

MICROSCOPIC DERIVATION OF THE GINZBURG-LANDAU EQUATIONS FOR AN ANISOTROPIC SUPERCONDUCTOR

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A microscopic derivation of the Ginzburg-Landau equations for an anisotropic superconductor is presented. The meaning of the constants encountered in the phenomenological theory and defined in terms of the gap values at absolute zero is clarified.

THE equations of superconductivity theory can be reduced to the Ginzburg-Landau equations<sup>[1]</sup> near the critical temperature  $T_c$ . This is possible because the theory is local in this region, i.e., all the quantities change noticeably over distances that are much larger than the correlation parameter ( $\delta \gg \xi_0$ , where  $\xi_0 \sim v/T$  and  $\delta$  is the depth of penetration of the field). It is interesting to ascertain what this can lead to in the anisotropic case. It turns out that this results in the equations proposed by Ginzburg<sup>[2]</sup>. The microscopic theory allows us to clarify the meaning of the quantities in these equations.

In the derivation we assume that the potential of the interaction, spectrum, etc are anisotropic. Such a procedure is a simple generalization of the usual one and was proposed in many places (in particular by Bogolyubov, Tolmachev, and Shirkov<sup>[3]</sup> and in the papers of Pokrovskii<sup>[4,5]</sup>). It is convenient to rewrite it in a form where the superconductor is described by three Green's functions<sup>[6]</sup>:

$$\begin{aligned} \delta_{\alpha\beta} \mathcal{G}(\mathbf{p}_1, \mathbf{p}_2; \tau_1 - \tau_2) &= -\langle T \{a_{\mathbf{p}_1, \alpha}(\tau_1) a_{\mathbf{p}_2, \beta}^+(\tau_2)\} \rangle, \\ I_{\alpha\beta} \mathfrak{F}^+(\mathbf{p}_1, \mathbf{p}_2; \tau_1 - \tau_2) &= \langle T \{a_{\mathbf{p}_1, \alpha}^+(\tau_1) a_{\mathbf{p}_2, \beta}^+(\tau_2)\} \rangle, \\ I_{\alpha\beta} \mathfrak{F}(\mathbf{p}_1, \mathbf{p}_2; \tau_1 - \tau_2) &= -\langle T \{a_{\mathbf{p}_1, \alpha}(\tau_1) a_{\mathbf{p}_2, \beta}(\tau_2)\} \rangle, \end{aligned}$$

where

$$\begin{aligned} a_{p\alpha}(\tau) &= e^{-(\mu N - H\tau)} a_{p\alpha} e^{(\mu N - H\tau)}, \\ a_{p\alpha}^+(\tau) &= e^{-(\mu N - H)\tau} a_{p\alpha}^+ e^{(\mu N - H)\tau}. \end{aligned}$$

The specific form of the Hamiltonian is of no interest to us.

Introducing the Fourier components for the functions  $\mathcal{G}$ ,  $\mathfrak{F}$ ,  $\mathfrak{F}^+$

$$\begin{aligned} \mathcal{G}(\mathbf{p}_1, \mathbf{p}_2; \tau) &= T \sum_{\omega} \mathcal{G}_{\omega}(\mathbf{p}_1, \mathbf{p}_2) e^{-i\omega\tau}, \\ \mathfrak{F}(\mathbf{p}_1, \mathbf{p}_2; \tau) &= T \sum_{\omega} \mathfrak{F}_{\omega}(\mathbf{p}_1, \mathbf{p}_2) e^{-i\omega\tau}, \end{aligned}$$

$$\mathfrak{F}^+(\mathbf{p}_1, \mathbf{p}_2; \tau) = T \sum_{\omega} \mathfrak{F}_{\omega}^+(\mathbf{p}_1, \mathbf{p}_2) e^{-i\omega\tau}, \quad \omega = 2\pi(n + 1/2),$$

we can write down for them equations<sup>[6]</sup> that account for the anisotropy in the presence of the magnetic field. The gauge is chosen such that the scalar potential is equal to zero. The corresponding equations are of the form

$$\{i\omega - \xi(\mathbf{p})\} \mathcal{G}_{\omega}(\mathbf{p}, \mathbf{p}') + e \sum_{\mathbf{q}} \mathbf{A}_{\mathbf{q}} \mathbf{v}_{\mathbf{p}} \mathcal{G}_{\omega}(\mathbf{p} - \mathbf{q}, \mathbf{p}') \quad (1)$$

