SOVIET PHYSICS JETP VOLUME 18, NUMBER 4 APRIL, 1964

EFFECT OF NUCLEAR DEFORMATIONS ON THE ISOTOPE SHIFT OF SPECTRAL LINES

F. A. BABUSHKIN

Submitted to JETP editor April 26, 1963

J. Exptl. Theoret. Phys. (U.S.S.R.) 45, 1479-1483 (November, 1963)

The effect of axial and nonaxial deformations on the isotope shift of spectral lines is calculated with an accuracy up to terms of order β^3. The deformation compressibility is taken into account to the same order. Satisfactory agreement between the theoretical and experimental values has been obtained for all pairs of isotopes for which the experimental values of the isotope shift constants and deformation parameters are known, with the exception of the pair $_{60}Nd^{146-148}$. The calculations show that the effect of the nonaxiality of the nucleus can be completely neglected.

In an earlier paper [1] the author has calculated the constants of the isotope shift (i.s.) of the spectral lines for spherical and deformed nuclei. The calculations were based on the method of Broch,[2] using wave functions obtained by the author.[3] The i.s. for deformed nuclei was calculated with an accuracy up to terms of order β^2, where β is the deformation parameter.

In comparing the theoretical and experimental data, discrepancies were observed in a number of cases (for the isotopic pairs $_{58}Ce^{140-142}$, $_{60}Nd^{146-148}$, $^{148-150}$, $_{62}Sm^{151-153}$, and $_{74}W^{184-186}$). It was proposed that this disagreement is due, first, to the inaccuracy in the determination of the deformation parameter and, second, to the unjustified neglect of terms of order β^3 in the case of strongly deformed nuclei.

In the present paper the author has made use of the latest results on nuclear Coulomb excitation to determine the deformation parameters of several nuclei. Furthermore, the calculations were carried to terms of order β^3. This allows us also to take into account the effect of the nonaxiality of atomic nuclei on the magnitude of the i.s.

Following Fradkin,[4] we introduce the notion of the compressibility of deformed nuclei which is characterized by the deformation compressibility parameter $\xi = -5/8\pi$. Since the calculations are carried to terms of order β^3, we must extend Fradkin's work to include the deformation compressibility to the same order, that is, we take

$$R_{\text{eq}}^{2\gamma} = R^{2\gamma}\left[1 - (\xi\beta)^2 \pm (\xi\beta)^3\right], \tag{1}$$

where R is assumed to be given by [5]

$$R = (1.115A^{1/3} + 2.151A^{-1/3} - 1.742A^{-1})\cdot 10^{-13}\ \text{cm}. \tag{2}$$

The sign in front of the last term is found empirically by comparing the theoretical and experimental data. For example, for the i.s. constant of $_{63}Eu^{151-153}$ we obtain 438, 384, and 331, respectively, for the three choices $+(\xi\beta)^3$, $(\xi\beta)^3 = 0$, $-(\xi\beta)^3$. Comparing with the experimental value 450 ± 50, we find that the agreement is best if we choose the positive sign in front of the last term of (1).

The i.s. constant for deformed nuclei is determined by the relation

$$C_{\text{def}}/C_{\text{sph}} = \delta\langle R_{\text{def}}^{2\gamma}\rangle/\delta\langle R_{\text{sph}}^{2\gamma}\rangle. \tag{3}$$

The method of calculating the ratio on the right-hand side is completely analogous to the method used by Fradkin. Taking into account (1), we obtain

$$\begin{aligned}\delta\langle R_{\text{def}}^{2\gamma}\rangle/\delta\langle R_{\text{sph}}^{2\gamma}\rangle = {} & 1 + [\gamma(2\gamma+3)/4\pi - \xi^2](\beta_1+\beta_2)^2/4 \\ & + \tfrac{1}{2}[f(\gamma,\gamma_1^0)\beta_1^3 + f(\gamma,\gamma_2^0)\beta_2^3] \\ & + (R/2\gamma\delta R)\{[\gamma(2\gamma+3)/4\pi - \xi^2](\beta_2^2-\beta_1^2) + f(\gamma,\gamma_2^0)\beta_2^3 \\ & - f(\gamma,\gamma_1^0)\beta_1^3\},\end{aligned} \tag{4}$$

where

$$\gamma = \sqrt{1-\alpha^2 Z^2},$$

$$f(\gamma,\gamma^0) = \frac{\gamma(2\gamma+3)^2}{42\pi}\sqrt{\frac{5}{4\pi}}\cos\gamma^0(1-4\sin^2\gamma^0) + \xi^3.$$

