ADIABATIC APPROXIMATION FOR DEGENERATE OR ALMOST DEGENERATE STATES

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The behavior of a system under the action of a time dependent perturbation is considered in the adiabatic approximation under the assumption that the system is degenerate in the absence of the perturbation. The state of the system is characterized by a density matrix in the space of eigenfunctions belonging to a given degenerate level. A unitary operator is constructed which relates the density matrix before the perturbation is turned on to the density matrix after the perturbation has been turned off. The case when there is no exact degeneracy in the absence of the perturbation but rather a set of closely spaced levels (almost degenerate states) is also considered. As an illustration of the application of the theory, the behavior of spin 1 and spin ½ particles in a variable magnetic field is examined.

1. INTRODUCTION

THE behavior of a nondegenerate system under the action of a slowly varying perturbation is well known.[1,2] Suppose a system (Hamiltonian $\mathcal{H}_0$) in the state ψ^0 with energy E_0 is subjected at the instant $t = 0$ to the perturbation V, which vanishes some time later at the instant τ (or V goes to zero asymptotically at $t \to \pm\infty$). The probability that, under the action of the perturbation, the system makes a transition from the state with energy E_0 to a state with energy E_0' will be small if V varies little during the time $\hbar(E_0 - E_0')^{-1}$. The time development of the state can be described by the adiabatic approximation,[1,2] i.e., by assuming that the wave function of the system has the form

$$\Phi(x, t) = e^{-i\varphi(t)}\psi(x, t), \tag{1.1}$$

where $\psi(\mathbf{x}, t)$ is the eigenfunction of the Hamiltonian $\mathcal{H}_0 + V$ at the time t,

$$(\mathcal{H}_0 + V)\psi = E(t)\psi, \tag{1.2}$$

which satisfies $(\psi, \dot{\psi}) = 0$, and φ has the form

$$\varphi(t) = \frac{1}{\hbar}\int_0^t E(t)\,dt. \tag{1.3}$$

For $V \to 0$ the function ψ and the energy E go over into ψ^0 and E_0, respectively.

If the level E_0 is degenerate, the situation is somewhat more complicated. As an introduction to the general discussion, we choose the simplest case, which has been investigated in various modifications by many authors (see, e.g., [3]): a two-fold degenerate system under the action of a time dependent perturbation. Until the perturbation is switched on, two eigenfunctions, ψ_1^0 and ψ_2^0, will be associated with a single level E_0. These functions are not uniquely defined, since an arbitrary linear combination of them is also an eigenfunction of the Hamiltonian $\mathcal{H}_0$ with energy E_0. For definiteness, we shall assume that some specific pair of functions has been chosen. In a number of cases, this choice of states is dictated by the physical conditions of the problem.

When the perturbation is turned on, the degeneracy is removed. The level E splits into two: E_1 and E_2, associated with two eigenfunctions of the Hamiltonian $\mathcal{H}_0 + V$, which we denote by $\psi_1(\mathbf{x}, t)$ and $\psi_2(\mathbf{x}, t)$.

In order to describe the time development of the state under the action of the slowly varying perturbation we set up a superposition of adiabatic functions of the form (1.1):

$$\Phi = a_1 e^{-i\varphi_1(t)}\psi_1(x, t) + a_2 e^{-i\varphi_2(t)}\psi_2(x, t). \tag{1.4}$$

The coefficients a_1 and a_2 will be practically constant under the condition[2]

$$|\hbar(\psi_1, \dot{\psi}_2)| / |E_1 - E_2| \ll 1. \tag{1.5}$$

A special consideration is required at the points where the perturbation is switched on or off and where, by assumption, $E_1 = E_2$. We shall assume that the inequality (1.5) holds even in the limiting case $E_1 \to E_2$, i.e.,

$$\lim_{E_1 \to E_2} |\hbar(\psi_1, \dot{\psi}_2)/(E_1 - E_2)| \ll 1. \tag{1.6}$$

The relation (1.6) can be satisfied by an appropri-

ate choice of the manner in which the perturbation is switched on and off.

The coefficients a_1 and a_2 in (1.4) are determined by the initial conditions. As an initial condition one may, for example, require that the system be in definite state,

$$\Phi(x, 0) = \psi_1^0 \tag{1.7}$$

say.