$$+ \sum_{\mathbf{k}} \Delta(\mathbf{p}, \mathbf{k}) \mathfrak{F}_{\omega}^+(\mathbf{k}, \mathbf{p}') = \delta_{\mathbf{p}, \mathbf{p}'},$$

$$\{i\omega + \xi(\mathbf{p})\} \mathfrak{F}_{\omega}^+(\mathbf{p}, \mathbf{p}') - e \sum_{\mathbf{q}} \mathbf{A}_{\mathbf{q}} \mathbf{v}_{\mathbf{p}} \mathfrak{F}_{\omega}^+(\mathbf{p} + \mathbf{q}, \mathbf{p}')$$

$$+ \sum_{\mathbf{k}} \Delta^*(\mathbf{p}, \mathbf{k}) \mathcal{G}_{\omega}(\mathbf{k}, \mathbf{p}') = 0,$$

$$\Delta^*(\mathbf{p}, \mathbf{k}) = gT \sum_{\mathbf{k}_1, \mathbf{k}_2, \omega} V(\mathbf{k}_1, \mathbf{k}_2; \mathbf{p}, \mathbf{k}) \mathfrak{F}_{\omega}^+(\mathbf{k}_1, \mathbf{k}_2),$$

$$\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{p} + \mathbf{k}. \quad (1')$$

We have used here the following notation:  $\xi(\mathbf{p}) = \epsilon(\mathbf{p}) - \epsilon_F$ , where  $\epsilon(\mathbf{p})$  — arbitrary electron dispersion law and  $\epsilon_F$  — Fermi energy,  $\mathbf{v}_{\mathbf{p}} = \nabla_{\mathbf{p}} \epsilon(\mathbf{p})$ ,  $\mathbf{A}_{\mathbf{q}}$  — Fourier component of the vector potential,  $V(\mathbf{k}_1, \mathbf{k}_2, \mathbf{p}, \mathbf{k})$  — dimensionless matrix elements of the interaction energy<sup>1</sup>

The summation in (1') is over those  $\mathbf{k}_1$  and  $\mathbf{k}_2$ , for which  $|\xi(\mathbf{k}_1)| \leq \tilde{\omega}(\mathbf{k}_1/k_1)$  and  $|\xi(\mathbf{k}_2)| \leq \tilde{\omega}(\mathbf{k}_2/k_2)$ , where  $\tilde{\omega}(\mathbf{k}/k)$  is a quantity on the order of the Debye energy. The magnetic field is introduced, as usual, by making the substitution  $\mathbf{p} \rightarrow \mathbf{p} - e\mathbf{A}$  in  $\epsilon(\mathbf{p})$  and by expanding  $\epsilon(\mathbf{p} - e\mathbf{A})$  in powers of  $\mathbf{A}$ . Only the term linear in  $\mathbf{A}$  was left in the series, since in the field of interest to us the electron torsion radius  $ep_0/H$  is large

<sup>1</sup>The fact that we make use of Fourier components of all quantities, disregarding the fact only a quasi-momentum and not a momentum exists in the lattice, will not affect the final results, since the field  $\mathbf{A}$  and  $\Delta$  change at distances that are large compared with the period of the lattice.

compared with the depth of penetration ( $p_0 \gg eA \sim eH\delta$ ). In addition, account is taken in (1) of the fact that the change in the field occurs over distances considerably larger than the inter-atomic distances. Equations (1) for an arbitrary potential are gauge-invariant, as are the corresponding equations of the theory of Bardeen, Cooper, and Schrieffer, accurate to terms  $T/\tilde{\omega}$ , which are always small in real superconductors. Therefore, by calculating all the quantities accurate to terms  $T/\tilde{\omega} \ll 1$ , we obtain the final expression in gauge-invariant form.

We introduce the Green's function of the electrons in the normal metal in the presence of the field  $\tilde{\mathcal{G}}_\omega(\mathbf{p}_1, \mathbf{p}_2)$ . The equation satisfied by this function can be written in two ways:

$$\{i\omega - \xi(\mathbf{p})\} \tilde{\mathcal{G}}_\omega(\mathbf{p}, \mathbf{p}') + e \sum_q \mathbf{A}_q \mathbf{v}_p \mathcal{G}_\omega(\mathbf{p} - \mathbf{q}, \mathbf{p}') = \delta_{\mathbf{p}, \mathbf{p}'}, \quad (2)$$

$$\{i\omega - \xi(\mathbf{p}')\} \tilde{\mathcal{G}}_\omega(\mathbf{p}, \mathbf{p}') + e \sum_q \mathbf{A}_q \mathbf{v}_{p'} \mathcal{G}_\omega(\mathbf{p}, \mathbf{p}' + \mathbf{q}) = \delta_{\mathbf{p}, \mathbf{p}'}. \quad (2')$$

