PRODUCTION OF HIGH ENERGY PIONS BY COSMIC RAY PARTICLES

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Production of high energy pions in the atmosphere as a result of various processes such as ionization, isobar decay ($J = T = 3/2$), and hyperon decay is considered. It is shown that these processes are not sufficient to explain the fluxes of high energy muons and γ quanta observed experimentally. The main features of the process responsible for high energy pion production are examined.

INTRODUCTION

A study of the interaction of high-energy cosmic rays with atomic nuclei has led us to the conclusion[1-3] that these interactions are characterized by large fluctuations of the inelasticity coefficient K and of the fraction $K_{\pi 0}$ of the energy transferred to the neutral pions. The experimental data[1,2] show that with probability of 10% the value of $K_{\pi 0}$ lies in the interval $0.5 \lesssim K_{\pi 0} \leq 1$.

The presence of large fluctuations in conjunction with a steeply decreasing spectrum of nuclear-active high-energy particles causes interactions with large values of $K_{\pi 0}$, i.e., interactions with anomalously large inelasticity coefficients close to unity, to predominate in many observed phenomena (large radiation bursts, generation of high-energy air showers, and possibly generation of extensive air showers[4]).

If the large role of the fluctuations of the inelasticity in the interaction is clearly manifest in the generation of the electron-photon high-energy component, a major role is also to be expected in the inelasticity fluctuations of the interaction in the generation of charged high-energy pions. We can proceed to clarify this question by comparing the experimental data with calculations made under the assumption that average interaction characteristics are realized for the different processes.

1. CHARACTERISTICS OF THE GENERATION OF HIGH-ENERGY PIONS IN THE ATMOSPHERE

A. Generation of π^0 mesons. The number of γ quanta with energy ϵ, $\epsilon + d\epsilon$ generated in 1 g/cm^2 per second in the atmosphere at a depth X by a current of nuclear-active particles moving in a vertical direction in a solid angle of one steradian, is equal to

$$n_\gamma(\varepsilon, X)\, d\varepsilon = \frac{2d\varepsilon}{L_{\text{int}}} \int_0^1 \frac{dy}{y} \int_{\varepsilon/y}^{\infty} n_0\left(\frac{\varepsilon}{y}, E_0\right) N_{\text{na}}(E_0, X)\, dE_0, \quad (1)$$

where $N_{na}(E_0, X)\, dE_0$ is the spectrum of the nuclear-active particles at a depth X of the atmosphere, which can be written in the form

$$N_{\text{na}}(E_0, X)\, dE_0 = B dE_0 e^{-\beta X}/E_0^\gamma$$

(henceforth X will be measured in units of the interaction range); $n_0(E, E_0)\, dE$ is the spectrum of the π^0 mesons generated in one event of interaction of a nuclear-active particle with energy E_0.

The available experimental data indicate that the γ-quantum spectrum coincides with the spectrum of the nuclear-active particles in the energy region $\epsilon \lesssim 10^{12}$ eV. (An analogous picture is observed for the $\pi^\pm$ meson spectrum reconstituted from the muon spectrum.) This means that the effective multiplicity of the high-energy pions, which are essentially responsible for the observed γ-quantum and muon spectra, depends quite weakly on the energy of the primary particles E_0, at least in the energy range $E_0 \lesssim 10^{12}$ eV, i.e., the form of the spectrum of the generated pions $n(E, E_0)\, dE$ does not depend in first approximation on E_0, at least in the region where $E/E_0 \lesssim 1$.

If we assume that $n(E, E_0)\, dE = \bar{n} f_1(E/E_0) E_0^{-1} dE$, and normalize $f_1(E/E_0)$ so that

$$\int_0^1 f_1(E/E_0)\, E_0^{-1} dE = 1,$$

then, by putting $E/E_0 = u$ we put

$$n_\gamma(\varepsilon, X)\, d\varepsilon = \frac{2d\varepsilon}{L_{\text{int}}} \int_0^1 \frac{dy}{y} \int_{\varepsilon/y}^{\infty} \frac{1}{3} \bar{n} f_1\left(\frac{\varepsilon}{yE_0}\right) \frac{BdE_0}{E_0^\gamma} e^{-\beta X}$$

$$= \frac{2B\bar{n}e^{-\beta X}\, d\varepsilon}{3\gamma L_{\text{int}} \varepsilon^\gamma} \int_0^1 u^{\gamma-1} f_1(u)\, du. \quad (2)$$

Here $\bar{n}$—average number of pions generated in one interaction.