When $V \to 0$, each of the functions ψ_1, ψ_2 goes over, in general, into a superposition of ψ_1^0 and ψ_2^0, so that we have for $t = 0$

$$\psi_i(\mathbf{x}, 0) = \sum_n C_{ni}\,\psi_n^0, \tag{1.8}$$

and also

$$\psi_i^0 = \sum_n C_{in}^*\,\psi_n(\mathbf{x}, 0). \tag{1.9}$$

There will be analogous relations for $t = \tau$ (but not necessarily with the same C_{ni} as for $t = 0$).

Taking account of the initial condition (1.7) and relation (1.9), we obtain from (1.4)

$$\Phi(x, \tau) = \sum_{k=1}^{2}\sum_{j=1}^{2} C_{1j}^*(0)\,C_{kj}(\tau)\,e^{-i\varphi_j(\tau)}\,\psi_k^0. \tag{1.10}$$

It follows from this expression that after the switching-on of the perturbation the function Φ will be represented by a superposition of the functions ψ_1^0 and ψ_2^0. In other words, the system makes a transition from the state ψ_1^0 to the state ψ_2^0 although there are no transitions between ψ_1 and ψ_2. The probability for the transition $\psi_1^0 \to \psi_2^0$ is equal to, according to (1.10),

$$P_{12} = |C_{11}^*(0)\,C_{21}(\tau)\,e^{-i\varphi_1(\tau)} + C_{12}^*(0)\,C_{22}(\tau)\,e^{-i\varphi_2(\tau)}|^2. \tag{1.11}$$

The determination of the coefficients C_{ij} is equivalent to the diagonalization of the matrix $\mathcal{H}$ in the space of functions ψ_1^0, ψ_2^0 for $V \to 0$. If, besides the states already considered, there are still other states describing the system, we are led to the problem of diagonalizing not the entire matrix, but only the part (submatrix) referring to the states under consideration.

The solution of this problem for a twofold degenerate system is well known.[3] Let $\mathcal{H}_{ik}$ be the matrix elements: $\mathcal{H}_{ik} = (\psi_i^0, \mathcal{H}\psi_k^0)$, which we shall assume to be real. Let us set

$$\lambda = (\mathcal{H}_{11} - \mathcal{H}_{22})/2\mathcal{H}_{12}. \tag{1.12}$$

We shall be interested in the limiting values of λ for $t \to 0$ and $t \to \tau$ (i.e., for $V \to 0$). Here $\mathcal{H}_{11} - \mathcal{H}_{22}$ as well as $\mathcal{H}_{12}$ tend to zero, but their ratio may have various nonvanishing values. Simple calculations lead to the following expressions for C_{ik}:

$$C_{11} = C_{22} = 2^{-1/2}\,[1 + \lambda\,(1 + \lambda^2)^{-1/2}]^{1/2},$$
$$C_{21} = -C_{12} = 2^{-1/2}\,[1 - \lambda\,(1 + \lambda^2)^{-1/2}]^{1/2}. \tag{1.13}$$

Substituting the values (1.13) in (1.11) for $t = 0$ and $t = \tau$, we find

$$P_{12} = \tfrac{1}{2}\,[1 - \lambda(\tau)\,\lambda(0)\,(1 + \lambda^2(\tau))^{-1/2}\,(1 + \lambda^2(0))^{-1/2}] - \tfrac{1}{2}\,[1 + \lambda^2(\tau)]^{-1/2}\,[1 + \lambda^2(0)]^{-1/2}\cos[\varphi_1(\tau) - \varphi_2(\tau)]. \tag{1.14}$$

(The special case $\lambda = 0$ is realized in the phenomenon of resonance charge exchange in atomic collisions.[4])

The preceding considerations can be applied, with small modifications, also to the case when there is a gap ΔE_1 between the two levels even before the perturbation is switched on, as long as this gap is much smaller than the distance ΔE_2 to the other levels of the system (case of almost degenerate states). Since $\Delta E_1 \ll \Delta E_2$, the perturbation can be imposed in such a way that the time of significant variation of the perturbation, T, satisfies the condition $\hbar/\Delta E_2 \ll T \ll \hbar/\Delta E_1$. Then the perturbation can be regarded as being switched on instantly with respect to the almost degenerate states while at the same time treating it adiabatically with respect to the other states. Under these conditions we can use the expressions given above with the difference that the perturbation V does not vanish at $t = 0$ and $t = \tau$, but has the values V_0 and V_τ, respectively. In accordance with well known considerations, the quantities V_0 and V_τ must not be too large.