With the aid of the Green's function  $\tilde{\mathcal{G}}_\omega(\mathbf{p}_1, \mathbf{p}_2)$  and Eqs. (2) and (2'), we write the system (1) for  $\mathcal{G}$ ,  $\mathfrak{F}$ , and  $\mathfrak{F}^+$  in integral form

$$\mathcal{G}_\omega(\mathbf{p}, \mathbf{p}') = \tilde{\mathcal{G}}_\omega(\mathbf{p}, \mathbf{p}') - \sum_{l, \mathbf{k}} \tilde{\mathcal{G}}_\omega(\mathbf{p}, l) \Delta(l, \mathbf{k}) \mathfrak{F}^+(\mathbf{k}, \mathbf{p}'), \quad (3)$$

$$\mathfrak{F}^+(\mathbf{p}, \mathbf{p}') = \sum_{l, \mathbf{k}} \tilde{\mathcal{G}}_\omega(l, \mathbf{p}) \Delta^*(l, \mathbf{k}) \mathcal{G}_\omega(\mathbf{k}, \mathbf{p}'). \quad (3')$$

From (2) we obtain  $\tilde{\mathcal{G}}_\omega(\mathbf{p}, \mathbf{p}')$  accurate to terms that are quadratic in  $\mathbf{A} \cdot \mathbf{v}$  inclusive:

$$\tilde{\mathcal{G}}_\omega(\mathbf{p}, \mathbf{p}') = \mathcal{G}_\omega(\mathbf{p}) \delta_{\mathbf{p}, \mathbf{p}'} - e \mathcal{G}_\omega(\mathbf{p}) \mathbf{A}_{\mathbf{p}-\mathbf{p}'} \mathbf{v}_p \mathcal{G}_\omega(\mathbf{p}') + e^2 \mathcal{G}_\omega(\mathbf{p}) \mathcal{G}_\omega(\mathbf{p}') \sum_q \mathbf{A}_q \mathbf{v}_p (\mathbf{A}_{\mathbf{p}-\mathbf{p}'-\mathbf{q}} \mathbf{v}_p) \mathcal{G}_\omega(\mathbf{p} - \mathbf{q}), \quad (4)$$

where  $\mathcal{G}_\omega(\mathbf{p}) = \{i\omega - \xi(\mathbf{p})\}^{-1}$  is the Green's function of the normal metal in the absence of the field.

Inasmuch as it is our aim to derive an equation relative to  $\Delta$  in the vicinity of  $T_C$ , where  $\Delta/T_C \sim \sqrt{1 - T/T_C} \ll 1$ , expressions (3) and (3') can be expanded in powers of  $\Delta$ . It is seen from (3') and (1') that the expansion of the function  $\mathfrak{F}^+$  can be carried out accurate to terms of third order in  $\Delta$ , exclusive. The function  $\mathcal{G}$  must be known accurate to second-order terms in  $\Delta$ :

$$\mathcal{G}_\omega(\mathbf{p}, \mathbf{p}') = \tilde{\mathcal{G}}_\omega(\mathbf{p}, \mathbf{p}') - \sum \tilde{\mathcal{G}}_\omega(\mathbf{p}, l) \Delta(l, \mathbf{k}_1) \tilde{\mathcal{G}}_\omega(l', \mathbf{k}_1) \Delta^*(l', \mathbf{k}_1') \tilde{\mathcal{G}}_\omega(\mathbf{k}_1', \mathbf{p}') \quad (5)$$

