclotron resonance takes place on the central cross section of the closed Fermi surface.

If the effective mass of ψ under consideration were isotropic, then the Fermi surface could be regarded as spherical. However, the clearly expressed anisotropy of the mass of ψ, as measured by experiment (Fig. 4a), establishes the existence of certain characteristic features of the shape of the Fermi surface.

For this purpose, we shall carry out a qualitative analysis of the anisotropy of the mass of ψ, starting out from the simple consideration that each departure in behavior of the mass of ψ from isotropic can be regarded as brought about by local distortions of the original Fermi surface which contains the orbit of the corresponding current carriers for a given direction of the magnetic field.

The character of these distortions can be determined in the following way. The definition of the effective mass [5]

$$\mu = (1/2\pi m_e)\, \partial S/\partial \varepsilon \qquad (2)$$

for a quadratic dispersion law, which is valid for the initial Fermi sphere, is transformed for the central section to the form

$$\mu = S / 2\pi m_e \varepsilon. \qquad (3)$$

Thus the effective mass μ is proportional to the area S of the extremal cross section of the Fermi surface enclosed by the resonance orbit. Consequently, the increase in the effective mass for any direction of the magnetic field must be associated with the presence of bosses on the initial Fermi sphere, around which the orbit of the current carrier passes for a given direction of the field.

The mass of ψ reaches a maximum value (Fig. 4a) for a magnetic field parallel to a four-fold axis **H** ‖ ⟨100⟩. Consequently, the orbits of current carriers lying (under these conditions) in the planes {100} ⊥ **H** pass along the distortions of the original Fermi sphere, thus producing an increase in the effective mass of the carriers. These distortions can be convex "fins," which protrude on the surface along the three great circles lying in the {100} planes. In this case, it is natural to expect the appearance of large polar bosses at six places of the surface close to the points of intersection of those circles which are located on the ⟨100⟩ axes.

For further analysis, we shall make use of the stereographic projection of Fig. 5, in which the

FIG. 5. Stereographic projection of the rotations of the vector of the magnetic field A(H), B(H), C(H), which are equivalent to the rotations denoted in similar fashion in Fig. 4a (A, B,C are the corresponding rotations of the plane of the orbit; a,b,c are the poles of the axes of rotation of the planes).

projection of the hodograph of the vector **H** is drawn for displacements which are equivalent to rotation in the (100) plane of the diagram of Fig. 4a — the parts A(**H**) — and rotation in the (01̄1) plane—the parts B(**H**) and C(**H**). Selection of the displacements is set by the clarity of the drawing. The successive locations of projections of plane orbits for rational directions of the vector **H**, their rotations A, B, C, and the projections of their axes of rotation, which are denoted by the points a, b, c, are also shown for the corresponding rotations of the vector.

For the rotation A, the mass of ψ changes from maximum to minimum value (Fig. 4a), corresponding to that orbit which passes, for **H** ‖ [010], along the fins and the four polar bosses (Fig. 5); for **H** ‖ [110], it intersects only two polar bosses.

In the rotation B, the mass of ψ has a small but clearly pronounced maximum for ≮ (**H**, [011]) = 16°; correspondingly, the direction of the plane of the orbit is shown in Fig. 5 by the dashed line. In this case, the location of the orbit should pass over a small local boss of the Fermi surface, which brings about an increase in the mass of ψ. This local boss cannot be situated on the intersection of the orbit with the (11̄0) plane, inasmuch as in such a case, as a consequence of the symmetry, there will also be two such bosses on the (110) plane close to the [001] pole over which would pass the orbit for **H** ‖ [110], i.e., for minimum value of the mass of ψ.