In Sec. 2, this theory will be generalized in two respects. First, we consider a system with an arbitrary number of degenerate (or almost degenerate) states. Second, the initial state of the system and its time development are described by a density matrix. The use of a density matrix allows one to extend the adiabatic approximation to such problems as the behavior of polarized beams of particles in magnetic fields which vary slowly in magnitude and direction and the related problem of the population of Zeeman levels in magnetic fields. Examples of this kind are considered in Sec. 3. Section 4 is devoted to the discussion of the magnetic resonance.

2. ADIABATIC APPROXIMATION. GENERAL DISCUSSION

Let ρ be a density matrix in the space of the eigenfunctions $\psi_1^0, \ldots, \psi_n^0$ of the Hamiltonian $\mathcal{H}_0$ corresponding to some degenerate level with energy E_0. Further, let $\rho_{\mathcal{H}}$ be the density matrix in the space of the eigenfunctions $\psi_1, \ldots, \psi_n$ of

the Hamiltonian $\mathcal{H} = \mathcal{H}_0 + V$ corresponding to the levels $E_1, \ldots, E_n$, which go over into E_0 when $V = 0$.

At the instant $t = 0$ at which the perturbation is switched on, the matrices $\rho_{\mathcal{H}}(0)$ and $\rho(0)$ are related by the unitary transformation

$$\rho_{\mathcal{H}}(0) = C^+(0)\,\rho(0)\,C(0), \tag{2.1}$$

where C is a unitary operator which diagonalizes the matrix (or an appropriate part of it) in the space of the functions $\psi_1^0, \ldots, \psi_n^0$ for $V \to 0$. The operator $C(0)$ does not, in general, reduce to the unit operator, although $\mathcal{H}$ coincides with $\mathcal{H}_0$ for $V = 0$.

The time development of $\rho_{\mathcal{H}}(\tau)$ can in the adiabatic approximation be represented as a unitary transformation,

$$\rho_{\mathcal{H}}(t) = U(t)\;\rho_{\mathcal{H}}(0)\;U^+(t), \tag{2.2}$$

by means of the diagonal matrix U with the elements

$$U_{m,n} = e^{-i\varphi_n(t)}\delta_{mn}, \qquad \varphi_n(t) = \frac{1}{\hbar}\int_0^t E_n(t)\,dt. \tag{2.3}$$

At the instant at which the perturbation is switched off, i.e., at $t = \tau$, we go again over to the space of the functions $\psi_1^0, \ldots, \psi_n^0$. The matrix $\rho(\tau)$ is obtained from the matrix $\rho_{\mathcal{H}}(\tau)$ by means of the transformation

$$\rho(\tau) = C(\tau)\,\rho_{\mathcal{H}}(\tau)\,C^+(\tau). \tag{2.4}$$

It follows from (2.4), (2.2), and (2.1) that

$$\rho(\tau) = M\rho(0)\,M^+, \tag{2.5}$$

where M is the transition operator,

$$M = C(\tau)\,U(\tau)\,C^+(0). \tag{2.6}$$

The matrix elements have the form

$$M_{mn} = \sum_k C_{mk}(\tau)\,C^*_{nk}(0)\,e^{-i\varphi_k(\tau)}. \tag{2.7}$$

As is seen from (2.5), the density matrix $\rho(\tau)$ coincides with $\rho(0)$ in the following cases:

1) if $C = 1$;

2) if $\rho(0) = I/\mathrm{Sp}\,I$, which corresponds to a uniform population of all degenerate states;

3) if $\rho(0)$ is composed of eigenvectors of the matrix C and, hence, commutes with M.

The probability for finding the system in the state ψ_m^0 after the perturbation has been switched off is equal to the diagonal element of the density matrix $\rho_{mm}(\tau)$. If the system was in a definite state n before the perturbation was switched on, the elements of the matrix $\rho(0)$ are equal to $\rho_{nn}(0) = 1$, $\rho_{\alpha\beta} = 0$ $(\alpha, \beta \neq 1)$. It then follows from (2.5) that

$$\rho_{mm}(\tau) = |M_{mn}|^2. \tag{2.8}$$

This quantity gives the probability for the transition $n \to m$. We shall denote it by P_{mn}.

It follows from (2.7) that P_{mn} can be written in the form

$$P_{mn} = A_{mn} + \sum_{k,k'} B_{mn}^{kk'} \cos\left[\varphi_k(\tau) - \varphi_{k'}(\tau) + \Omega_{mn}^{kk'}\right], \tag{2.9}$$

where A_{mn}, $B_{mn}^{kk'}$, and $\Omega_{mn}^{kk'}$ are expressions composed of $C_{mn}(0)$ and $C_{mn}(\tau)$.