(the summation is over  $l, l', \mathbf{k}_1$ , and  $\mathbf{k}_1'$ ). Substituting (5) in (3') we obtain the required expansion for  $\mathfrak{F}^+$ , with the aid of which we get from (1') an equation for  $\Delta^*(\mathbf{p}_1, \mathbf{p}_2)$ :

$$\begin{aligned} \Delta^*(\mathbf{p}_1, \mathbf{p}_2) = & gT \sum V(\mathbf{k}_1, \mathbf{k}_2; \mathbf{p}_1, \mathbf{p}_2) \tilde{\mathcal{G}}_\omega(l, \mathbf{k}_1) \Delta^*(l, \mathbf{k}_3) \\ & \times \tilde{\mathcal{G}}_\omega(\mathbf{k}_3, \mathbf{k}_2) - gT \sum V(\mathbf{k}_1, \mathbf{k}_2; \mathbf{p}_1, \mathbf{p}_2) \\ & \times \tilde{\mathcal{G}}_\omega(l, \mathbf{k}_1) \Delta^*(l, \mathbf{k}_3) \tilde{\mathcal{G}}_\omega(\mathbf{k}_3, l') \Delta(l', \mathbf{k}') \tilde{\mathcal{G}}_\omega(l', \mathbf{k}') \\ & \times \Delta^*(l', \mathbf{k}') \tilde{\mathcal{G}}_\omega(\mathbf{k}', \mathbf{k}_2) \end{aligned} \quad (6)$$

(the summation is over all the repeated indices, and  $\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{p}_1 + \mathbf{p}_2$ ). Using expression (4) for  $\tilde{\mathcal{G}}_\omega(\mathbf{p}_1, \mathbf{p}_2)$ , we obtain for the first term of the right half of (6)

$$\begin{aligned} T \sum_{\mathbf{k}_1, \mathbf{k}_2} V(\mathbf{k}_1, \mathbf{k}_2; \mathbf{p}_1, \mathbf{p}_2) \{ & \mathcal{G}_\omega(\mathbf{k}_1) \Delta^*(\mathbf{k}_1, \mathbf{k}_2) \mathcal{G}_\omega(\mathbf{k}_2) \\ & - e \sum_{\mathbf{k}_3} \mathcal{G}_\omega(\mathbf{k}_1) \Delta^*(\mathbf{k}_1, \mathbf{k}_3) \mathcal{G}_\omega(\mathbf{k}_3) (\mathbf{A}_{\mathbf{k}_3-\mathbf{k}_2} \mathbf{v}_{\mathbf{k}_3}) \mathcal{G}_\omega(\mathbf{k}_2) \\ & - e \sum_l \mathcal{G}_\omega(l) (\mathbf{A}_{l-\mathbf{k}_1} \mathbf{v}_l) \mathcal{G}_\omega(\mathbf{k}_1) \Delta^*(l, \mathbf{k}_2) \mathcal{G}_\omega(\mathbf{k}_2) \\ & + e^2 \sum_{\mathbf{k}_3, \mathbf{q}} \mathcal{G}_\omega(\mathbf{k}_1) \Delta^*(\mathbf{k}_1, \mathbf{k}_3) \mathcal{G}_\omega(\mathbf{k}_3) (\mathbf{A}_q \mathbf{v}_{\mathbf{k}_3}) \\ & \times (\mathbf{A}_{\mathbf{k}_3-\mathbf{k}_2-\mathbf{q}} \mathbf{v}_{\mathbf{k}_3}) \mathcal{G}_\omega(\mathbf{k}_2) + e^2 \sum_{l, \mathbf{q}} \mathcal{G}_\omega(l) (\mathbf{A}_q \mathbf{v}_l) (\mathbf{A}_{l-\mathbf{q}-\mathbf{k}_1} \mathbf{v}_l) \\ & \times \mathcal{G}_\omega(l - \mathbf{q}) \mathcal{G}_\omega(\mathbf{k}_1) \mathcal{G}_\omega(\mathbf{k}_2) \Delta^*(l, \mathbf{k}_2) + e^2 \sum_{\mathbf{k}_3, l} \mathcal{G}_\omega(l) \\ & \times (\mathbf{A}_{l-\mathbf{k}_1} \mathbf{v}_l) (\mathbf{A}_{\mathbf{k}_3-\mathbf{k}_2} \mathbf{v}_{\mathbf{k}_3}) \mathcal{G}_\omega(\mathbf{k}_1) \mathcal{G}_\omega(\mathbf{k}_3) \Delta^*(l, \mathbf{k}_3) \mathcal{G}_\omega(\mathbf{k}_2) \}. \end{aligned} \quad (7)$$