It then follows, with account of symmetry, that the local bosses should lie on the intersection of the plane of the orbit with the planes of the great circles {100}; the projections of these places are shaded in Fig. 5. It is clear that, by virtue of
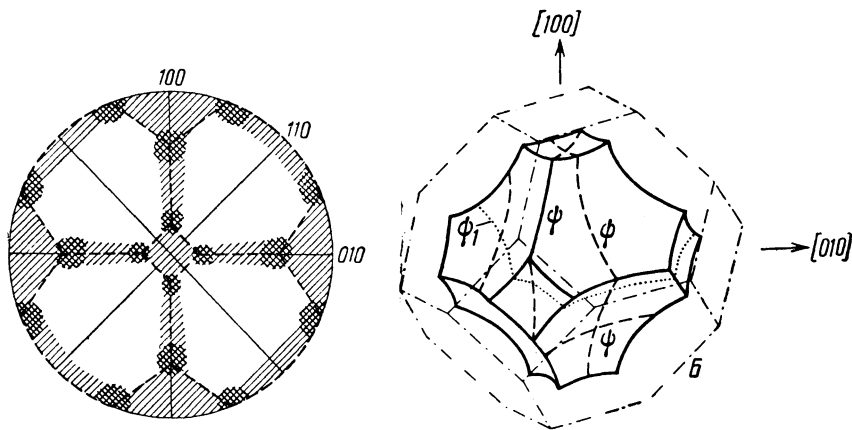
FIG. 6

FIG. 7

FIG. 6. Stereographic projection (indicated by the dotted lines) of the fins of the model of the Fermi surface of the second zone of lead (see Fig. 7). Shading to the right is the schematic representation of the convex fins and the polar bosses of the closed Fermi surface; shading to the left indicates the local bosses.

FIG. 7. Model of the closed hole Fermi surface of lead: the dash-dot lines are the boundaries of the fundamental Brillouin zone; the dotted lines are the positions of the orbits ψ for rational directions of the magnetic field.

symmetry, four such local bosses are distributed about each $\langle 100 \rangle$ pole.

For the rotation C, the mass increases because of the fact that at $\sphericalangle$ (H, [111]) = 24° the orbit intersects two local bosses (this location of the orbit is shown in Fig. 5 by the dash-dot line), and at the end of the rotation it passes along the convex fins, the four polar bosses and the eight local bosses. This location of the orbit, which is equivalent to the initial one for the rotation A, when H ‖ [010], corresponds to the absolute maximum of the effective mass of ψ.

The projections of the convex fins and the polar bosses of the Fermi surface are denoted in the stereographic projection of Fig. 6 by shading to the right and the projections of the local bosses located in correspondence with the conclusions given above are shaded to the left. The projection of the model of the hole Fermi surface of the second zone is shown there by a dotted line (see Fig. 7), constructed under the assumption of almost complete freedom of the electrons.[2,3] The obvious similarity of these two projections in their characteristic features serves as the basis for assuming the mass of ψ to belong to the hole Fermi surface of the second zone and its model close to the shape of the actual Fermi surface.

We note that the depth of the cyclotron resonance of ψ increases several fold as the field H rotates by ~5° from the direction H ‖ [100] to the [011] axis. This is brought about by the fact that for H ‖ [100] the orbit travels along the fins and local bosses, and only holes belonging only to a narrow belt on the Fermi surface act at resonance. When the field is rotated, the orbit falls into the more sloping parts of the Fermi surface, as a consequence of which, the number of resonant holes increases. This experimental fact is also in excellent agreement with the picture described above.

The model of a closed hole Fermi surface of the second zone of lead, constructed by the Harrison method,[3] is shown in Fig. 7;[2,3] the radius of the sphere of free electrons in wave number space is equal to $p_S = 1.24 \, (2\pi/a)$, where the lattice constant a = 4.939 Å. The positions of the orbit ψ of holes for rational directions of the magnetic field are shown on the surface. The form of the orbits ψ for the same directions of the field can be seen more clearly in Fig. 8, where the construction of three rational cross sections of the model of the Fermi surface of lead has been carried out.