Since $Be^{-\beta X} d\epsilon/\epsilon^{\gamma} = N_{na}(\epsilon, X)\, d\epsilon$ and $n/3 = n_0$, we get

$$n_{\gamma}(\varepsilon, X)\, d\varepsilon = \frac{2}{\gamma L_{int}} a_0 N_{na}(\varepsilon, X)\, d\varepsilon,$$

$$a_0 = \bar{n}_0 \int_0^1 u^{\gamma-1} f_1(u)\, du. \tag{3}$$

Thus, a characteristic of the intensity of the generation of π^0 mesons in an elementary interaction act is the quantity a_0, if the condition $n(E, E_0)\, dE = \bar{n} f_1(E/E_0)\, E_0^{-1} dE$ is satisfied.

B. Generation of charged pions. An analogous characteristic of the generation of charged pions can be obtained from the known muon intensity[5].

If we denote the vertical flux of pions with energy E, E + dE at a depth X of the atmosphere by $P_{\pi}(E, X)\, dE$, and if we assume that the spectrum of the charged pions generated in an elementary act $n_1(E, E_0)\, dE$ does not depend on the energy of the generating principle, i.e., the spectrum can be written in the form

$$n_1(E, E_0)\, dE = \bar{n}_1 f_1(E/E_0)\, E_0^{-1} dE,$$

then $P_{\pi}(E, X)\, dE$ is given by the expression[5]

$$P_{\pi}(E, X)\, dE = \frac{Ba_1 dE}{E^{\gamma}} e^{-X} \int_0^X \left(\frac{\xi}{X}\right)^{C/E} e^{(1-\beta)\xi}\, d\xi \tag{4}$$

where

$$a_1 = \bar{n}_1 \int_0^1 u^{\gamma-1} f_1(u)\, du, \qquad \int_0^1 f_1(u)\, du = 1,$$

$$C = \frac{m_{\pi} c^2 P_0}{c\tau_{\pi} \rho_0} = 1.8 \cdot 10^{11} \text{ eV}.$$

But $Be^{-\beta X} dE/E^{\gamma}$ is the flux of the nuclear-active component particles at a depth X of the atmosphere, and therefore

$$P_{\pi}(E, X)\, dE = a_1 \varphi(E, X)\, N_{na}(E, X)\, dE,$$

$$\varphi(E, X) = X \int_0^1 (y)^{C/E} e^{X(1-\beta)(y-1)}\, dy.$$

In order to obtain the flux of muons with energy E_{μ}, $E_{\mu} + dE_{\mu}$ at sea level (assuming that all muons result from $\pi - \mu$ decay) it is necessary to calculate the total number of pions with energy in the interval $1.3\, E_{\mu}$, $1.3(E_{\mu} + dE_{\mu})$ which have decayed in the entire atmosphere

$$N_{\pi\,\text{dec}}(E, X_0)\, dE = dE \int_0^{X_0} P_{\pi}(E, X) \frac{C dX}{EX}$$

$$= \frac{BCa_1 dE}{E^{\gamma+1}} \int_0^1 (y)^{C/E} \left\{ \frac{e^{X[y(1-\beta)-1]} - 1}{(1-\beta)y - 1} \right\} dy.$$

For sea level $X_0 \approx 14$ and since $(1-\beta)y - 1 < 0$, we have

$$N_{\mu}(E_{\mu})\, dE_{\mu} = \frac{BCa_1 dE}{E^{\gamma+1}} \int_0^1 (y)^{C/E} \frac{dy}{1-(1-\beta)y}$$

$$= \frac{Ba_1 dE_{\mu}}{(1.3)^{\gamma-1} E_{\mu}^{\gamma}} \sum_{n=0}^{\infty} \frac{(1-\beta)^n}{1 + 0.72(n+1)E/10^{11}}. \tag{5}$$