In the absence of a field  $\Delta(\mathbf{p}_1, \mathbf{p}_2) = \Delta(\mathbf{p}_1) \delta_{\mathbf{p}_1, -\mathbf{p}_2}$ , so that  $\mathbf{k} = \mathbf{p}_1 + \mathbf{p}_2$  characterizes the inhomogeneity of the gap. Starting from this, we shall henceforth write  $\Delta(\mathbf{p}_1, \mathbf{p}_2) = \Delta_{\mathbf{k}}(\mathbf{p}_1)$ . Since the important quantities in  $\Delta_{\mathbf{k}}$  and  $\mathbf{A}_q$  are  $k, q \sim 1/\delta$ , and  $\xi \sim T_C$ , we can use the fact that  $T/v\delta \ll 1$  in the case of a London superconductor in which we are interested and expand (7) up to second powers of  $\mathbf{v} \cdot \mathbf{k}$  and  $\mathbf{v} \cdot \mathbf{q}$  inclusive. Recognizing also that the changes in all the quantities occur at distances much larger than interatomic, so that  $k \ll p$  and  $q \ll p$ , we obtain as a result

$$\begin{aligned} \Delta_{\mathbf{k}}^*(\mathbf{p}) = & \frac{gT}{(2\pi)^3} \sum_{\omega} \int U(\mathbf{k}_1, \mathbf{p}) \{ \mathcal{G}_\omega(\mathbf{k}_1) \mathcal{G}_\omega(\mathbf{k}_1) \\ & - \mathcal{G}_\omega(\mathbf{k}_1) \mathcal{G}_\omega(\mathbf{k}_1) (\mathbf{v}_{\mathbf{k}_1} \mathbf{k})^2 \Delta_{\mathbf{k}}^*(\mathbf{k}_1) + e \sum_q (\mathbf{A}_q \mathbf{v}_{\mathbf{k}_1}) \\ & \times (\mathbf{q} \mathbf{v}_{\mathbf{k}_1} + 2\mathbf{k} \mathbf{v}_{\mathbf{k}_1}) \mathcal{G}_\omega^2(\mathbf{k}_1) \mathcal{G}_\omega^2(\mathbf{k}_1) \Delta_{\mathbf{k}+\mathbf{q}}^*(\mathbf{k}_1) \\ & + e^2 \sum_{\mathbf{q}, \mathbf{q}'} (\mathbf{A}_q \mathbf{v}_{\mathbf{k}_1}) (\mathbf{A}_{q'} \mathbf{v}_{\mathbf{k}_1}) \Delta_{\mathbf{k}+\mathbf{q}+\mathbf{q}'}^*(\mathbf{k}_1) [2\mathcal{G}_\omega^3(\mathbf{k}_1) \mathcal{G}_\omega(\mathbf{k}_1) \\ & - \mathcal{G}_\omega^2(\mathbf{k}_1) \mathcal{G}_\omega^2(\mathbf{k}_1)] - \mathcal{G}_\omega^2(\mathbf{k}_1) \mathcal{G}_\omega^2(\mathbf{k}_1) \\ & \times \sum_{\mathbf{p}_1, \mathbf{p}_2} \Delta_{\mathbf{p}_1}^*(\mathbf{k}_1) \Delta_{\mathbf{p}_2}^*(\mathbf{k}_2) \Delta_{\mathbf{p}_1+\mathbf{p}_2-\mathbf{k}}(\mathbf{k}_1) \} \frac{d\xi_1 d\sigma_{1F}}{v_{1F}}. \end{aligned} \quad (8)$$

Here  $U(\mathbf{k}_1, \mathbf{p}) \equiv V(\mathbf{k}_1, -\mathbf{k}_2; \mathbf{p}, -\mathbf{p})$  and the integration with respect to  $\sigma_F$  is carried over the Fermi surface.