If the perturbation contains some parameter, for instance, the rate at which the interaction is switched on and off, then P_{mn} will be a function of this parameter. It is seen from (2.9) that the variation of P_{mn} due to variations of the parameter consists of two parts: a smooth term coming from the dependence of A and B on the parameter and an oscillating one related to the term $\cos(\varphi - \varphi_{k'} + \Omega_{mn}^{kk'})$. If the last term is a rapidly oscillating function of the parameter, averaging over a small region of variation of the parameter gives

$$\bar{P}_{mn} = A_{mn}. \tag{2.10}$$

In some cases, however, the oscillating part of the variation of the probability due to variations of the parameter is observed experimentally. Thus, in the resonance charge exchange in hydrogen the probability as a function of the velocity of the particle at a fixed scattering angle shows distinct maxima and minima,[5] in correspondence with the theory.[4]

In conclusion we give a formula which expresses C_{ik} in terms of the matrix elements $\mathcal{H}_{ik} = (\psi_i^0, \mathcal{H}\psi_k^0)$ and the roots of the secular equation $\|\mathcal{H}_{ik} - E\delta_{ik}\| = 0$:

$$C_{ik} = e^{i\varepsilon_{ik}}(D^{ik}_{i})^{1/2}(D)^{-1/2}. \tag{2.11}$$

Here D is the Vandermonde determinant composed of the roots of the secular equation; D_{ij}^k is the determinant obtained from D by replacing the k-th column by the quantities δ_{ij}, $\mathcal{H}_{ij}$, $(\mathcal{H}^2)_{ij}$, $\ldots$, $(\mathcal{H}^{n-1})_{ij}$. The phases ϵ_{ij} must satisfy the condition

$$\varepsilon_{ik} - \varepsilon_{jk} = \lambda_{ij}^k,$$

where λ_{ij}^k is the phase of the determinant D_{ij}^k. Otherwise the ϵ_{ik} are arbitrary.

3. PARTICLES WITH SPIN ½ AND 1 IN A VARIABLE MAGNETIC FIELD

Let us consider first a particle with spin ½. The energy operator is

$$\mathcal{H} = -\tfrac{1}{2}\hbar g\,\mathbf{H}\sigma,$$

where g is the gyromagnetic ratio, **H** is the field intensity, which varies in magnitude as well as direction. The condition for the applicability of the adiabatic approximation reduces in this case to the requirement that the rates of change of the magnitude and direction of the field be small as compared to the Larmor frequency.

It is convenient to choose a coordinate system such that the operator $\mathcal{H}$ is diagonal at the moment at which the perturbation is switched on. For this purpose the z axis must be directed along the initial direction of the field. Then the matrix C will be a 2×2 unit matrix at $t = 0$: $C(0) = I$. At $t = \tau$ the field is switched off, where we assume that at this instant the field makes an angle θ with the original direction. Through the initial and final field directions we can put a plane which we shall take as the z, x plane. Thus we have for $\mathcal{H}$ immediately before the switching-on of the field

$$\mathcal{H} = -\tfrac{1}{2}\hbar g H\,(\sigma_z \cos\theta + \sigma_x \sin\theta). \tag{3.1}$$

The matrix $C(\tau)$ which diagonalizes the Hamiltonian (3.1) is

$$C(\tau) = I\cos(\theta/2) - i\sigma_y \sin(\theta/2). \tag{3.2}$$

The matrix U defined by (2.3) can be written in the form

$$U = I\cos(\mu/2) + i\sigma_z \sin(\mu/2), \tag{3.3}$$

$$\mu = g\int_0^\tau H dt. \tag{3.4}$$

Substituting (3.2) and (3.3) in the general formula (2.6), we obtain the transition operator M.

