DIMENSION EFFECT IN THE PRESENCE OF ELECTRON DRIFT INSIDE A METAL

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We study a new type of size effect in metals in a magnetic field, connected with drift of electrons from the sample surface. It is shown that a set of field peaks which are narrow, periodic and slowly diminishing in amplitude occurs within the metal. These peaks may be due to electrons on open trajectories or near elliptic limiting points on the Fermi surface (in an inclined magnetic field). They arise as a result of focusing of electrons moving in the skin layer parallel to the metal surface by the field. The periodicity of the peaks is determined by the mean displacement of the electrons inside the metal. Anomalous penetration of the field leads to a periodic H-dependence of the impedance of a plane-parallel plate. The effect has been observed experimentally by measuring the surface impedance of tin at $f \approx 3$ Mc. Periodic oscillations of the impedance with the field in the presence of open trajectories as well as of limiting points were observed. Open trajectories can be detected and local values of the Gaussian curvature of the Fermi surface and mean free path of electrons in the metal may be measured by studying the period and amplitude of the size effect.

1. INTRODUCTION

IN recently published papers [1-5] it is shown that dimensional effects are observed in measurements of the surface impedance of metals when the electron mean free path is large. The gist of these effects is that singularities of various types are observed on the impedance vs. H curve when the characteristic dimensions of the electron trajectories in the magnetic field H are equal to the sample thickness d. The dimensional effects are much more pronounced in a high-frequency field than in the corresponding measurements with direct current. This is due to the presence of the small parameter with the dimension of length, namely the skin-layer depth δ.

On the other hand, it is shown in recent theoretical papers [6-8] that anomalous penetration of the electromagnetic field in the metal is possible in many cases. Narrow and rather slowly damped peaks of fields and currents are produced within the metal at distances that are multiples of the diameters D_{ext} of the extremal electron trajectories. These anomalies in the propagation of the electromagnetic waves are due to 'chains' of electron orbits. A trajectory passing in the skin layer near the surface, produces a new 'skin layer' at a depth $z = D_{ext}$, which in turn is a source for the occurrence of a similar 'skin layer' at a depth $z = 2D_{ext}$, etc. This anomalous penetration of the field into the metal leads to a unique dimensional effect in the plane-parallel metal plate. It is characterized by the occurrence of impedance singularities that are periodic in the magnetic field. This phenomenon is experimentally observed in tin [9].

The above-mentioned dimensional effects are due to electrons which move in closed orbits in both momentum and coordinate space, without drifting inside the metal (i.e., $\bar{v}_z = 0$, where $\bar{v}_z$ is the average projection of the electron velocity on the normal to the surface of the metal). Yet a metal in a constant magnetic field contains electrons with $\bar{v}_z \neq 0$. Electron drift inside the metal is possible in two cases. For closed orbits the drift motion occurs for all electrons of non-central sections of the Fermi surface in a magnetic field which is not parallel to the surface. For closed Fermi surfaces drift is possible also in a parallel field, owing to the presence of open orbits. In the present paper we show that in both cases there is anomalous penetration of the electromagnetic field in the metal and a corresponding dimensional effect.

2. PHYSICAL PICTURE OF ANOMALOUS PENETRATION OF THE FIELD IN A METAL IN THE PRESENCE OF ELECTRON DRIFT

Let the metallic half-space z > 0 be situated in an external magnetic field oriented at an angle φ to the surface. The electrons with velocities $\bar{v}_z \neq 0$ will move along a spiral trajectory with period of revolution $T = 2\pi/\Omega = 2\pi mc/|e|H$ (Ω— cyclotron frequency, m—effective mass). The average displacement of the electron inside the metal during the period T is $u = |\bar{v}_z T|$ $= |2\pi cm \bar{v}_z/eH|$. On the Fermi surface $\bar{v}_z T$ varies with p_H, the projection of the electron quasimomentum on the H direction. The relative number of electrons having a given displacement u will be largest for the extremal values $u(p_H)$ $= u_{ext}$.

In the case of interest to us, the anomalous skin effect, the following inequalities are satisfied

$$l \gg u_{ext} \gg \delta. \tag{2.1}$$

Consequently relatively many electrons reach a depth $z = nu_{ext}$ (n—integer), having the same state in momentum space as at the instant of leaving the metal surface. In particular, the extremum of $u(p_H)$ is reached when $p_H = p_{H\,max}$, where $p_{H\,max}$ corresponds to the elliptical limiting point on the Fermi surface. This phenomenon is well known in electron optics under the name 'focusing of electrons by a longitudinal magnetic field.' At the limiting point, the velocity of the electron v_0 is parallel to H, $|\bar{v}_z| = v_0 \times$ sin φ, and $mv_0 = K^{-1/2}$ (K is the Gaussian curvature of the Fermi surface $\varepsilon_0 = \varepsilon(p)$ at the limiting point). Consequently,

$$u_{lim} = (2\pi c/|e|H) K^{-1/2} \sin \varphi. \tag{2.2}$$

The 'focusing' effect should occur also in the presence of open trajectories with $\bar{v}_z \neq 0$. The electron orbit in momentum space and the projection of its trajectory in coordinate space on a plane perpendicular to H are similar, with a similarity coefficient eH/c, and are turned relative to each other through $\pi/2$. Therefore the direction of the average drift velocity depends not only on the direction of H, but also on the orientation of the normal n to the surface of the sample relative to the crystallographic axes. The average electron displacement u inside the metal is constant in magnitude for all open periodic trajectories and is given by

$$u = cb \cos \theta/|e|H, \tag{2.3}$$

where b is the period of the open orbit (reciprocal lattice) in momentum space and θ the angle between the direction of the opening and the surface of the metal.