Summing over  $\omega$  and integrating over  $\xi_1$  we get

$$\begin{aligned}
 \Delta_k^*(\mathbf{p}) = & \frac{g}{(2\pi)^3} \ln \frac{2\tilde{\omega}\gamma}{T\pi} \int U(\mathbf{k}_1, \mathbf{p}) \frac{d\sigma_{1F}}{v_{1F}} \Delta_k^*(\mathbf{k}_1) \\
 & + \frac{k_i k_j g \lambda}{2(2\pi)^3} \int U(\mathbf{k}, \mathbf{p}) v_i v_j \Delta_k^*(\mathbf{k}_1) \frac{d\sigma_{1F}}{v_{1F}} + \frac{e\lambda g}{(2\pi)^3} \\
 & \times \sum_{\mathbf{q}} (\mathbf{A}_{\mathbf{q}})_i (\mathbf{q} + 2\mathbf{k})_j \int U(\mathbf{k}_1, \mathbf{p}) v_i v_j \Delta_{\mathbf{k}+\mathbf{q}}^*(\mathbf{k}_1) \frac{d\sigma_{1F}}{v_{1F}} \\
 & - \frac{2e^2 \lambda}{(2\pi)^3} \sum_{\mathbf{q}, \mathbf{q}'} (\mathbf{A}_{\mathbf{q}})_i (\mathbf{A}_{\mathbf{q}'})_j \int U(\mathbf{k}_1, \mathbf{p}) v_i v_j \Delta_{\mathbf{k}+\mathbf{q}+\mathbf{q}'}^*(\mathbf{k}_1) \frac{d\sigma_{1F}}{v_{1F}} \\
 & - \frac{g\lambda}{(2\pi)^3} \sum_{\mathbf{p}_1, \mathbf{p}_2} \int U(\mathbf{k}_1, \mathbf{p}) \Delta_{\mathbf{p}_1}^*(\mathbf{k}_1) \Delta_{\mathbf{p}_2}^*(\mathbf{k}_1) \Delta_{\mathbf{p}_1+\mathbf{p}_2-\mathbf{k}}(\mathbf{k}_1) \frac{d\sigma_{1F}}{v_{1F}}, \\
 & \lambda = \frac{7\zeta(3)}{8(\pi T)^2}. \quad (9)
 \end{aligned}$$

Here  $\ln \tilde{\omega}$  is connected with  $\tilde{\omega}(\mathbf{p}/p)$  by relation (9) of [4]

$$\ln \tilde{\omega} = \int \psi^2(\mathbf{p}) \ln \tilde{\omega} \left( \frac{\mathbf{p}}{p} \right) \frac{d\sigma_F}{v_F} \left/ \int \frac{d\sigma_F}{v_F} \right.$$

(the meaning of the quantity  $\psi(\mathbf{p})$  will be clarified later).

Pokrovskii [4,5] has shown for the equilibrium case when  $\Delta(\mathbf{p})$  does not depend on  $\mathbf{k}$ , that in the weak-coupling model we have for arbitrary temperatures  $\Delta^*(\mathbf{p}) = Q^* \psi(\mathbf{p})$ , where  $Q^* \equiv Q^*(T)$  is a function of the temperature only. Here  $T_c = (2\tilde{\omega}\gamma/\pi) e^{-\Lambda/g}$ , and  $\psi(\mathbf{p})$  and  $\Lambda$  are determined from the integral equation [4]

$$\psi(\mathbf{p}) = \Lambda \int U(\mathbf{k}_1, \mathbf{p}) \psi(\mathbf{k}_1) \frac{d\sigma_{1F}}{v_{1F}}.$$

The function  $\psi(\mathbf{p})$  characterizes the anisotropy gap at absolute zero  $\Delta(\mathbf{p}, T=0) = 2\tilde{\omega}\psi(\mathbf{p})e^{-\Lambda/g}$ , where  $\psi(\mathbf{p})$  is normalized by the condition

$$\int \psi^2(\mathbf{p}) \frac{d\sigma_F}{v_F} = \int \frac{d\sigma_F}{v_F}.$$

We analogously seek  $\Delta\mathbf{k}(\mathbf{p})$  in the form  $\Delta\mathbf{k}(\mathbf{p}) = Q^*(\mathbf{k})\psi(\mathbf{p})$ . To obtain an equation for  $Q^*(\mathbf{k})$ , it is necessary to seek for  $\Delta\mathbf{k}(\mathbf{p})$  the following approximation

$$\Delta_k^*(\mathbf{p}) = Q^*(\mathbf{k})\psi(\mathbf{p}) + \psi_1(\mathbf{p}).$$