Here γ^0 is the parameter of nonaxiality, and the indices 1 and 2 characterize the isotopes for which the i.s. is calculated. The expression for C_{sph} is given by (4) of [1]. The deformation parameters β are determined from

$$Q_0 = 3(5\pi)^{-1/2}ZR^2\beta\left[\cos\gamma^0 + 0.36\beta(1-2\sin^2\gamma^0)\right], \tag{5}$$

where R is given by (2).

The values of the deformation parameter β of nonaxial nuclei computed according to (5) are listed in Table I. For comparison, we have given

Table I. Nuclear Deformation Parameters

Element	A	β_{axial}	γ^0, deg	$\beta_{nonaxial}$
$_{44}$Ru	98	0,196	27,2 [6]	0,224
	100	0,212	24,4 [6]	0,236
	102	0,235	26.0 [6]	0,266
	104	0,259	24.4 [6]	0,290
$_{46}$Pd	106	0.208	26,8 [6]	0,238
	108	0,220	26.8 [6]	0,251
	110	0.234	26.8 [6]	0.266
$_{48}$Cd	108	0,182	25,4 [6]	0.204
	110	0,175	26.4 [6]	0.198
	112	0,179	27.6 [6]	0,206
	114	0,184	26.8 [6]	0,209
	116	0.186	26.0 [6]	0.208
$_{58}$Ce	142	0,137 [8]		
$_{60}$Nd	148	0.127 [9]		
$_{62}$Sm	150	0.188 [8]	26.0 [6]	0,211
	152	0.286	13,2 [6]	0,296
$_{64}$Gd	154	0.285	13.8 [7]	0,294
	156	0,311	11.0 [7]	0,318
$_{70}$Yb	170	0.303 [10]		
$_{74}$W	182	0,247 [11]	11.3 [7]	0,252
	184	0,241 [11]	13.8 [7]	0,250
	186	0.224 [11]	16.5 [7]	0,234
$_{75}$Re	185	0,197 [12]		
	187	0,182 [12]		
$_{76}$Os	186	0.197 [12]	16.5 [7]	0,206
	188	0.189 [11]	19.2 [7]	0.201
	190	0,180 [11]	22.2 [7]	0,196
	192	0.161 [11]	25.2 [7]	0,181
$_{78}$Pt	194	0.145	30.0 [7]	0.170
	196	0.104	30.0 [7]	0.122
$_{81}$Tl	203	0.186 [13]		
	205	0,189 [13]		

Table II. Isotope Shifts

Element	$A_1 - A_2$	C_{theor}, $10^{-3}cm^{-1}$		C_{exp}, $10^{-3}cm^{-1}$
		$C_{def.ax.}$	$C_{def.nonax.}$	
$_{37}$Rb	85—87	15.8		8±12 [14]
$_{44}$Ru	96—98	43.0		
	98—100	34.4	33,8	34±9 [14] average
	100—102	38.2	39.7	
	102—104	40.0	40.0	
$_{46}$Pd	106—108	37.7	38.4	42±9 [14]
	108—110	38.7	39,6	36±5 [14]
$_{47}$Ag	107—109	38.8		38±6 [14]
$_{48}$Cd	106—108	41.0		
	108—110	34.2	34.3	32±4 [14]
	110—112	38.6	40.0	
	112—114	38.5	37.4	
	114—116	36.6	36.1	
$_{50}$Sn	112—114	42.9		40±10 [14]
	114—116	40.7		40±10 [14]
	116—118	41.1		30±10 [14]
	118—120	40.7		30±10 [14]
	120—122	40.5		15±10 [14]
	122—124	40.4		15±10 [14]
$_{58}$Ce	140—142	89.5		147±30 [14]
$_{60}$Nd	146—148	98.4		202±38 [15]
	148—150	219		286±54 [15]
$_{62}$Sm	148—150	156		217±46 [15]
	150—152	255	249	317±67 [15]
	152—154	176		170±35 [4]
$_{63}$Eu	151—153	438		450±50 [14]
$_{64}$Gd	154—156	164	167	162±26 [16]
	155—157	110		109±18 [16]
	156—158	154		121±19 [16]
	158—160	130		125±20 [16]
$_{70}$Yb	170—172	145		110±10 [14]
	171—173	102		112±12 [4]
	172—174	133		99±8 [14]
	174—176	115		94±8 [14]
$_{72}$Hf	178—180	134		114±13 [14]
$_{74}$W	182—184	157	161	132±23 [17]
	184—186	115	114	117±20 [17]
$_{75}$Re	185—187	146		157±40 [14]
$_{76}$Os	186—188	178	182	176±27 [15]
	188—190	175	183	150±23 [15]
	190—192	149	153	130±20 [15]
$_{77}$Ir	191—193	136		130±30 [14]
$_{78}$Pt	194—196	128	118	135±25 [14]
$_{80}$Hg	198—200	233		243±27 [15]
$_{81}$Tl	203—205	294		280±40 [14]