Assume now that some of the focusing electrons moving in the skin layer have a velocity projection $v_z = 0$. Such electrons interact most effectively with the external electromagnetic field and make the principal contribution to the impedance. Being focused at a depth $z_n = nu_{ext}$, these 'effective' electrons reproduce the velocity increments Δv which they acquire from the field in the skin layer, i.e., in final analysis, they reproduce the high-frequency current which they produce on the surface. Therefore, field and current peaks appear at the depth $z_n = nu_{ext}$. It must be emphasized that, unlike the aforementioned anomalous penetration of the field with the aid of the 'chain of orbits,' these peaks are due to the drift of the electrons that move directly from the surface.

In the case of open orbits, in order to satisfy the requirement that focusing electrons include effective ones, it is necessary that some of the open trajectories be such that $v_z = 0$ at least in one point. In the case of the limiting point this requirement leads to the inequality $\psi > \varphi$, where ψ is the characteristic angular dimension of the vicinity of the limiting point, determining the velocity scatter of the focusing electrons. The quantity ψ is determined from the condition $|u(\psi) - u_{ext}| \approx \delta$. Hence

$$\psi \approx (\delta/v_0 T\varphi)^{1/2} > \varphi. \tag{2.4}$$

From (2.1) and (2.4) we get the following limitations on the angle φ:[1]

$$(\delta/v_0 T)^3 \ll \varphi^3 \ll \delta/v_0 T. \tag{2.5}$$

The occurrence of field peaks inside the metal can be explained in a different way which, incidently, is illustrative of the nature of the corresponding mathematical calculation. The sharp inhomogeneity of the field near the surface of the metal can be described with the aid of a superposition of monochromatic plane waves whose wave vectors k have a continuous spectrum of width $\Delta k \sim \delta^{-1}$.

The electrons drifting inside the metal interact most effectively with those field-spectrum harmonics whose wavelength λ is contained an inte-

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[1] In the opposite limiting case $\psi < \varphi$ (for example in a magnetic field normal to the surface) one should also observe anomalous penetration of the electromagnetic field into the metal. This phenomenon, however, is not connected with the 'effective' electrons and has a different character; its study will be the subject of a separate communication.

ger number of times in the distance u_{ext}: u_{ext} = $N\lambda$. This condition separates from the continuous spectrum a discrete series of wavelengths which can penetrate to an anomalously large depth in the metal. The interference of these waves gives rise to quasiperiodic field maxima at a depth $z_n = nu_{ext}$. The widths of these maxima are determined in final analysis by the width of the initial wavelength spectrum and are therefore of the order of $\delta \ll u_{ext}$. It is easy to see that their amplitude decreases with depth quite slowly, as $\exp(-z/l \sin \varphi)$.

It is obvious that the indicated field-distribution anomalies in a semi-infinite metal lead to periodical impedance singularities in a plane-parallel plate. The period is determined from the condition

$$d = nu_{ext} \qquad (2.6)$$

and is found to be

$$\Delta H = (2\pi c/|e|d)|m\bar{v}_z|_{ext}. \qquad (2.7)$$

In the case of an elliptical limiting point[2] it follows from (2.7) and (2.2) that

$$\Delta H = 2\pi c \sin \varphi \, K^{-1/2}/|e|d, \qquad (2.8)$$

and in the presence of open trajectories (2.3)

$$\Delta H = bc \cos\theta/|e|d. \qquad (2.9)$$

3. THEORY

The complete system of equation consists of Maxwell's equations and the kinetic equation for the distribution function of the electrons in the metal

$$\partial^2 E_\alpha(z)/\partial z^2 = -4\pi i\omega c^{-2}j_\alpha(z), \quad \alpha = (x, y), \qquad (3.1)$$

$$-i\omega f + v_z \partial f/\partial z + \Omega \partial f/\partial \tau + \nu f = eE(z)\mathbf{v} \, \partial f_0/\partial \varepsilon, \qquad (3.2)$$

$$\mathbf{j} = \frac{2e}{h^3} \int \mathbf{v} f d^3p, \qquad (3.3)$$

$$j_z(z) = 0. \qquad (3.4)$$

Here $E(z)$—intensity of the alternating electric field ($\sim e^{-i\omega t}$), $j(z)$—current density, ω—frequency of external field, f—non-equilibrium addition to the Fermi distribution function $f_0(\varepsilon - \varepsilon_0)$, $\varepsilon(p)$—energy, ε_0—chemical potential of the electrons, ν—frequency of collision between the electrons and the scatterers, $\tau = \Omega t$—dimensionless time (phase) of orbital electron motion, and h—

Planck's constant. Equation (3.4) is identical with the condition of electrical quasineutrality of the metal.

Equations (3.1)—(3.4) must be supplemented with boundary conditions on the metal surface. For the fields this calls for continuity of the tangential components E_α (α = x, y), while the distribution function call for diffuseness of the electron scattering. Under conditions of the anomalous skin effect (2.1), the character of the reflection of the electrons from the boundary does not exert a decisive influence on the field distribution and on the surface impedance in the half-space. This is physically connected with the fact that the main contribution to the current density is made by the 'effective' electrons, which move parallel to the surface in the skin layer. Therefore in the calculation of current density it is possible to disregard completely the boundary condition for f and to use the expression for the distribution function in an unbounded metal. (An account of the boundary, as is well known [10,6], does not change the dependence of the impedance on the external parameters and leads only to an inessential numerical factor of the order of unity.)

Equation (3.4) determines the longitudinal component of the field $E_z(z)$ in terms of E_x and E_y. If conditions (2.1) are satisfied, the quantity E_z in the kinetic equation (3.2) can be neglected, and (3.4) can be disregarded completely (see also [10,6]).

Thus, in the case of the anomalous skin effect, when δ is the smallest parameter with the dimension of length, the problem in a half-space can be reduced to the problem of finding the distribution of the field in an unbounded metal. To this end it is necessary to continue the field and the current outside the metal in suitable fashion.