The equation obtained in this manner for  $\psi_1(\mathbf{p})$  can be solved if

$$\begin{aligned}
 \frac{T_c - T}{T_c} Q_k^* \int \psi^2(\mathbf{k}_1) \frac{d\sigma_{1F}}{v_{1F}} + \lambda \int v_i v_j \psi^2(\mathbf{k}_1) \frac{d\sigma_{1F}}{v_{1F}} \left[ \frac{k_i k_j}{2} Q_k^* \right. \\
 \left. + e \sum_{\mathbf{q}} (\mathbf{A}_{\mathbf{q}})_i (\mathbf{q} + 2\mathbf{k})_j Q_{\mathbf{k}+\mathbf{q}}^* - 2e^2 \sum_{\mathbf{q}, \mathbf{q}'} (\mathbf{A}_{\mathbf{q}})_i (\mathbf{A}_{\mathbf{q}'})_j Q_{\mathbf{k}+\mathbf{q}+\mathbf{q}'}^* \right] \\
 - \lambda \sum_{\mathbf{p}_1, \mathbf{p}_2} \int Q_{\mathbf{p}_1}^* Q_{\mathbf{p}_2}^* Q_{\mathbf{p}_1+\mathbf{p}_2-\mathbf{k}} \psi^4(\mathbf{k}_1) \frac{d\sigma_{1F}}{v_{1F}} = 0. \quad (10)
 \end{aligned}$$

Introducing the notation

$$\begin{aligned}
 \frac{1}{m_{ij}} = \frac{2}{2e_F} \int v_i v_j \psi^2(\mathbf{k}_1) \frac{d\sigma_{1F}}{v_{1F}} \left/ \int \frac{d\sigma_{1F}}{v_{1F}} \right., \\
 f = \frac{7\zeta(3)}{8(\pi T)^2} \int \psi^4(\mathbf{k}_1) \frac{d\sigma_{1F}}{v_{1F}}, \quad \eta = \frac{7\zeta(3) e_F}{12(\pi T)^2} \quad (11)
 \end{aligned}$$

and taking Fourier transforms with respect to  $\mathbf{k}$ , we obtain ultimately an equation for  $Q(\mathbf{x})$ :

$$\begin{aligned}
 \sum_{j,k} \frac{1}{2m_{jk}} (\nabla_j + 2ieA_j(\mathbf{x})) (\nabla_k + 2ieA_k(\mathbf{x})) Q^*(\mathbf{x}) \\
 + \frac{1}{\eta} \left\{ \frac{T_c - T}{T_c} - f |Q(\mathbf{x})|^2 \right\} Q^*(\mathbf{x}) = 0.
 \end{aligned}$$

Let us proceed to calculate the current density. In anisotropic conductors the current density is given by

$$\begin{aligned}
 j_i(\mathbf{q}) = 2eT \sum_{\omega, \mathbf{p}} \mathfrak{G}_{\omega}(\mathbf{p} + \mathbf{q}, \mathbf{p}) v_i(\mathbf{p}) \\
 - 2e^2T \sum_{\mathbf{p}, \mathbf{k}, \omega} w_{ij}(\mathbf{p}) A_j(\mathbf{k}) \mathfrak{G}_{\omega}(\mathbf{p} + \mathbf{q} - \mathbf{k}, \mathbf{p}), \quad (12)
 \end{aligned}$$

where  $w_{ij}(\mathbf{p}) = \partial^2 \epsilon(\mathbf{p}) / \partial p_i \partial p_j$ . Since in a normal metal the current in a constant magnetic field is zero, we have

$$\begin{aligned}
 2eT \sum_{\omega, \mathbf{p}} \mathfrak{G}_{\omega}(\mathbf{p} + \mathbf{q}, \mathbf{p}) v_i(\mathbf{p}) \\
 - 2e^2T \sum_{\mathbf{p}, \mathbf{k}, \omega} w_{ij}(\mathbf{p}) A_j(\mathbf{k}) \mathfrak{G}_{\omega}(\mathbf{p} + \mathbf{q} - \mathbf{k}, \mathbf{p}) = 0. \quad (13)
 \end{aligned}$$

Subtracting (13) from (12) we obtain for current density

$$\begin{aligned}
 j_i(\mathbf{q}) = 2eT \sum_{\omega, \mathbf{p}} \mathfrak{G}_{\omega}^{(2)}(\mathbf{p} + \mathbf{q}, \mathbf{p}) v_i(\mathbf{p}) \\
 - 2e^2T \sum_{\omega, \mathbf{p}, \mathbf{k}} w_{ij}(\mathbf{p}) A_j(\mathbf{k}) \mathfrak{G}_{\omega}^{(2)}(\mathbf{p} + \mathbf{q} - \mathbf{k}, \mathbf{p}), \quad (14)
 \end{aligned}$$

where  $\mathfrak{G}_{\omega}^{(2)}$  is a correction quadratic in  $\Delta$  to  $\mathfrak{G}_{\omega}$ .