The location of the noncentral orbit ψ_1, which arises for a direction of the magnetic field for which $\sphericalangle$ (H, [100]) = 30°, has been drawn in the (011) plane by the dotted lines. The corresponding effective mass is minimal in p_H (the component of the momentum parallel to H), since the
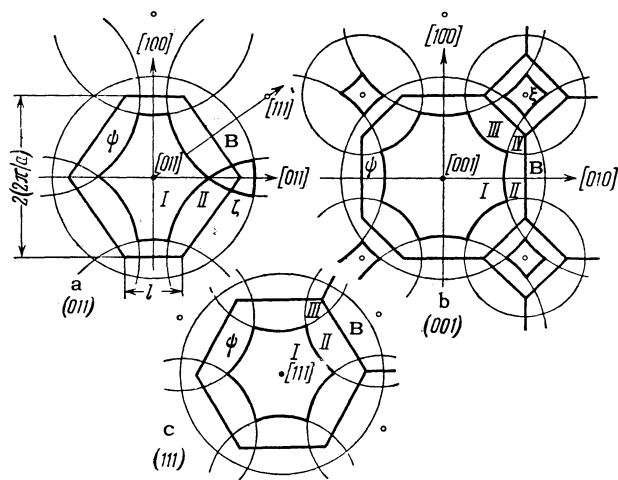

FIG. 8. Cross section of the model of the Fermi surface of lead. The solid circles are the lines of intersection with the given plane of the spheres of free electrons, B—boundaries of the fundamental Brillouin zone. The Roman numerals denote which spheres the given region belongs to at a single time.

neighboring orbits in p_H pass along the local bosses located close to the [010] and [001] poles. As follows from the model of Fig. 7, this extremal orbit ψ_1 can exist in the range $45° \gtrsim \sphericalangle(H, [100])$ $\gtrsim 10°$. Cyclotron resonance corresponding to these conditions, which is similar in amplitude and shape to the resonance ψ, is observed experimentally, and noted by the lines ψ_1 in Fig. 4a.

The effective mass, which corresponds to motion of the holes along these orbits, and also for intermediate directions of the field, can be computed from the Fermi surface model in the following way. The effective mass m* is determined by the formula

$$m^* = \frac{1}{2\pi} \oint_l \frac{dl}{v_\perp},$$ (4)

where the integral is taken over the closed orbit l in momentum space; $v_\perp$ is the component of the velocity of the electron perpendicular to the element of orbit dl and lying in the plane of the orbit.[5] Since an arbitrary orbit in the model constructed under the approximation of almost free electrons (Fig. 8) is made up of different radii, while $v_\perp$ is constant on each arc and is proportional to its radius, calculation of the relative effective mass $\mu = m^*/m_e$ reduces to the determination of the angular dimensions of the arc φ making up the orbit. Thus

$$\mu/\mu_c = \sum_n \varphi_n/2\pi, \qquad \mu_c = p_c/v_c m_e.$$ (5)

where the summation is carried out over all arcs making up the orbit, and μ_c is the relative mass of the electron moving with velocity v_c.

For the orbit ψ, under the condition H ‖ [111], calculation of the effective mass yields $\mu_{\psi[111]}$ = 0.77 μ_c. The chosen direction of the field for which the effective mass is a minimum is distinguished by the fact that the orbit corresponding to it passes along a minimum number of singularities of the Fermi surface model. This orbit only intersects six fins of the surface, if we omit the polar regions in which the distortions of the model should not be great. Because of this circumstance, calculation of the effective mass based on the knowledge of the shape of the orbit for the chosen direction of the field H ‖ [111] is much more reliable than for any other. Taking it into account (Fig. 4a) that $\mu_{\psi[111]}$ = 1.14, we get μ_c = 1.48. But since p_c = 1.24 $(2\pi/a)$, then the Fermi velocity of the current carriers of lead v_c = 1.24 × 10^8 cm/sec. Naturally, this is no more than a rough estimate of the value of v_c, justified only by the fact that the measurements of the Fermi velocity of electrons

of lead (in contrast to those of tin[6]) have not to date been carried out.