As in the papers by Azbel' and one of the authors [6,7,10], we continue the field and the current into the region z < 0 in even fashion. This makes it possible to use the Fourier transformation for the solution of the system (3.1)—(3.4). The equations for the Fourier components of the electric field

$$\mathscr{E}_\alpha(k) = \frac{1}{\pi} \int_0^\infty E_\alpha(z) \cos kz \, dz \qquad (3.5)$$

are algebraic and can be readily solved. The result is

$$\mathscr{E}_\alpha(k) = -2 \, [k^2 \hat{I} - i4\pi\omega \, c^{-2}\hat{\sigma}(k)]_{\alpha\beta}^{-1} E'_\beta(0), \qquad (3.6)$$

where $\hat{I}$—unit matrix, $\hat{\sigma}$—conductivity tensor, the prime denotes the derivative with respect to z, and the index β denotes summation from 1 to 2.

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[2]In the case of a hyperbolic limiting point, the oscillations of the dimensional effect are missing, since the cyclotron frequency $\Omega = |e| H/mc$ at this point vanishes (the effective mass $m \to \infty$). In a parabolic limiting point Ω is finite, but K = 0 so that there are no oscillations of the dimensional effect (the corresponding period ΔH is infinite).

The Fourier components of the elements of the conductivity tensor $\sigma_{\alpha\beta}(k)$ in bulk metal are of the form

$$\sigma_{\alpha\beta}(k) = \frac{2e^2}{h^3}\int_0^\infty \left(-\frac{\partial f_0}{\partial\varepsilon}\right)d\varepsilon \int \frac{mdp_H}{\Omega}\int_0^{2\pi}d\tau\,v_\alpha(\tau)\int_{-\infty}^\tau d\tau'v_\beta(\tau')$$

$$\times\exp\left[\frac{\nu-i\omega}{\Omega}(\tau-\tau')\right]\cos\left(\frac{k}{\Omega}\int_\tau^{\tau'}v_z(\tau'')\,d\tau''\right). \qquad (3.7)$$

Owing to the condition (2.1) we get $kv/\Omega \sim v/\Omega\delta \gg 1$, and we can use the stationary-phase method [10] in the calculation of the integrals with respect to τ and τ'. As a result of simple calculations we arrive at the formula

$$\sigma_{\alpha\beta} = \frac{\pi e^2}{h^3|k|}\int_0^{2\pi}\frac{d\chi}{|K|}n_\alpha n_\beta + \frac{2\pi e^2}{h^3|k|}\,\mathrm{Re}\int_0^{2\pi}\frac{d\chi}{|K|}$$

$$\times\frac{n_\alpha n_\beta}{\exp(2\pi\nu/\Omega + ik\bar{v}_z T)-1}. \qquad (3.8)$$

Here n_α are the components of the unit velocity vector on the Fermi surface $\varepsilon(p) = \varepsilon_0$ and χ is the azimuthal angle in velocity space with polar axis along z; the integration with respect to χ is along the line of the stationary phase points $v_z(\tau, p_H) = 0$. In the derivation of (3.8) we have neglected ω compared with ν, and also the interference terms from the different points of stationary phase.

The occurrence of field peaks inside the metal is connected with periodic δ-like singularities of the $\sigma_{\alpha\beta}$ as functions of k. We shall show that such singularities can occur in the presence of open periodic trajectories, and also in the case of closed Fermi surfaces in an oblique magnetic field.

We consider first closed Fermi surfaces. The first term in (3.8) gives the well known expression for the Fourier component of the high-frequency conductivity tensor of a metal at H = 0. The singularities of $\sigma_{\alpha\beta}$ of interest to us are connected with the second term of (3.8):

$$\Delta\sigma_{\alpha\beta}(k) = \frac{2\pi e^2}{h^3|k|}\,\mathrm{Re}\int_0^{2\pi}\frac{d\chi}{|K|}\frac{n_\alpha n_\beta}{\exp(2\pi\bar{v}/\Omega + ik\bar{v}_z T)-1}. \qquad (3.9)$$

The main contribution to $\Delta\sigma_{\alpha\beta}(k)$ is made by electrons in which the displacement $u(\chi) = |\bar{v}_z T|$ is extremal, with $u_{ext}\neq 0$. The extremum of $u(\chi)$ must be attained on the central section, where $u(\chi) = 0$, and at the limiting point[3]. The vicinity of the central section gives

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[3]When the Fermi surface has a complicated shape, $u(\chi)$ can have an extremum also at other sections. The corresponding calculation does not differ at all from the case of the limiting point given below.

a non-oscillating addition to $\Delta\sigma_{\alpha\beta}$ (3.9). We therefore confine ourselves to the calculation of the contribution to $\Delta\sigma_{\alpha\beta}$ from the electrons near the limiting point. Resonance maxima at $ku_{lim} = 2\pi n$ are possessed only by the element $\Delta\sigma_{yy}$ (the y axis is directed along the projection of $\vec{H}$ on the plane z = 0). The remaining elements of $\Delta\sigma_{\alpha\beta}$ have no singularities, since $n_x \approx 0$ near the limiting point. For this reason only the $E_y(z)$ component will have corresponding anomalies in the metal.

We introduce the symbols $\Delta = ku_{lim}/2\pi - n$ and $\bar{\nu}/\Omega = \gamma$. For small values of $|\Delta|$ and γ, the expression (3.9) can be rewritten as

$$\Delta\sigma_{yy} = \frac{2e^2}{h^3kK|ku''|^{1/2}}\frac{[(\gamma^2+\Delta^2)^{1/2}+\Delta]^{1/2}}{(\gamma^2+\Delta^2)^{1/2}}. \qquad (3.10)$$

The values of all the quantities in (3.10) are taken at the limiting point, $u'' = \partial^2 u(\chi_{lim})/\partial\chi^2$. From (3.10) it follows that $\Delta\sigma_{yy}$ has as a function of k a series of narrow maxima located at $|ku_{lim}| \approx 2\pi n$, with a relative width $|\Delta| \sim \gamma$. The value of $\Delta\sigma_{yy}$ at the maximum is on the order of $\sigma_{yy}(0)(n\gamma)^{-1/2}$.