We seek the current density accurate to terms linear in  $\mathbf{q} \cdot \mathbf{v}$ ,  $\mathbf{k} \cdot \mathbf{v}$ , and  $\mathbf{A} \cdot \mathbf{v}$ , inclusive. With this accuracy, the second term in (14) vanishes and

$$j_i(\mathbf{q}) = 2eT \sum_{\omega, \mathbf{p}} \mathfrak{G}^{(2)}(\mathbf{p} + \mathbf{q}, \mathbf{p}) v_i(\mathbf{p}). \quad (15)$$

Substituting  $\mathfrak{G}^{(2)}$  from (5) in (15) and carrying out calculations similar to those before, we obtain for  $j(\mathbf{x}) = \sum_{\mathbf{q}} j_{\mathbf{q}} e^{i\mathbf{q} \cdot \mathbf{x}}$  the expression

$$\begin{aligned}
 j_k(\mathbf{x}) = \left\{ \frac{ie}{m_{kj}} [Q(\mathbf{x}) \nabla_j Q^*(\mathbf{x}) \right. \\
 \left. - Q^*(\mathbf{x}) \nabla_j Q(\mathbf{x})] - \frac{4e^2 |Q(\mathbf{x})|^2 A_j(\mathbf{x})}{m_{kj}} \right\} C, \\
 C = \frac{7\zeta(3) e_F}{12(2\pi)^3 T^2} \int \frac{d\sigma_F}{v_F}.
 \end{aligned}$$

The system of equations obtained corresponds to the phenomenological scheme obtained by Ginzburg [2]. The connection between the function  $\psi(\mathbf{x})$  introduced in [2] and our  $Q(\mathbf{x})$  is given by the relation  $\psi(\mathbf{x}) = \sqrt{C} Q(\mathbf{x})$ . An important factor in the equations obtained, however, is the doubled charge  $2e$ , which corresponds to the fact that  $\psi(\mathbf{x})$  has the meaning of a wave function of a Cooper pair. Taking this circumstance into account, we can use directly Ginzburg's formulas [2] for the physical

quantities, in which we make the substitution  $e \rightarrow 2e$ .

The best method of determining the mass tensor seems to be measurement of the anisotropy of the depth of penetration near  $T_c$ . There are apparently no such data at present. The data on the anisotropy of the surface tension  $\Delta$  are less definite in this respect. According to Sharvin and Gantmakher<sup>[7]</sup> the quantity  $(\Delta_{[110]} - \Delta_{[001]})/\Delta$  for tetragonal tin is 20–25%. Inasmuch as  $\Delta \propto m^{-1/2}$ , we get  $\Delta m/m \sim 50\%$ .

Several recent results (for example, data on the absorption of ultrasound<sup>[8]</sup>) show that the anisotropy of the energy gap itself is quite small. From Hohenberg's estimates<sup>[9]</sup> of the impurity dependence of  $T_c$  it follows that for aluminum  $[\Delta^2(n) - (\overline{\Delta(n)})^2]/(\overline{\Delta(n)})^2$  is of the order of  $10^{-2}$ . In this case the anisotropy of the mass tensor characterizes the anisotropy of the energy spectrum of the normal metal. In light of this, the large anisotropy  $m_{ijk}$  obtained by Sharvin and Gantmakher<sup>[8]</sup> becomes quite understandable.

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<sup>1</sup>V. L. Ginzburg and L. D. Landau, JETP 20, 1064 (1950).

<sup>2</sup>V. L. Ginzburg, JETP 23, 236 (1952).

<sup>3</sup>Bogolyubov, Tolmachev, and Shirkov, *Novyi metod v teorii sverkhprovodimosti (New Method in Superconductivity Theory)* AN SSSR, 1958.

<sup>4</sup>V. L. Pokrovskii, JETP 40, 641 (1961), Soviet Phys. JETP 13, 447 (1961).

<sup>5</sup>V. L. Pokrovskii and N. S. Ryvkin, JETP 43, 92 (1962), Soviet Phys. JETP 16, 67 (1963).

<sup>6</sup>L. P. Gor'kov, JETP 34, 735 (1958), Soviet Phys. JETP 7, 505 (1958).

<sup>7</sup>Yu. V. Shavrin and V. F. Gantmakher, JETP 38, 1456 (1960), Soviet Phys. JETP 11, 1052 (1960).

<sup>8</sup>Morse, Olsen, and Gavenda, Phys. Rev. Lett. 3, 15 (1959).

<sup>9</sup>P. Hohenberg, JETP 45, 1208 (1963), Soviet Phys. JETP 18, 834 (1964).

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