If we assume that the high-energy pions may constitute an appreciable fraction of the nuclear-active particles and that upon interaction with the atomic nuclei they generate pions with spectrum

$$n_2(E, E_0)\, dE = \bar{n}_2 f_2(E/E_0)\, E_0^{-1} dE,$$

different from the spectrum of the pions generated by the nucleons of the same energy E_0, then the expression for N_{μ} will have a somewhat different form:

$$N_{\mu}(E_{\mu})\, dE_{\mu} = \frac{Ba_1 dE_{\mu}}{(1.3)^{\gamma-1} E_{\mu}^{\gamma}} F(E_{\mu}, a_2),$$

$$F(E, a_2) = \frac{1}{1-a_2} \sum_{n=0}^{\infty} \frac{b^n}{1 + 0.72(n+1)E/10^{11}},$$

$$b = \frac{1 - a_2 - \beta}{1 - a_2}, \quad a_2 = \bar{n}_2 \int_0^1 u^{\gamma-1} f_2(u)\, du. \tag{6}$$

It must be noted that formulas (5) and (6) give the spectrum of the muons at sea level in the energy region in which the ionization losses and muon decay in the atmosphere can be neglected (i.e., in the energy region $E \gtrsim 10$ BeV).

Since $BdE/E^{\gamma} = N_{na}(E, X = 0)\, dE$ is the flux of primary cosmic particles with energy E, E + dE, we get from (6)

$$N_{\mu}(E)\, dE = a_1 \frac{N_{na}(E, X=0)\, F(E, a_2)\, dE}{(1.3)^{\gamma-1}}. \tag{7}$$

The values of the function $F(E, a_2)$ for different E and a_2 are given in Table I.

Table I

a_2	E, BeV: 10	10^2	10^3	$5 \cdot 10^3$	10^4
0	1.27	0.73	0.147	0.031	0.0158
0.1	1.29	0.77	0.153		
0.2	1.31	0.79	0.163		
0.3	1.33	0.82	0.175	0.0385	0.0196

Since $F(E, a_2)$ depends very little on the parameter a_2, the exact value of which we do not know, we can determine the quantity a_1 with sufficient reliability if we know the spectrum of the primary cosmic particles, since it follows from (7) that

$$a_1 = \frac{(1.3)^{\gamma-1} N_{\mu}(E)}{F(E, a_2)\, N_{na}(E, X=0)}. \tag{8}$$

2. ENERGY SPECTRUM OF PRIMARY PARTICLES IN THE ENERGY REGION $10^{10} - 10^{13}$ eV

No direct measurement of the flux of primary particles in the energy region above 10^{10} eV were made, and the spectrum in this region was constructed from the results of various indirect measurements (emulsion measurements, ionization bursts, and extensive air showers). Whereas such measurements do give a correct description of the form of the spectrum, they cannot claim to determine sufficiently accurately the absolute value of the flux of particles with given energy, because in none of these methods is there a unique connection between the measured parameters and the primary-particle energy. The presence of fluctuations with a decreasing primary-particle spectrum will lead to an overestimate of the measured primary-particle intensity, unless the fluctuations are taken into account. This can apparently explain the fact that a comparison of the fluxes of nuclear active particles with energies $10^{10} - 10^{12}$ eV at mountain altitudes[1] with the flux of primary particles[6-8] leads to absorption ranges in air of 100—85 g/cm^2 for the nuclear active particles, in sharp contrast with the experimental data.

We have therefore decided to proceed in a different fashion, and reconstitute the spectrum of the primary cosmic particles from the known spectrum of the nuclear-active particles at mountain altitudes[1,9], using the fact that the absorption range remains constant at 120—125 g/cm^2 in the broad energy range $10^{10} - 10^{12}$ eV[10,11].