However, fundamental interest here attaches not to the value of v_c, but to the comparison of the anisotropy of the calculated and measured mass of ψ. The results of the calculation of μ_ψ for different directions of the magnetic field are shown in Fig. 4a by the circles. As is seen from this drawing, the character of the anisotropy of the computed mass is in full qualitative agreement with the experimental curve ψ.

Values of the effective masses found by Gold[2] as a function of the amplitude of the α oscillators of the magnetic susceptibility of lead on the temperature are shown in Fig. 4a. Keeping in mind the low accuracy of such a method of mass determination, it must be assumed that these data practically coincide with the values of the mass of ψ. The area of the corresponding orbit, computed from the period of oscillations, amounts to 1.05 $(2\pi/a)^2$, whence, assuming the orbit to be circular, we get its transverse dimension equal to ~ 1.02 $(2\pi/a)$, which agrees with the dimensions of the hole surface of the second zone (Fig. 7), confirming the interpretation of the α oscillations given by Gold. The results of magnetoacoustic measurements also practically agree with these numbers.[7]

Thus the results of analysis of experimental studies of the anisotropy of the mass of ψ and calculation of the anisotropy according to a model give, with sufficient accuracy, satisfactory testimony in support of the position that the model of the hole Fermi surface of the second zone of lead, constructed in the approximation of almost free electrons, is very close to the actual hole Fermi surface of lead.

THE MULTIPLY-CONNECTED ELECTRON FERMI SURFACE OF THE THIRD ZONE OF LEAD

The results of the experimental study of cyclotron resonance given in [1] confirmed the correctness of the surface model of the third zone shown in Fig. 9. On the other hand, it is interesting to carry out an analysis of experimental data with the purpose of obtaining evidence on the Fermi surface independent of any model.

According to Eq. (4) of [1], we have

$$\mu(\varphi) = \mu(0)/\cos\varphi.$$ (6)

In the polar set of coordinates (μ, φ), Eq. (6) describes a straight line perpendicular to the axis of the cylinder if the axis is parallel to the plane
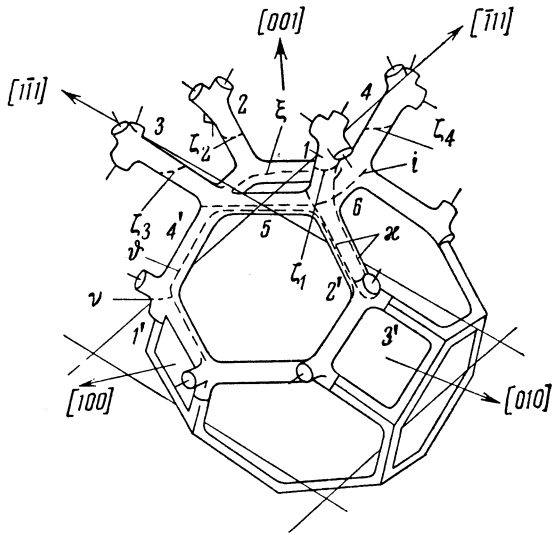
FIG. 9. Scheme of construction of the model of the multiply-connected electronic surface of the third zone. The dotted lines are the orbits of the electrons in the rational directions of the field, denoted by the same symbols that correspond to their masses in Fig. 3.

of the coordinates, or the projection of the axis on the plane of the coordinates if the axis is inclined at an angle with this plane.[1] Evidently, the absolute minimum of the value of $\mu(\varphi)$, i.e., of the line (6) from the origin, occurs in a direction parallel to the axis of the cylinder.

The group of lines $\zeta_1 - \zeta_6$, corresponding to the depth of the cyclotron resonances, is shown in the polar diagram of Fig. 4. They differ by their relatively small curvature, while the average segments of these lines are practically identical with straight lines. In particular, this applies to the line ζ_1 in the limits of rotation of the field $\sim 40°$ from the [011] axis in the (100) plane and $\sim 30°$ in the (0$\bar{1}$1) plane; this same mass of ζ_1 reaches a minimum value relative to all other masses of $\zeta_1 - \zeta_6$ for H ∥ [011]. The masses ζ_3, ζ_6 have a relative minimum in the (0$\bar{1}$1) plane for $\sphericalangle$(H, [100]) = 35°.

