NEGATIVE ABSORPTION IN A NONEQUILIBRIUM HYDROGEN PLASMA

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Conditions are examined for which a nonequilibrium hydrogen plasma can be regarded as a medium with a negative absorption sufficient for the creation of a generator operating close to the frequencies of ionization of hydrogen from the ground or low-lying levels.

THE creation of generators making use of new materials and working in new frequency ranges, including the ultraviolet, is of interest for physics and technology. In the present paper, the possibility is considered (in principle) of the creation of a generator working on the basis of the recombination of protons and electrons in a nonequilibrium hydrogen plasma which makes it possible to obtain quasimonochromatic radiation at several frequencies in the ultraviolet, visible and infrared regions.

In the problem under consideration, a dense, supercooled hydrogen plasma serves as a medium with negative absorption; the source of negative absorption are the induced transitions from states located close to the edge of the continuous spectrum to a lower discrete level of hydrogen.

The smallness of the mean time between collisions makes it possible in principle to reduce rapidly the electron temperature of a dense plasma; relaxation to lower discrete levels takes place comparatively slowly. A highly ionized plasma with rapidly decreasing kinetic temperature is a medium with a large inversion population, inasmuch as the only levels populated in it lie close to the edge of the continuous spectrum. If such a medium is placed between two mirrors (as is usually done in optical generators), generation takes place near frequencies corresponding to transitions from the edge of the continuous spectrum to one or several lower discrete levels of the hydrogen.[1)]

Broadening and splitting of levels in a dense low-temperature hydrogen plasma lead to the fusion of discrete levels with high quantum numbers $n > n_1$ in a quasicontinuous spectrum (the Ingliss-Teller effect). The number n_1 of the last discrete level is determined by the formula (see [1])

$$\log N_p = 23.3 - 7.5 \log n_1, \qquad (1)$$

where N_p is the number of protons in 1 cm^3. The mean time τ_{st} between electron collisions determines the rate of establishment of the electron temperature; it is estimated by the formula (see [2])

$$\frac{1}{\tau_{st}} = N_e v \sigma_{st} \sim N_e v \left(\frac{e^2}{mv^2}\right)^2 \ln \Lambda,$$

in which N_e is the number of electrons in 1 cm^3, v is the mean velocity, and $\ln \Lambda$ is the Coulomb logarithm. For the plasma densities of interest to us here, $N_p \sim N_e \sim 10^{-15} - 10^{-16}$ cm^{-3}, the time of transition of an electron from the continuous spectrum to the quasicontinuous part as a consequence of triple collisions is shown to be a quantity of the same order as $\tau_{st} \sim 10^{-10} - 10^{-11}$ sec. Thus after a time of the order of $\tau \sim 10^{-10} - 10^{-11}$ sec, a single distribution of electrons $N_e(E)$ is established in the dense plasma in the continuous and quasicontinuous portions of the spectrum. As a consequence of the nonradiative transitions, a rapid filling of the upper discrete levels also takes place.

We now proceed to numerical estimates, which show in principle the possibility of the preparation of a medium of hydrogen plasma with negative absorption sufficient for the creation of a generator. We shall discuss transitions to the ground state in more detail; here, without making concrete the methods of rapid cooling of the plasma and its removal from the active volume produced in the generation of neutral hydrogen, we shall not analyze the kinetics of development of the generation. In particular, we shall not discuss here the condition for continuity of generation.[2)]

[1)]In such a medium, negative absorption also takes place at frequencies corresponding to transitions between discrete excited levels with small quantum numbers, but generation by such transitions would be less intense.

[2)]At relatively small rates of removal of heat and of neutral atoms, discontinuous generation in small pulses can take place.

The principal characteristic of the amplifying medium is the coefficient of negative absorption $\kappa^+(E)$, which indicates how much the flux of the quanta increases as a consequence of induced transitions with radiation as they traverse a path of 1 cm in the medium. Assuming that the intensity of the plane monochromatic wave passing through the medium is not too great, and that it does not significantly change the distribution function $N_e(E)$ for transitions from quasicontinuous spectrum to the ground level, we have

$$\varkappa_1^+(E) \approx \frac{1}{4\Gamma}\lambda_1^2 \sum_n A_{n1} N_n = \frac{1}{4}\lambda_1^2 hB\left(1 - \frac{E}{\mathrm{Ry}}\right)^{3/2} \frac{dN_e}{dE}(E), \qquad (2)$$

where $E = h\nu = hc/\lambda_1$, λ_1 is the wavelength, $A_{n1} = B/n^3$ is the probability of spontaneous transition from the n-th level to the ground state, $hB = 8.5 \times 10^{-17}$ erg, N_n is the number of electrons in the n-th level. Summation is carried out over all n within the width Γ of the ground level.