The spectrum of protons in the energy range up to 30 BeV was sufficiently well measured at an altitude of 3200 meters above sea level[9]. The integral spectrum of the nucleons at this altitude can be represented from these data in the form

$$N_{\text{na}}(\geqslant E) = \frac{3{,}56 \cdot 10^{-3}}{(2+E)^{1{,}8}} \text{ cm}^{-2}\text{ sec}^{-1}\text{ sr}^{-1}. \qquad (9)$$

We have extrapolated this spectrum in the region of high energies—up to 10^{12} eV (curve 1 on Fig. 1).

At the same altitude, we measured with the aid of an ionization calorimeter[1] the absolute intensity of the nuclear-active particles with energies in the region $2 \times 10^{11} - 10^{12}$ eV. As can be seen from Fig. 1, the results of these measurements coincide well with the spectrum (9), extrapolated to the energy region 10^{12} eV.

In order to proceed to higher energies, we used measurements of the spectrum of ionization bursts, made under conditions in which the influence of simultaneous incidence on the apparatus of several

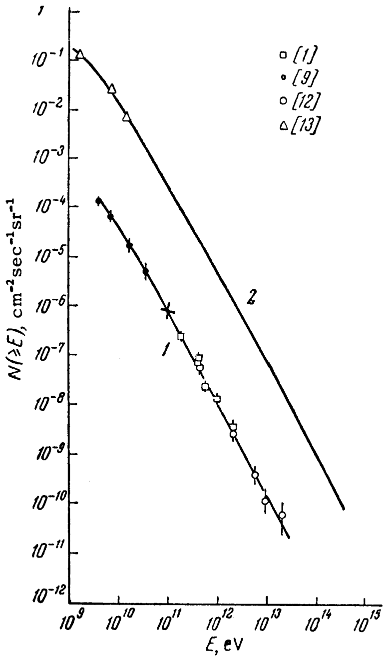

FIG. 1. Curve 1—integral energy spectrum of nuclear-active component at altitude 3200 meters above sea level; curve 2—integral energy spectrum of primary cosmic particles, reconstituted from curve 1; the cross denotes the normalization point.

nuclear active particles on the spectrum has been reduced to a minimum[12]. This spectrum of ionization bursts (see Fig. 1) was normalized at one point (at $E \approx 10^{11}$ eV) to the measured spectrum of nuclear active particles (Fig. 1 shows the normalization point marked by a cross). In the energy region up to 10^{13} eV, the spectrum was continued to that of the ionization bursts. We consider this procedure to be justified by the experimental fact that in the energy region $2 \times 10^{11} - 2 \times 10^{12}$ eV at mountain altitudes the ionization-burst spectra and the spectra of the nuclear-active components have the same slope[1].

The spectrum of nuclear-active particles obtained by the described procedure is not purely power-law: in the energy region $10^{10} - 10^{11}$ the exponent of the integral spectrum is $\gamma - 1 = 1.7$; in the energy region $10^{11} - 10^{12}$ eV we have $\gamma - 1 = 1.8$, while in the energy region $10^{12} - 10^{13}$ eV we have $\gamma - 1 = 1.9$.

In order to obtain from curve 1 of Fig. 1 the spectrum of the primary cosmic particles, we have assumed that the absorption range of all the particles in the energy interval $10^{10} - 10^{13}$ eV is equal to 120 g/cm^2, and increased the ordinates of curve 1 by a factor $e^{700/120} = 350$.

The primary cosmic particle spectrum obtained in this manner (curve 2 of Fig. 1) does not contradict the known data on the exponent of the spectrum in the corresponding primary-particle energy interval. Thus, the emulsion method has yielded for the exponent of the integral spectrum of the nuclear-active particles in the energy region $5 \times 10^{11} - 5 \times 10^{12}$ eV values $\gamma - 1 = 2.1 \pm 0.1$ and 1.9 ± 0.2[14] for an altitude of 12 km and for balloon altitudes, respectively.

3. COMPARISON OF CALCULATIONS WITH EXPERIMENT

Figure 2 shows the dependence of $N_\mu(E)/a_1$ on the muon energy E, calculated for two values of a_2. The same figure shows the experimental differential muon spectrum for sea level[13,16]. It is seen from the figure that the calculated spectrum is parallel to the experimental one. It follows from this that in the energy range $10 \text{ BeV} < E < 10^3$ BeV we have $a_1 = \text{const} = 0.125$ for $a_2 = 0$ and $a_1 = 0.107$ for $a_2 = 0.3$.