Analogous singularities in a magnetic field parallel to the surface of the metal are possessed by the elements of the conductivity tensor in the presence of open periodic trajectories. In this case the quantity $\sigma_{\alpha\beta}(0)$ contributes a small correction to $\Delta\sigma_{\alpha\beta}$. The expression for the conductivity can be represented in the form

$$\sigma_{\alpha\beta}(k) = \frac{e^2}{h^3|k|}\int_{closed}\frac{n_\alpha n_\beta\,\Omega d\chi}{|K||\bar{v}|} + \frac{e^2}{h^3|k|}\int_{open}\frac{n_\alpha n_\beta\,d\chi}{|K|}$$

$$\times\left[\frac{\bar{v}/\Omega}{(\bar{v}/\Omega)^2+\pi^{-2}\sin^2(ku/2)}\right]. \qquad (3.11)$$

In the first term of (3.11) the integration is over the part of the $v_z = 0$ curve belonging to the closed sections of the Fermi surface. The second term is due to the contributions from the electrons on the open periodic trajectories. It is different from zero when the open trajectories have points at which the electron velocity is parallel to the surface of the metal. Since the value of u does not depend on χ the cyclotron frequency Ω varies slowly with χ, the factor in the square brackets can be taken outside the integral, with $\bar{\nu}/\Omega$ replaced by some characteristic value $(\bar{\nu}/\Omega)_0 = \gamma_0$.

To simplify the subsequent calculations, we assume that the conductivity tensor $\sigma_{\alpha\beta}^{closed}$ [the first term in (3.11)], due to the closed trajectories, reduces to the principal axes simultaneously with $\sigma_{\alpha\beta}^{open}$ (this assumption does not affect the final results). Expression (3.11)

can be written in terms of the principal axes in the form

$$\sigma_\mu(k) = \frac{3\pi}{4}\frac{B_\mu}{|k|}\left(1 + \beta_\mu\frac{\gamma_0^2}{\gamma_0^2 + \pi^{-2}\sin^2(ku/2)}\right), \quad (3.12)$$

where

$$B_\mu = \frac{4e^2}{3\pi\,h^3}\int_{\text{closed}}\frac{n_\mu^2\,\Omega d\chi}{|K|\,\bar{v}}$$

are the corresponding principal value of the tensor $B_{\alpha\beta}(\sim\sigma_{\alpha\beta}^{\text{closed}})$, and β_μ determines the relative 'weight' of the open periodic trajectories. It is obvious from (3.12) that as in the case of a limiting point, the quantity $\sigma_\mu(k)$ has for $ku = 2\pi n$ many sharp and narrow maxima with widths $|\Delta| = |ku - 2\pi n| \sim \gamma_0$.

Since the character of the singularities of $\sigma(k)$ is the same in both considered cases, only one need be investigated to calculate the field distribution in the metal. The calculations are best carried out for the case of open trajectories, where there is a single analytic expression for $\sigma(k)$ (we shall henceforth omit the index μ) both near and far from the maxima. The distribution of the field in the metal is described by the function

$$J(\zeta) = -\frac{\pi}{2}\frac{E(\zeta)}{E'(0)} = u\int_0^\infty x\cos x\zeta dx$$

$$\times\left\{x^3 - iM^3\left[1 + \beta\frac{\gamma_0^2}{\gamma_0^2 + \pi^{-2}\sin^2(x/2)}\right]\right\}^{-1}, \quad (3.13)$$

where we have introduced the notation

$$\zeta = z/u; \quad ku = x; \quad M = (3\pi^2\omega Bu^3/c^2)^{1/3} \approx u/\delta \gg 1;$$
$$\delta \approx (c^2 u/3\pi^2\omega\sigma_0)^{1/3}.$$

Recognizing that the integrand in (3.13) has several narrow minima for $x \approx 2\pi n$, the expression for $J(\zeta)$ can be reduced to the form

$$J(\zeta) = J_0(\zeta) + \Delta J(\zeta), \quad (3.14)$$

where the function

$$J_0(\zeta) = u\int_0^\infty\frac{x\cos x\zeta}{x^3 - iM^3}dx$$

differs from zero only for $|\zeta| \lesssim M^{-1}$, (i.e., $z \lesssim \delta$) and describes the sharp damping of the field near the surface of the metal.

The function $\Delta J(\zeta)$ represents a series of narrow slowly damped field peaks, separated by distances $z = nu(\zeta = n)$:

$$\Delta J(\zeta) = 2\gamma_0 iM^3 u$$

$$\times\sum_{m=1}^{\infty}\frac{\cos(2\pi m\zeta)\,2\pi m\exp\{-2\pi\gamma_0\zeta[1 - i\beta M^3((2\pi m)^3 - iM^3)^{-1}]\}}{[(2\pi m)^3 - iM^3]^{3/2}[(2\pi m)^3 - iM^3(1 + \beta)]^{1/2}}$$
$$(3.15)$$

Near the n-th maximum $|\zeta - n| \ll M^{-1}$, i.e., $|z - nu| \ll \delta$:

$$\Delta J_{max}(n) \approx CM^{-1}u\gamma_0\beta\exp(-2\pi\gamma_0 nu + \pi i/6),$$
$$C \sim 1. \quad (3.16)$$

Far from the maximum at $|\zeta - n| \gg M^{-1}$

$$\Delta J(\zeta) \approx \Delta J_{max}(n)/|M(\zeta - n)|^2. \quad (3.17)$$

Thus, the width of each maximum is $|z - nu| \sim \delta$, and the relative height is $\Delta J(n)/J_0(0) \sim (\nu/\Omega)_0 e^{-z/l}$ (l—characteristic mean free path of the electrons on the open periodic trajectories). The schematic form of the field distribution in the metal is shown in Fig. 1.
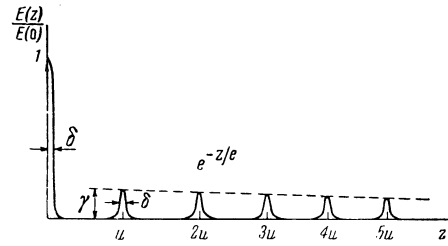