On the basis of these experimental facts, and of Eq. (6) established above, we can draw the following conclusions.

1. The anisotropy of the mass of ζ_1, measured in the two crystallographic planes, testified to the existence of an almost cylindrical tube with an axis parallel to the [011] direction—a tube which is part of the Fermi surface of lead, to which the current carriers yielding the cyclotron resonance ζ belong.

2. Since, as a consequence of the cubic symmetry in the reciprocal lattice of lead, there are at

least five directions $\langle 011 \rangle$ which are equivalent to the direction [011], the Fermi surface should have six cylindrical tubes that are identical in shape with axis parallel to the $\langle 011 \rangle$ directions. All six resonances of ζ belong to these tubes.

There is also a series of experimental facts which supports the following deductions. In the first place, there is the relatively large amplitude of resonances of ζ, which is brought about by the fact that the shape of the tubes is close to cylindrical. In the second place, there is the following circumstance: the orientation of the axis of the tube to which this or that effective mass of ζ belongs can be determined by analyzing the conditions for the excitation of cyclotron resonance, corresponding to this mass. From this point of view, the cyclotron resonance of the greatest depth should yield electrons which have a Fermi velocity in the spin layer parallel to the high frequency field. Such a velocity is achieved by electrons which belong to the tube, the axis of which is perpendicular to the current, i.e., the length of the specimen, as, for example, the tube 1 (Fig. 9) specimen 5. On the other hand, a resonance does not at all have to be excited in the tube, the axis of which is parallel to the current, as for example, in tube 2 of specimen 5 and tube 1 of specimen 2 (in this connection, the line in Fig. 4a is drawn according to measurements with specimen 1). The considerations that have been set forth are completely confirmed by experiment.

In addition to the two conclusions described above, one can make another important derivation on the basis of the experimental data. The fact that for considerable angles φ the mass $\mu(\varphi)$ increases more rapidly than according to the law (6) means that the area of the oblique cross section of the tube increases with increasing inclination more rapidly than the area of the cross section of the cylinder. Thus the tube, within the measurement accuracy is a cylinder in the middle region and broadens toward its ends. The broadening of the tube, and also the vanishing of the cyclotron resonance for large angles φ (Fig. 4a), points to the fact that each tube joins with the other parts of the Fermi surface at its end; it is natural to suppose that these parts are tubes, the axes of which are parallel to the other two-fold axes.

We now turn to the model of the singly-connected electron Fermi surface of the third zone of lead, constructed under the approximation of almost free electrons and simplified for the purpose of clarity. This model is shown in Fig. 9 in the form in which it was drawn by Gold.[2] It is

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[1] As a consequence of this, Fig. 5 in [1] is unnecessary, since everything which it demonstrates follows directly from the polar diagram of Fig. 3 in [1].

clear from Fig. 9 that the model of the surface of the third zone consists, as a whole, of identical tubes, the axes of which are the fins of the fundamental Brillouin zone, parallel to the six directions $\langle 011 \rangle$.

The tubes are linked at their ends, which are located close to the vertices of the fundamental zone and form a single multiply-connected surface which extends throughout the entire reciprocal lattice of lead.

The complete correspondence of the fundamental features of the model of the Fermi surface of the third zone with the derivations given above give grounds for attributing the cyclotron resonances of ζ to the electrons of the third zone. One can compare each of the resonances of the specific tube of the model and establish the fact that a one-to-one relation can be set up between them. The comparison was given in [1] for tubes 1, 2, 3, 4. It is not difficult to carry out a similar analysis of the behavior of the masses of ζ_1, $\zeta_3 - \zeta_6$ in the $(0\bar{1}1)$ plane of the diagram of Fig. 4a; in this case, there is also complete agreement with the model of the Fermi surface given in Fig. 9.