The intensity of π^0-meson generation is determined by the value of a_0 [see formula (3)].

Figure 3 shows the dependence of the quantity $(2/\gamma L_{int}) N_{na}(\epsilon, X)$ on the γ-quantum energy for different depths of the atmosphere. In the calculation of these curves the corresponding values of the spectra of the nuclear-active particles $N_{na}(\epsilon, X)$ were determined from the following integral spectra: from the spectra of the primary particles (see Fig. 1) for curve 1, from the spectrum of the primary particles recalculated to an altitude of 12 km by multiplying the ordinates of curve 2 on Fig. 1 by $\exp(-200/L_p)$ ($L_p = 120$ g/cm^2) for curve 2, and from the spectrum measured at mountain altitudes (Fig. 3) for curve 3. The experimental values were taken from the integral γ-quantum spectra measured at different altitudes.

Bowler et al[17] give the vertical flux of γ quanta with energy $\epsilon \geq 470$ BeV at an altitude where the pressure is 25 g/cm^2:

$$n_\gamma(\geqslant \varepsilon, X) = (3.3 \pm 0.8) \cdot 10^{-7} \left(\frac{470}{\varepsilon}\right)^{2 \pm 0.5} \text{ cm}^{-2}\text{sec}^{-1}\text{sr}^{-1}.$$

Neglecting the cascade processes in the layer of atmosphere 25 g/cm^2 thick, we can assume that the measured flux of the γ quanta is determined only by the number of γ quanta generated by the primary cosmic-ray particles in this layer. Therefore, dividing the measured flux by 25 g/cm^2, we obtain the intensity of generation of the γ quanta with energy ≥ 470 BeV per g/cm^2. Differentiating the integral spectrum we obtain an expression for $n_\gamma(\epsilon, X)$, from which we calculate the points shown in Fig. 3:

$$n_\gamma(\varepsilon, X) = (5.6 \pm 2.0) \cdot 10^{-11} \left(\frac{470}{\varepsilon}\right)^{3.0 \pm 0.5} \text{g}^{-1}\text{sec}^{-1}\text{sr}^{-1} \cdot \text{BeV}^{-1}.$$

Comparing these points with curve 1, we see that in the energy interval $100 \text{ BeV} < \epsilon < 1000$ BeV we have $a_0 = \text{const}$. The point at $\epsilon = 470$ BeV has the smallest error, and for it $a_0 = 0.08^{+0.025}_{-0.031}$.

Baradzeĭ et al[6] give the integral spectrum of the γ quanta generated in one cascade unit (casc.un.) of the atmosphere at an altitude where the pressure is 200 g/cm^2. It can be represented in the form

$$n_\gamma(\geqslant \varepsilon, X) = (7.7 \pm 1.2) \cdot 10^{-7} \left(\frac{190}{\varepsilon}\right)^{1.92^{+0.08}_{-0.16}} \text{sec}^{-1}\text{sr}^{-1}(\text{casc.un.})^{-1}$$