FIG. 1

In the case of a limiting point, the quantity γ_0 is replaced by the value of $\bar{v}/\Omega$ at this point. The decrease in the flashes on going inside the metal is characterized by $\exp(-z/l_{\lim}\sin\varphi)$ ($l_{\lim}$—mean free path of the electron at the limiting point). The relative amplitude of the flash is $|\Delta J_{max}/J_0(0)| \approx \gamma_0\exp(-2\pi\gamma nu)$ in a strong field with $(u\nu/\Omega)_{\lim} \ll \delta$, and of the order of $(\delta/l\varphi)^{1/2}u_{\lim}$ in a weak field, when $(u\nu/\Omega)_{\lim} \gg \delta$.

It is curious that the line widths and heights of the maxima are determined by different parameters. The width $\Delta z \sim \delta$ is connected with the characteristic width of the spectrum of the plane waves ($\Delta k \sim \delta^{-1}$) arising in the skin layer. The heights of the maxima and their decrease with distance are governed by the non-monochromaticity of the waves that penetrate a large distance in the metal. The damping of these waves is determined by the electron mean free path l, since their interaction with the electron has a resonant character. Therefore, with increasing mean free path, the heights of the first field peaks decrease because of the increase in their number inside the metal.

This clarifies the character of the field distribution in a plane-parallel plate. If the electrons cannot cover the distance between the plate surface during the mean free path time, then the field distribution in the plate should be the same as in a half-space. In the opposite case, that of a

thin plate, the role of the mean free path is played here by the path covered by the electrons in the plate, i.e., l is replaced by d for open trajectories and by $d/\sin \varphi$ for the limiting point. In the latter case an exact calculation of the distribution of the field in the plate is rather difficult, since it is necessary to take into account the influence of the surfaces of the sample on the metal conductivity operator.

4. EXPERIMENT

We used the previously developed [5] modulation procedure to measure the imaginary part of the surface impedance of a metallic sample, due to the magnetic field. The investigated sample was placed inside the coil of a resonant circuit. The oscillation frequency f was determined by the inductance of the resonating circuit with the sample, so that its variation was proportional to the change in the effective depth of penetration δ_{eff} of the alternating magnetic field $H_\sim$ in the metal

$$\delta_{eff} = \mathrm{Re} \frac{\int_0^d H_\sim (z)\, dz}{H_\sim (0)} = - \mathrm{Re} \frac{E(0) - E_d}{E'(0)}. \qquad (4.1)$$

Here E_d—electric field on the second side of the plate, due to the anomalous penetration of the wave in the metal.

Since the distribution of the field in the plate is in the first approximation the same as in a half space, the quantity $E(0)/E'(0)$ is proportional to the impedance of the half space, and varies smoothly with variation of the magnetic field. The dimensional effect of interest to us is connected with $E_d/E'(0)$ and is determined by the emergence of the next field peak to the other surface of the sample z = d. Using (3.13) and (3.14) we can readily see that the experimentally measured derivative $\partial f/\partial H$ is connected with the distribu-

tion function of the field in the half space by the relation

$$\frac{\partial f}{\partial H} = G \frac{\partial}{\partial H} \mathrm{Re}\, \Delta J \left(\frac{d}{u}\right), \quad G > 0. \qquad (4.2)$$

The experiments were carried out with tin specimens grown, in dismountable quartz molds [1], from a metal containing $\sim 10^{-4}$ percent of impurities. The samples were discs 17.8 mm in diameter and d = 0.4 mm thick.

To study the dimensional effects due to open periodic trajectories, we used a specimen with a normal to the surface n $\parallel$ [001]. According to the data of Alekseevskiĭ and Gaĭdukov [12], the Fermi surface of tin is open along the $\langle 100 \rangle$ and $\langle 110 \rangle$ axes. Therefore, when the magnetic field H lies in the plane of the plate and is directed along one of the axes $\langle 110 \rangle$ (or $\langle 100 \rangle$) the electron moving on the Fermi surface along the second axis $\langle 110 \rangle$ (or $\langle 100 \rangle$) on an open orbit will drift from one surface of the sample to the other.

The plot of $\partial f/\partial H$ for H $\parallel$ [110] (Fig. 2) shows clearly the sequence of narrow periodic singularities in the fields $H_n = nH_1$ (n = 1, 2, ..., 12). The period ΔH measured from the positions of the peaks is approximately 3 per cent larger than the corresponding value of ΔH calculated from formula (2.9). This discrepancy must apparently be attributed to the error in the measurement of the sample thickness. In the interval between the main lines, approximately half way between them, there is seen a second weaker system of lines having the same period but observed in fields $H_0 + nH_1$ ($H_0 \approx 0.5\ H_1$).

The curve on Fig. 2 corresponds to polarization of the alternating field E $\perp$ H. Analogous curves for E $\parallel$ H have the same periodicity in H and differ only slightly from that shown in Fig. 2.