The canonical form of the density matrix for particles with spin $\frac{1}{2}$ is

$$\rho = \tfrac{1}{2}(I + \sigma \mathbf{P}),$$

where **P** is the polarization vector. Using (2.5), we can express the components of the polarization vector after the switching-off of the field, $P(\tau)$, in terms of the components of the polarization vector before the switching-on of the field, $P(0)$. The corresponding expressions have the form

$$\begin{aligned} P_x(\tau) &= P_r \cos(\mu-\delta)\cos\theta + P_z(0)\sin\theta,\\ P_y(\tau) &= -P_r \sin(\mu-\delta),\\ P_z(\tau) &= -P_r \cos(\mu-\delta)\sin\theta + P_z(0)\cos\theta; \end{aligned} \tag{3.5}$$

$$\delta = \tan^{-1}(P_y(0)/P_x(0)),\quad P_r = (P_x^2(0) + P_y^2(0))^{1/2}. \tag{3.5'}$$

As is seen, the expressions for the components of the polarization vector contain two parts: a smoothly varying part representing simply the rotation of the z component together with the field and an oscillating part containing $\cos(\mu-\delta)$ and $\sin(\mu-\delta)$. After averaging over the oscillating part only the smoothly varying part remains, while the other terms reduce to zero.

Let us now turn to the case of a particle with spin 1. As above, we shall take the Hamiltonian diagonal at the instant at which the field is switched on. Before the switching-off of the field we write the Hamiltonian, analogous to (3.1), in the form

$$\mathcal{H} = -\hbar g H\,(S_z\cos\theta + S_x \sin\theta). \tag{3.6}$$

S_X, S_Y, and S_Z are the spinor matrices for spin 1:

$$S_x = \frac{1}{\sqrt{2}}\begin{pmatrix}0&1&0\\1&0&1\\0&1&0\end{pmatrix},\quad S_y = \frac{i}{\sqrt{2}}\begin{pmatrix}0&-1&0\\1&0&-1\\0&1&0\end{pmatrix},$$

$$S_z = \begin{pmatrix}1&0&0\\0&0&0\\0&0&-1\end{pmatrix}. \tag{3.7}$$

The matrix C which diagonalizes the Hamiltonian (3.6) is equal to

$$C = I - iS_y \sin\theta - 2S_y^2 \sin^2(\theta/2). \tag{3.8}$$

The matrix U can be written in the form

$$U = I + iS_z \sin\mu - 2S_z^2\sin^2(\mu/2), \tag{3.9}$$

where μ is given, as before, by (3.4).

The canonical form of the density matrix for particles with spin 1 is

$$\rho = \tfrac{1}{3}\Big[I + \sqrt{\tfrac{3}{2}}\sum_i a_i S_i + \frac{1}{\sqrt{2}}\sum_i b_i (3S_i^2 - 2) + \sqrt{\tfrac{3}{2}}\sum_{i\neq k} a_{ik}(S_iS_k + S_kS_i)\Big]. \tag{3.10}$$

Here the term containing $\sum_i a_i S_i$ characterizes the vector polarization and the remaining terms, the tensor polarization.

As an example we consider the special case where before the switching-on of the field

$$\rho(0) = S_z^2/2 \tag{3.11}$$

or, in matrix form,

$$\rho(0) = \begin{pmatrix}1/2&0&0\\0&0&0\\0&0&1/2\end{pmatrix}. \tag{3.12}$$

Such a density matrix corresponds to an equal population of the states with eigenvalues $S_Z = \pm 1$ and to an unpopulated state with the eigenvalue $S_Z = 0$. In this case we obtain for $\rho(\tau)$ the expression

$$\rho(\tau) = \tfrac{1}{2}[S_z^2\cos^2\theta + (S_xS_z + S_zS_x)\sin\theta\cos\theta + S_x^2\sin^2\theta]. \tag{3.13}$$

Here all three states are populated. The relative population depends on the angle of rotation of the field θ. The relative populations can be changed by altering the angle θ.

4. MAGNETIC RESONANCE

The adiabatic approximation can also be applied to problems of the type of the magnetic resonance in a rotating magnetic field.

Let us consider a particle with spin $\frac{1}{2}$. We assume that the states with the eigenvalues of σ_z equal to $+1$ and -1 have the energies E_1 and E_2. (A degeneracy can be removed by some constant field.) The corresponding Hamiltonian is denoted by $\mathcal{H}_0$. It is given by the diagonal matrix

$$\mathcal{H}_0 = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}. \tag{4.1}$$

Let us impose on the system a magnetic field which rotates in the x, y plane with the angular velocity ω,

$$H_x = H_0 \cos\omega t, \qquad H_y = H_0 \sin\omega t. \tag{4.2}$$

Then the Hamiltonian will have the form

$$\mathcal{H} = \mathcal{H}_0 - \tfrac{1}{2}\hbar g H_0 \,(\sigma_x \cos\omega t + \sigma_y \sin\omega t) \tag{4.3}$$

or, in matrix form

$$\mathcal{H} = \begin{pmatrix} E_1 & -\frac{1}{2}\hbar g H_0 e^{-i\omega t} \\ -\frac{1}{2}\hbar g H_0 e^{i\omega t} & E_2 \end{pmatrix}. \tag{4.4}$$