in the same table the deformation parameters for axially symmetric nuclei ($\gamma^0 = 0$), taken from the earlier paper of the author.[1] The deformation parameters of some nuclei were calculated anew using the latest data on nuclear Coulomb excitation. The remaining deformation parameters not listed in the table but used in the calculation of the i.s. were taken from[1].

The calculated values of the i.s. constants are listed in Table II. A comparison of the theoretical and experimental values of the i.s. constants shows that the inclusion of terms of order β^3 and the introduction of the deformation compressibility in these terms improve the agreement between theory and experiment. A large discrepancy between the theoretical and experimental values of the i.s. constants remains for the isotope pair $_{60}Nd^{146-148}$. This discrepancy can hardly be explained by an inaccuracy in the determination of the deformation parameters of these isotopes, since these isotopes are relatively well investigated by the method of Coulomb excitation, and various authors have obtained almost identical values for the reduced electric quadrupole transition probabilities in these nuclei, from which the deformation parameters are determined.

An analysis of Table II shows that the nonaxiality of atomic nuclei has a very insignificant effect on the magnitude of the i.s. and can be completely neglected in view of the limits of error of present-day experimental data.

1 F. A. Babushkin, JETP 44, 1661 (1963), Soviet Phys. JETP 17, 1118 (1963).

2 E. K. Broch, Arch. Math. o. Nat. 48, 25 (1945).

3 F. A. Babushkin, JETP 42, 1604 (1962), Soviet Phys. JETP 15, 1113 (1962).

4 É. E. Fradkin, JETP 42, 787 (1962), Soviet Phys. JETP 15, 550 (1960).

5 A. R. Bodmer, Nucl. Phys. 9, 371 (1959).

6 J. Hiura and S. Suekane, Progr. Theoret. Phys. 24, 462 (1960).

7 E. P. Grigoriew and M. P. Avotina, Nucl. Phys. 19, 248 (1960).

8 O. Natan and V. I. Popov, Yadernye reaktsii pri malykh i srednikh energiyakh (Nuclear Reactions at Small and Intermediate Energies), AN SSSR (1962).

9 Andreev, Grinberg, Gusinskiĭ, Erokhina, and Lemberg, Izv. AN SSSR, ser fiz. 24, 1474 (1960), Columbia Tech. Transl. p. 1466.

10 M. S. El-Nesr and E. Bashandy, Z. Physik 168, 349 (1962).

11 F. K. McGowan and P. H. Stelson, Phys. Rev. 109, 901 (1958).

12 Alder, Bohr, Huus, Mottelson, and Winther, Revs. Modern Phys. 28, 432 (1956) [Russ. Transl. in Deformatsiya atomnykh yader (Deformation of Atomic Nuclei), IIL (1958), p. 9].

13 P. H. Stelson and F. K. McGowan, Phys. Rev. 99, 112 (1955).

14 P. Brix and H. Kopfermann, Revs. Modern Phys. 30, 517 (1958).

15 Meligy, Tadros, and El-Wahab, Nucl. Phys. 16, 99 (1960).

16 Kopfermann, Krüger, and Steudel, Ann. Physik (6) 20, 258 (1957).

17 G. Gluck and J. Blaise, J. Phys. Radium 20, 466 (1959).

Translated by R. Lipperheide
243