A change up to 2—3° in the angle of inclination of the field relative to the surface of the sample

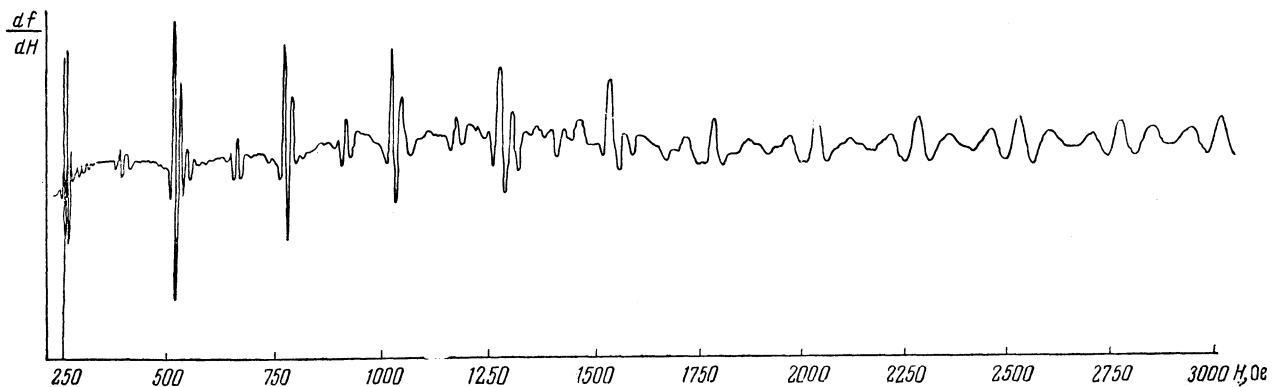FIG. 2. Dimensional effect lines due to open periodic trajectories; n $\parallel$ [001], variable electrical field E $\parallel$ [$\bar{1}$10], H $\parallel$ [110], temperature 2.0°K, f = 3.2 Mc, d = 0.40 mm.
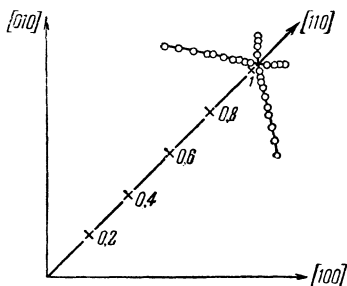
FIG. 3. Angular dependence of the position of the first line of the dimensional effect, due to the open trajectories. The scale is in units of b (b is the reciprocal lattice period in [110] direction).

does not change the picture of the effect. At the same time, a small (~ 1°) rotation of the field in the plane of the sample immediately smears out all the lines with n > 1. Only the line in the field H_1 remains and splits when the field is rotated (see Fig. 3), with one of the lines disappearing rapidly, and the other remaining up to $\kappa \sim 15°$. The angular dimension of the region of the directions of H where the dimensional effect from the open trajectories is observed agrees well with the data of Alekseevskiĭ and Gaĭdukov[12], According to [12] (see Fig. 4 of that paper), open periodic orbits remain when the field direction is varied only if H remains in the (110) plane. In our experiments this condition is satisfied when the field is inclined.

An analogous effect, but in much weaker form, is observed for H ‖ [100]. Here, too, two systems of lines are seen, with $H_0' = 0.4H_1'$. Like in the [110] direction, H_1' exceeds the calculated value by approximately 2 per cent. It became possible to observe three principal maxima and two additional maxima. The second system of lines may be due to the rather complicated form of the open electron trajectories in tin; in particular, the trajectories can have several points with $v_z = 0$. This circumstance can greatly complicate the line shape, and also lead to the appearance of additional singularities on the curves.

The existence of such orbits in tin is quite probable, since it is predicted by the model of almost-free electrons (see Fig. 13b in [5]). Nor can we exclude the possibility that the appearance of the second system of lines is due to the small fraction of electrons which are specularly reflected from the surface of the metal (the authors are indebted to M. Ya. Azbel' for the last remark). .

The dimensional effect connected with the electrons near the limiting points was investigated with the same set-up but with H making a small angle with the surface of the sample. The inclination of the field could be produced and regulated by inclining the dewar (i.e., the sample with the coil) relative to the electromagnet and also with the aid of an additional vertical solenoid

inside the dewar. Part of the current flowing through the electromagnet was diverted to the solenoid. Samples with n ‖ [001] (sample 1) and with $\bar{n}$ ‖ [010] (sample 2) were investigated. (In both samples the angle between the normal and the crystallographic axis was approximately 40'.) The dimensional effect was observed in field directions close to [100].

A typical plot of $\partial f/\partial H$ against H for different angles φ is shown in Fig. 4. The presence of narrow impedance singularities that are periodic with the field is observed, with an amplitude and period that increase with increasing φ. Unlike the dimensional effects on open trajectories, the amplitudes of the maxima hardly decrease with the field (and in some cases even increase somewhat). In agreement with the theory, the effect was observed at small angles between the vectors E and H (longitudinal polarization).
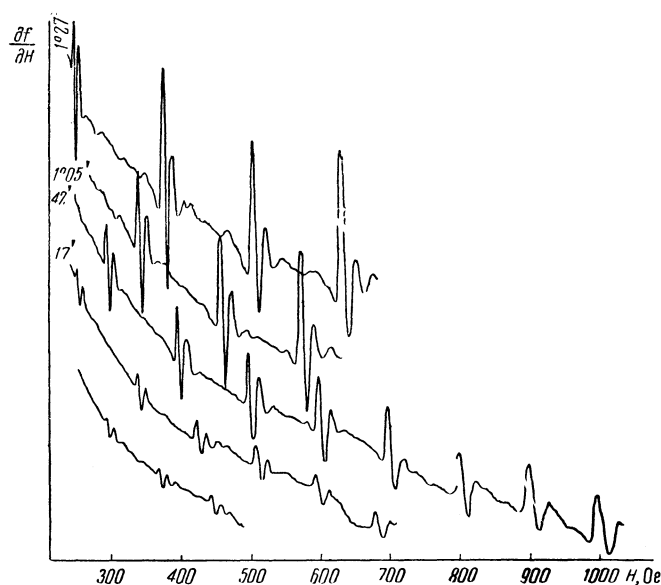

FIG. 4. Dimensional effect lines due to the limiting point; n ‖ [010], E ‖ [100], H ‖ (001), temperature 1.9°K, f = 3.2 Mc, d = 0.39 mm. To the left of the curves is indicated the variation of the angle of inclination $\Delta\varphi$ referred to the inclination of the field for the lower curve.