Let us now go over to a coordinate system which rotates together with the field. This is effected by a unitary transformation of the spinor ψ:

$$\tilde{\psi} = e^{i\omega t \sigma_z/2}\psi. \tag{4.5}$$

The Hamiltonian has in the rotating system the form

$$\tilde{\mathcal{H}} = \begin{pmatrix} E_1 - \hbar\omega/2 & -\frac{1}{2} g\hbar H_0 \\ -\frac{1}{2} g\hbar H_0 & E_2 + \hbar\omega/2 \end{pmatrix}. \tag{4.6}$$

The eigenvalues of $\tilde{\mathcal{H}}$ are equal to

$$\tilde{E}_{1,2} = \tfrac{1}{2}(E_1 + E_2) \pm \tfrac{1}{2}[(E_1 - E_2 - \hbar\omega)^2 + g^2\hbar^2 H_0^2]^{1/2}, \tag{4.7}$$

so that the difference between the energy levels in the rotating system of coordinates is

$$\tilde{E}_1 - \tilde{E}_2 = [(E_1 - E_2 - \hbar\omega)^2 + g^2\hbar^2 H_0^2]^{1/2}. \tag{4.8}$$

The difference $\tilde{E}_1 - \tilde{E}_2$ will be smallest at resonance, i.e., when

$$\hbar\omega = E_1 - E_2,$$

where it is equal to

$$\tilde{E}_1 - \tilde{E}_2 = g\hbar H_0.$$

For $H_0 \to 0$ the difference $\tilde{E}_1 - \tilde{E}_2$ vanishes at resonance, i.e., there is exact degeneracy. Near the resonance the quantity $\tilde{E}_1 - \tilde{E}_2$ is different from zero for $H_0 \to 0$, but is so small that the states are almost degenerate.

In correspondence with the discussion of Sec. 1, we shall assume that the field H_0 is switched on instantaneously at $t = 0$, whereupon the amplitude of H_0 changes adiabatically, and is switched off instantaneously at $t = \tau$. We can then use expression (1.14) for the transition probability P_{12}, with

$$\lambda = (\tilde{\mathcal{H}}_{11} - \tilde{\mathcal{H}}_{22})/2\,\tilde{\mathcal{H}}_{12} = -(E_1 - E_2 - \hbar\omega)/g\hbar H_0. \tag{4.9}$$

In the special case $H_0(0) = H_0(\tau)$ we obtain

$$P_{12} = \frac{g^2\hbar^2 H_0^2(0)}{(E_1 - E_2 - \hbar\omega)^2 + g^2\hbar^2 H_0^2(0)} \sin^2\left\{\frac{1}{2\hbar}\int_0^\tau (\tilde{E}_1 - \tilde{E}_2)\,dt\right\}. \tag{4.10}$$

If H_0 is constant during the time from 0 to τ, (4.10) together with (4.8) agrees with the known results.[6]

It is seen that our approach to this problem permits the generalization of known results to the case of a variable amplitude of the rotating field. We note further that, if we consider a process with constant H_0 and slowly varying frequency ω, it can be interpreted as an adiabatic transition through the point of "pseudo-crossing" of the levels in the rotating coordinate system.

Let us assume, in particular, that $\omega(0)$ and $\omega(\tau)$ are located on opposite sides of the resonance frequency $\omega_0 = (E_1 - E_2)/\hbar$ at the same distance ϵ:

$$\omega(0) = \omega_0 - \varepsilon, \qquad \omega(\tau) = \omega_0 + \varepsilon.$$

Then $\lambda(0)$ and $\lambda(\tau)$ are equal in magnitude, but have opposite sign. From (1.14) and (4.9) we find then for the transition probability

$$P = \frac{(gH_0)^2}{\varepsilon^2 + (gH_0)^2} \sin^2 \frac{1}{2\hbar}\int_0^\tau (\tilde{E}_1 - \tilde{E}_2)\,dt + \frac{\varepsilon^2}{\varepsilon^2 + (gH_0)^2}.$$

If ϵ exceeds greatly the width of the resonance curve gH_0, i.e., if $\epsilon \gg gH_0$, then $P \approx 1$, which corresponds to a spin flip during the transition through the resonance.

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