Thus all the fundamental properties of the behavior of the cyclotron resonances of ζ entirely agree with the most characteristic geometric features of the model of a multiply-connected electron Fermi surface of the third zone, constructed in the approximation of almost free electrons.

It was shown in a previous paper [1] that the presence of bulges at the points of union of the tubes, of which the multiply-connected surface is composed, successfully explains both the α oscillations discovered by Gold, [2] and also the value and the weak anisotropy of the mass of ν. A comparison of the mass of ν and the orbit ν in Fig. 9 has already been given in [1]. In the present paper the anisotropy of this mass is measured; in the $(0\bar{1}1)$ plane it turned out to be almost the same as in the (100) plane. This also confirms the conclusion drawn earlier. [1] Actually, the orbit of ν lies in the bulging position of the joining of the tubes. This fact, that the anisotropy of the mass of ν is also small in the different crystallographic planes, means that these bulges have a spheroidal shape.

Because of the existence of a bulge at points of joining of the tubes, the multiply-connected surface has [8,9] open directions along the $\langle 111 \rangle$ axis. As a consequence of the presence of the open directions, hole orbits must exist on this surface, capable of having a cyclotron resonance. The origin of such an orbit can be made clear with the help of Fig. 9. The magnetic field is oriented along the [110] axis, then the (110) surface perpendicular to

it gives a section of the Fermi surface that is open in two directions: $[1\bar{1}1]$ and $[\bar{1}11]$ (noncentral section). The straight lines shown in Fig. 9 of these directions are the axes of the "spirals" which consist of the tubes 1, 5, 4', 1' etc., and 3, 5, 2', 3' etc., the axes of which have the $\langle 011 \rangle$ direction. The entire Fermi surface can be represented as consisting of such "spirals" having open axial cross sections.

The closed hole orbit χ arises inside the "rhombus," which is formed by two pairs of "spirals," the axes of which are shown in Fig. 9 (two orbits in the limits of one zone for each direction of the field $\langle 110 \rangle$). The dimensions of the orbit are close to the dimensions of the orbit of ψ which lies on the surface of the second zone and, consequently, the effective mass of χ is close to the value of the mass of ψ, while it should increase together with increase in the parameter of the orbit χ upon departure of the field H from the [110] axis.

The curve of χ is shown in Fig. 4a, the character of which corresponds entirely to the characteristics described (the axis [011] is apparently equivalent to the axis [110]). The anisotropy of the amplitude of the cyclotron resonance corresponding to the mass of χ, and the character of its perturbations also confirm the comparison of this mass of the noncentral orbit of χ. The amplitude of the cyclotron resonance in sample 2, when $\sphericalangle(J, [011]) = 0$ (Fig. 2), is comparable with the strongest resonances of χ. Upon rotation of the magnetic field in the $(0\bar{1}1)$ plane by 4—5°, the amplitude of the resonance of χ decreases sharply and the resonance vanishes. Only in specimen 4, at $\sphericalangle(J, [0\bar{1}1]) = 28°$, was it possible to trace the anisotropy of the mass over a wide range of angles. The depth of the resonance also depends strongly on the angle of the field with respect to the surface of the specimen: an inclination of 10—15' is sufficient to decrease the amplitude of resonance by one order of magnitude. The facts thus enumerated are in agreement with the results of the research of Young, [9] who showed that the condition for optimal excitation of the resonance of χ on a specimen having a surface (100) is $\sphericalangle(J, [011]) = 60°$. Thus the orbit of χ in coordinate space is not a plane; this corresponds to the model of Fig. 9.

Investigations of the cyclotron resonance in specimen 5 with other orientations of the crystallographic axes (relative to the high frequency current) than in specimen 1 have made it possible to clear up the error in the determination of the value of the effective mass of the cyclotron resonance.

A one-third smaller mass of ξ was ascribed earlier to the third peak of this resonance. The resonance ι is observed also in the $(0\bar{1}1)$ plane, although of lesser amplitude and in a smaller range of angles. The character of the anisotropy of the mass of ι and its large value permit one to ascribe it tentatively to the orbit ι in Fig. 9.