Figure 5 shows the variation of the period ΔH with the angle of inclination φ for sample 2 (curve 1). Since the quantity measured in the experiment is merely the change in angle of inclination $\Delta\varphi$, the absolute value of the angle φ is obtained by linear extrapolation to $\Delta H(\varphi) = 0$. Using (2.2), we can calculate the Gaussian curvature of the Fermi surface in the [100] direction. The same limiting point causes the dimensional effect in sample 1. The only difference is that when the magnetic field is inclined relative to the
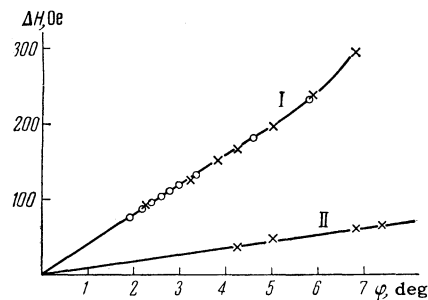
FIG. 5. Dependence of the period of oscillations on the inclination of the field for sample 2, with H || (001). Different symbols designate different series of measurements. Some of the data obtained in the series (designated by the circle) are shown in Fig. 4.

surface, the field H moves in the other crystallographic plane. Therefore the measurements of the periods ΔH should yield identical values of K for both samples. It was found however, that $K^{-1/2} = 2.37 \times 10^{-19}$ g/cm-sec for sample 1 and 2.19×10^{-19} for sample 2, i.e., a difference of approximately 8 per cent. (For comparison we indicate that the radius of the Fermi sphere in the known model of almost-free electrons for tin is $p_0 = 1.73 \times 10^{-19}$ g/cm-sec).

Such a discrepancy cannot be attributed to the error in the measurement of the thickness of the plates. Actually, the difference is connected with the displacement of the limiting point over the Fermi surface with varying angle of inclination φ. The dependence of K on φ leads to an error in the determination of absolute value of φ in linear extrapolation of the curve $\Delta H(\varphi) \to 0$.

The discrepancy in the values of K can be decreased somewhat by introducing suitable corrections. If we fix the angle of inclination φ and rotate H about the normal n to the surface, then the variation of the period of oscillation ΔH is proportional to the change of $K^{-1/2}$ as a function of the angle of rotation κ. In sample 1 at $\varphi \approx 2°$, the period ΔH does not depend on κ if $|\kappa| < 4°$. In sample 2, the period of ΔH varies as a function of κ by approximately 6 per cent when κ is changed by 3° (see Fig. 6); for $\kappa > 3°$, the lines due to the dimensional effect drop out. This is connected with the angular region of existence of the limiting point on the Fermi surface. Accordingly, in sample 2 the effect was observed only for $\varphi < 3°$. If we now take into account the dependence $\Delta H(\kappa)$ (obtained with sample 2) on the extrapolated plot of $\Delta H(\varphi)$ for sample 1, then the discrepancy in the values of $K^{-1/2}$ can be reduced to 3.5 per cent.
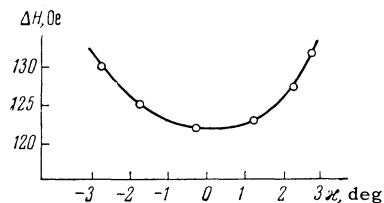

FIG. 6. Dependence of the period of oscillations of the dimensional effect for limiting point I on the rotation of H about the normal to the surface of specimen 2. $\varphi = 3°$; $\kappa = 0$ for H || (001).
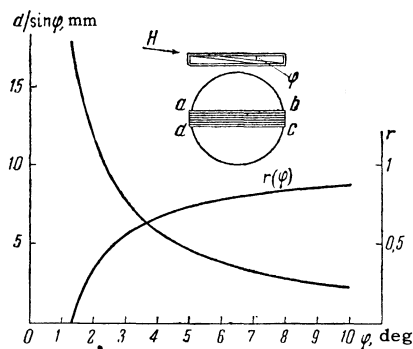

FIG. 7. Variation of the conditions of observation of the dimensional effect on a limiting point with increasing inclination of the magnetic field relative to the sample surface. abcd—oscillator coil.

To increase the accuracy with which K is measured, it is convenient to decrease the angle of inclination φ. However, with decreasing φ the amplitude of the maxima increases very rapidly for two reasons. First, the path length d/sin φ, which the electrons must cover from one surface of the plate to the other, increases. Second, as can be seen from Fig. 7, the 'useful' area on the surface of the sample decreases with decreasing φ. The coefficient of reduction of the useful area is $r(\varphi) = 1 - d\rho^{-1} \cot \varphi$, where ρ is the diameter of the disc. Nonetheless, on sample 1 the effect is clearly seen up to an angle $\varphi = 1°30'$ when d/sin $\varphi = 15.3$ mm and r = 0.14.