The lower limit E_1 of the energy for which this estimate is applicable is given by Eq. (1) with account of $E_1 = \mathrm{Ry}/n_1^2$. The upper limit E_2 of applicability of Eq. (2) is determined by the value of n_2 of the principal quantum number, corresponding to the Debye screening radius:[1]

$$n_2 = a_0^{-1}(kT/4\pi N_p e^2)^{1/2}, \qquad E_2 = \mathrm{Ry}/n_2^2,$$

where a_0 is the radius of the first Bohr orbit, T is the electron temperature. In particular, for $N_p \approx 10^{16}\ \mathrm{cm}^{-3}$ and $T \approx 1000°$ we have $n_1 = 9$ and $n_2 = 30$.

The coefficient of negative absorption for transitions from the continuous spectrum is estimated by the formula

$$\varkappa_1^+(E) = \frac{\lambda_1^2}{4} v N_p \sigma_{\mathrm{sp}} \frac{dN_e}{dv} = \frac{\lambda_1^2}{4} hDN_p\left(\frac{E_0}{E-\mathrm{Ry}}\right)^{1/2} \frac{dN_e}{dE}(E). \qquad (3)$$

Here $\sigma_{\mathrm{sp}}(E)$ is the corresponding cross section of spontaneous recombination, $D = \sigma_{\mathrm{sp}}(E_0)\sqrt{2E_0/m}$, $E_0 = 1.6 \times 10^{-12}$ erg, $hD = 8 \times 10^{-40}$ erg-cm^3.

We find the lower limit E_3 of applicability of Eq. (3) by means of the uncertainty relation

$$N_p^{-1}\, 4\pi p^2 \Delta p \sim h^3; \qquad (4)$$

whence we have for (3)

$$E > E_3, \qquad E_3 \approx h^2 N^{2/3}/m.$$

We note that Eqs. (2) and (3) transform into one another if we start with the relation (4) and make the substitution

$$2\mathrm{Ry}/n^3 \leftrightarrow h^2 N_p/4\pi m^2 v.$$

For a simplified estimate of the character of the variation of $\kappa_1^+(E)$ with E, we shall assume an insignificant filling of the discrete levels, the distribution of electrons over the continuous and quasicontinuous spectrum being assumed to be similar to Maxwellian:

$$dN(E)/dE = 2\pi^{-1/2} N_e (E - E_1)^{1/2} (kT)^{-3/2} e^{-(E-E_1)/kT}. \qquad (5)$$

Here the maximum of the coefficient $\kappa_1^+(E)$ corresponds to an energy of the amplified quantum equal to

$$E = E_1 + \tfrac{1}{4}(\mathrm{Ry} - E_1) = E_1 + \Delta/4.$$

The maximum value of the coefficient of negative absorption is seen to be equal to

$$\varkappa_{1\,max}^+ = 0.1\lambda_1^2 hB\Delta^2 (kT\mathrm{Ry})^{-3/2} e^{-\Delta/4kT} N_p.$$

For $T = 1000°$ we then find: $\kappa_{1\max}^+ \sim 0.1\ \mathrm{cm}^{-1}$ for $N_p = 10^{16}\ \mathrm{cm}^{-3}$, and $\kappa_{1\max}^+ \sim 1\ \mathrm{cm}^{-1}$ for $N_p = 10^{17}\ \mathrm{cm}^{-3}$. Upon increase in E, the coefficient $\kappa_1^+(E)$ falls off rapidly so that for $E - \mathrm{Ry} \sim 0.1$ eV, $T = 1000°$, $N_p = 10^{16}\ \mathrm{cm}^{-3}$, we get $\kappa_1^+ \sim 10^{-4}\ \mathrm{cm}^{-1}$.

In addition to the induced radiation, those processes are important for the given set of problems which correspond to absorption of quanta by the medium. The principal contribution to the absorption is borne by transitions from the ground state to the quasicontinuous parts of the spectrum. Here we have for the absorption coefficient

$$\varkappa_1^- = \tfrac{1}{8}\lambda_1^2 N_{\mathrm{H}}\, hB/\mathrm{Ry} = 4.6 \cdot 10^{-17} N_{\mathrm{H}}.$$

Thus, for the estimated values of the parameters, the absorption is equal to the radiation for concentrations of hydrogen atoms N_H found in the ground state amounting to $\sim 0.1\ N_p$.