THE FERMI SURFACE OF THE FOURTH ZONE OF LEAD

Let us now consider the problem of the electron surface of the fourth zone. This surface arises in the construction of the model of the Fermi surface of lead (Fig. 8); its elements are small closed surfaces, having a shape close to the surface of a tetrahedron, and distributed about the vertices of the fundamental Brillouin zone (six in each zone). The phase volume of the model of the fourth zone surface amounts to about 10 per cent of the volume of the model of the third zone surface. However, if it is taken into consideration that the actual third zone surface differs from the model by an appreciable (almost twofold) increase in the diameters of its nodes, then it is established as valid that the phase volume of the actual electron surface of the third zone is sufficiently large to contain the electrons assigned to the fourth zone in the model.

Thus it is quite probable that the fourth zone of the actual Fermi surface of lead is empty, which agrees with the absence of any experimental proofs of its existence. In this case, the requirement of equality (compensation) of the total phase volumes of the electron and hole surfaces, which is obligatory for a metal with even valence, applies only to the volumes of surfaces of the second and third zones.

However, it is impossible to confirm categorically that the fourth zone is empty on the basis of the results of this research, inasmuch as it is not possible to prove the equality of the volumes of surfaces of the second and third zones, since it is not possible to compute them with sufficient accuracy for this purpose.

At the same time, the effective masses of the electrons of the fourth zone, if such exists at all, can be shown to be small, and to fall in the region $\mu < 0.3$, in which the cyclotron resonance cannot be observed, because of the superconductivity of the lead. However, investigations of the cyclotron resonance in lead performed by Young[9] at a frequency of 36 Gc have shown the absence of resonances corresponding to the effective mass $0.3 > \mu > 0.08$. Moreover, since they possess a small mass and a correspondingly large mobility, the electrons

of the fourth zone should manifest themselves in the de Haas–van Alphen effect, such as takes place in bismuth, zinc, and tin. However, no long-period oscillations of the susceptibility of lead were observed by Gold.[2]

On the basis of the experimental facts thus enumerated, the absence of electrons from the fourth zone of lead is probable.

CONCLUSION

The experimental investigations of the cyclotron resonance in lead just described lead to the conclusion that all the fundamental and most characteristic features of the model of the Fermi surfaces of the second and third zones of lead, constructed in the approximation of almost free electrons, agree completely with the results of experiments.

The study of the closed hole surface of the second zone graphically and convincingly proves its very close similarity to the model. However, because of the comparatively small anisotropy of the effective masses of the holes and the absence of any sort of clearly demonstrated (experimentally) features of the behavior of the masses or directions of the magnetic field, quantitative corrections to the model were not obtained. Another reason for this is the relatively low resolving power of the measurements of hole cyclotron resonances, inasmuch as the latter are rather broad, a fact which is also connected with the small anisotropies of the hole masses.

The situation is different in the study of the multiply-connected electron surface of the third zone. The structure of this surface determines the existence of a series of strongly anisotropic masses and sharply marked characteristic directions of the magnetic field. The great depth and narrowness of the lines of electron cyclotron resonance made it possible to carry out the measurements with high resolving power and with great accuracy. As a result, not only was qualitative agreement obtained with the model, but also important dimensions of the third zone surface were found, which define the quantitative differences of the actual surface from its model.

It must be emphasized that the experimental results of this research, touching on the surfaces of the second and third zones of lead, make it possible to draw quite definite conclusions on the geometry of the Fermi surface, independent of any sort of model. This circumstance has an important methodological significance.

Figure 4a shows the curves of ϑ and κ consid-

ered in [1], and several other sections of curves whose interpretation has not yet been discussed. This does not mean, of course, that it is impossible to give any probable hypothesis on their occurrence. The fact, is, however, that the volume of the existing experimental data is insufficient for reliable testing of such assumptions. On the other hand, no interpretation of these masses can change the conclusions derived above in the course of discussion of the results of the experiments.

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