Owing to the complexity of the Fermi surface of tin, one given direction of H may correspond to not one but several different elliptical limiting points. Indeed, with starting with $\varphi \approx 4°$, sample 2 shows still another system of lines due to the dimensional effect from the limiting point with $K^{-1/2} \approx 0.48 \times 10^{-19}$ g/cm-sec. When H is inclined to the (001) surface, the lines split (see Fig. 8); one of the branches disappears after 2° and the other can be traced up to angles on the order of 20°. The dependence of the period ΔH on φ for this limiting point (for $\kappa = 0$) is shown in Fig. 5 (curve II).
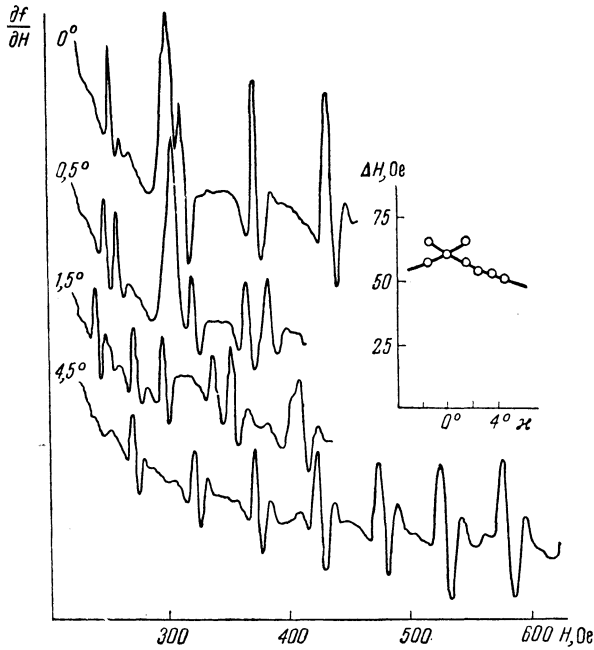
FIG. 8. Splitting of the lines of the dimensional effect of limiting point II upon rotation of H about the normal n || [010]; $\varphi = 6°50'$; E || [100]; temperature 2.2°K, f = 3.2 Mc; d = 0.39 mm. To the left of the curves is indicated the angle κ; $\kappa = 0$ for H || (001). Single peak on the upper two curves—effect of limiting point I (for given φ it exists only if $\kappa < 1°$).

5. DISCUSSION OF RESULTS

There is no doubt that the size-effect experiments described above are connected with the drift motion of the electrons into the metal. Narrow lines, which are periodic in the direct field, are observed, in full accord with the physical picture and with the theoretical analysis. The investigation of the periodicity of the oscillations permits reliable identification of the effects due both to the open trajectories and to the limiting points. Most experimental facts agree splendidly with the deductions of the theory. An exception is that the slower decrease in the amplitude of the observed maxima decreases more slowly with the field than would follow from the theory ($\partial f/\partial H \sim n^{-1}$ for the limiting points at $(u\nu/\Omega)_{lim} \ll \delta$, $\partial f/\partial H \sim n^{-2}$ for open trajectories—see Sec. 3). It is possible that this discrepancy is due to the same factors that cause the disagreement with the predictions of the theory of monotonic dependence of the impedance of tin on H at these frequencies.

From the point of view of the possibilities of determining the characteristics of the energy spectra of the electrons in metals, the most interesting is the dimensional effect due to the limiting points. From measurements of the period of the oscillations it is possible to determine the local

values of the Gaussian curvature of the Fermi surface in all its elliptical points. On the other hand, a study of cyclotron resonance gives the value of the effective mass at the limiting point [10,13]. The aggregate of these data makes it possible to calculate directly the local velocity by means of the formula $v = (m\sqrt{K})^{-1}$.

Khaĭkin[13] observed five mass values at H || [100] (in Fig. 4c of [13] they are numbered 10, 11, 13, 17, and 19 with two of them (17 and 19) probably coinciding on the [100] axis). If we assign different masses to the limiting points observed in the present paper, we obtain the possible values of the velocities (see Table I). For comparison we indicate that in the model of almost free electrons $v = p_0/m_0 = 1.9 \times 10^8$ cm/sec.

Table I

Effective masses from [13]		$v \cdot 10^{-8} = (m\sqrt{K})^{-1} \cdot 10^{-8}$, cm/sec	
No. of mass	m/m_0	$K_I^{-1/2} = 2.2 \cdot 10^{-19}$, g/cm-sec	$K_{II}^{-1/2} = 0.5 \cdot 10^{-19}$, g/cm-sec
13	~0.20	12	2.6
17,19	~0.7	3.5	0.75
11	1.15	2.1	0.46
10	1.60	1.53	0.33

Cyclotron resonance is the most intense for mass 10 and is observed in the same angle interval as the size effect from the limiting point I. We can therefore assume with assurance that they are identical. The lines of the dimensional effect from the limiting point II split when H is inclined away from the (001) plane, the same as in the case of cyclotron resonance for masses 17 and 19.
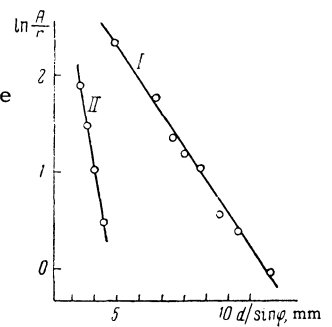

FIG. 9. Relative change in the amplitude of the dimensional effect with variation of the angle of inclination φ (with allowance for the coefficient $r(\varphi)$).

Table II

Limiting point	$K^{-1/2} \cdot 10^{19}$, g/cm-sec	m/m_0	$v \cdot 10^{-8}$, cm/sec	l, mm
I	2.23	1.60	1.53	2.8
II	0.48	0.7	0.75	0.7

Consequently, we can assume that the limiting point II corresponds to masses 17 and 19. Therefore the values of the velocity 0.75 and 1.53 are shown in the table in italics.

The dependence of the relative amplitude of the dimensional effect on the angle φ (for H = const) makes it possible to estimate the electron mean free path near the given limiting point. The change of the amplitude with increasing angle of inclination for $d \gg l \sin \varphi$ is essentially like $\exp(-d/l \sin \varphi)$, see Fig. 9. By determining the mean free path from the slope of the line, we obtain for the limiting point I a value $l = 2.8$ mm while for point II we have $l \approx 0.7$ mm. These values of l agree well with the results of [14], in which $l \approx 3$ mm was obtained from measurements of the static conductivity of tin of the same purity. It is interesting to note that the mean free path, in accordance with our measurements, is different for electron groups on the Fermi surface—see Table II, in which are gathered the characteristic of the electrons for the two limiting points which we investigated in the [100] direction.

Thus, the dimensional effect in the presence of drift of the electrons is a convenient and reliable method of measuring such important characteristics as the local value of the Gaussian curvature and the mean free path. In addition, it makes it possible to observe open electron trajectories in the same manner as with galvanomagnetic effects [12] and magnetoacoustic resonance [15].

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