Without discussing here the methods of removal of the neutral atoms, we shall not dwell on the kinetics of filling the ground level. We only note that the characteristic time of spontaneous transition of the plasma to the ground state for the estimated values of the parameters is not lower than 10^{-7} sec; the nonradiative transitions to the ground state for triple collisions do not change this time estimate of filling the low level.[3]

The more significant role is played by nonradiative transitions in the population of the excited discrete levels. In accord with the approximation of Bethe (see, for example, [5], page 576), the probabilities of nonradiative transitions for collisions of excited atoms with electrons fall off rapidly with decrease in n. In this case the time of a nonradiative transition for the given plasma between neighboring levels and the times of spontaneous radiative transitions are equal in order of magnitude only for $n \sim 5$. This leads, mainly to the impossibility of utilization of transitions to the levels $n \geq 5$ for generation. Moreover, the rapid scattering of electrons to the upper discrete levels leads to a certain decrease in the population in the quasicontinuous part of the spectrum. Here it is convenient to use for generation transitions from the upper discrete levels; such transitions correspond to a κ^+ materially exceeding the value computed above for transitions from the quasicontinuous part of the spectrum.

The absorption of quanta for transitions of electrons from the more populated portions of the spectrum is characterized by small cross sections in the problem under consideration. The probability density of a transition with absorption from the quasicontinuous region is equal to

$$W(E) = (2\pi e^2 h/mc\,\mathrm{Ry})(1 - E/\mathrm{Ry})^{3/2}, \quad (6)$$

similarly for free-free transitions with absorption we have[4]

$$W(E) = \frac{4\pi}{3\sqrt{6}}\,\frac{e^6 h^2}{m^{3/2} c\,\mathrm{Ry}^3}\left(1 - \frac{\mathrm{Ry}}{E}\right)^{-1/2}. \quad (7)$$

Integrating (6) and (7) over the spectrum (5), we get the result that at T = 1000° the absorption coefficient corresponding to these transitions amounts to $\sim 10^{-4}\kappa_{1\,max}^{+}$.

Inasmuch as the natural frequency of the plasma oscillations $\nu_0 = e(N/\pi m)^{1/2} = 10^4\sqrt{N}$ is much lower than the generation frequency $\nu \approx \mathrm{Ry}/h$, the absorption of photons associated with collective vibrations is negligible.

Thus, cooling the dense, highly ionized plasma for a time $\tau \sim 10^{-9} - 10^{-8}$ sec, we obtain a medium which is suitable for generation from the ground level at the ionization frequency. Here the concentration of the plasma must amount to $N_p \sim N_e \sim 10^{14} - 10^{17}$; the number of atoms in the ground state must be less than 10%.

No less intensive generation by means of nonequilibrium plasma can be obtained at frequencies corresponding to transitions to excited levels. The probability of spontaneous transition to a level with a quantum number n_0 can be represented in the form [4] $A_{nn_0} \approx n_0^{-1} B/n^3$. We have for the coefficient of negative absorption

$$\kappa_{n_0}^{+}(E) = \tfrac{1}{4}\, n_0^3 \lambda_1^2 h B (1 - E/\mathrm{Ry})^{3/2} dN(E)/dE.$$

With increase of n_0, the spontaneous transition probability falls as n_0^{-1}, while the coefficient $\kappa_{n_0}^{+}(E)$ increases as n_0^3. Therefore, for isolation of a particular frequency, one must use a dielectric mirror in the generator. It should be noted that in generation which makes use of transitions to an excited level, the problems associated with the removal of the hydrogen thus formed become simplified. Here we have discussed the conditions which make possible generation by means of a supercooled plasma in hydrogen; clearly, the fundamental qualitative features of the analysis that has been given hold also in the general case.

One of the difficulties in the creation of a generator is the practical realization of a method for preparation of a highly supercooled plasma. We shall pause briefly on the method in which bunches of cold electrons and protons, combining with a small difference in average velocities, are compressed to sufficient densities in the active volume. The removal of the hydrogen gas formed becomes here automatic.

For electron and proton concentrations of $N = 10^{16}\ \mathrm{cm}^{-3}$ and average velocity of each bunch $v^0 \sim 10^8$ cm/sec, the current densities of the bunches are estimated at the value $j = Nve \sim 10^4\ \mathrm{A/cm^2}$. Such a density of electrons is not impossible. At the present time, as a peak one obtains "cold" stationary current density $j \sim 10^6\ \mathrm{A/cm^2}$, and in pulses even up to $j \sim 10^8\ \mathrm{A/cm^2}$. The situation here is made easier by the fact that the recombination cross sections depend only on the relative velocities; from this view point the achieved temperatures estimated above for the electrons are recalculated for protons in the ratio $T_p/T_e = m_p/m_e$; on the other hand, the time of transfer of energy from the protons to the electrons is much greater than the time for establishment of the electron temperature.

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237