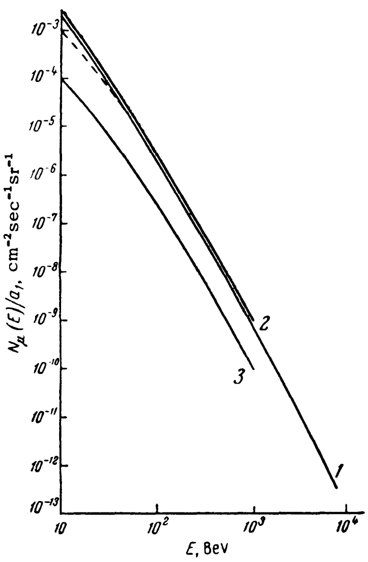

FIG. 2

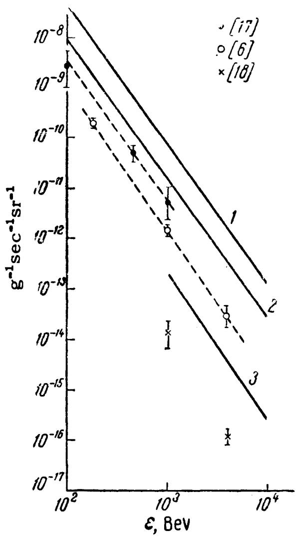

FIG. 3

FIG. 2. Calculated differential spectra of vertical flux of muons at sea level for the following cases: curve 1—$a_2 = 0$, curve 2—$a_2 = 0.3$ without account of decay and ionization losses of the muons in the atmosphere; dashed segment of curve—with account of these effects; curve 3—experimental spectrum of muons from [15,16].

FIG. 3. Differential spectra of γ quanta generated in one gram of the atmosphere at different depths: ●—X = 25 g/cm^{-2}, o—X = 200 g/cm^2[6], ×—X = 700 g/cm^{-2}[18]. The continuous curves show the dependence on the γ-quantum energy of the quantity $(2/\gamma L_{int})N(\epsilon,X)$ at different depths: 1—X = 0, 2—X = 200 g/cm^2, 3—X = 700 g/cm^2.

Dividing the values of $n_\gamma(\geq \epsilon, X)$ by 38.4 g/cm^2 (1 casc.un.in air) and differentiating, we obtain $n_\gamma(\epsilon, X)$—the intensity of generation of γ quanta with given energy in 1 g/cm^2 at an altitude of 12 km, shown in Fig. 3. In order to determine a_0, these experimental data must be compared with curve 2 of Fig. 3: within the limits of measurement error we have a_0 = const in the energy interval 100 BeV < ϵ < 1000 BeV, and at ϵ = 1000 BeV we have $a_0 = 0.083^{+0.023}_{-0.020}$.

Experimental data are available[18] on the spectrum of γ quanta with energy $\epsilon \geq 4 \times 10^{12}$ eV, generated in one cascade unit of air at mountain altitudes, at a pressure of 735 g/cm^2. This spectrum can be written in the form

$$n_\gamma(\geqslant \varepsilon, X) = (1.5 \pm 0.5)\cdot 10^{-13}\cdot\left(\frac{4\cdot 10^3}{\varepsilon}\right)^{2.5\pm 0.3}$$
$$\text{sec}^{-1}\,\text{sr}^{-1}\,(\text{casc.un.})^{-1}$$

We know the spectrum of nuclear-active particles at an altitude where the pressure is 700 g/cm^2 (curve 1 on Fig. 1). Assuming that the intensity of the γ quanta decreases with the depth of the atmosphere X like $\exp(-X/L_p)$, we have recalculated the spectrum measured by Nishimura et al[18] to an atmosphere of 700 g/cm^2. Further, we obtained the differential spectrum from the integral γ-quantum spectrum and calculated the intensity of generation of γ quanta per g/cm^2, shown in Fig. 3. These points must be compared with curve 3, which gives the value of $n_\gamma(\epsilon, X)$ for 3200 meters above sea level, calculated from the integral spectrum of the nuclear-active particles at this altitude (curve 1 on Fig. 1). From this comparison we find that $a_0 = 0.069^{+0.041}_{-0.034}$ at ϵ = 1000 BeV.

Thus, the available aggregate of experimental data on the generation of high-energy γ quanta ($\epsilon \lesssim 10^3$ BeV) gives for different altitudes practically the same value of a_0 (within the limits of measurement errors) with an average result

$$a_0 = 0.080^{+0.016}_{-0.015}.$$

If the high-energy γ quanta and the muons are due only to pions, and the generation of neutral and charged pions occurs in the same processes, in which the ratio $\bar{n}_{\pi_0} : \bar{n}_{\pi_\pm} = 1:2$ is conserved in the mean, then we should get $a_0/a_1 = 1/2$.

The flux of γ quanta at high altitudes, like the flux of muons at sea level, is determined by the intensity of the primary cosmic particles [see (3) and (8)]. Therefore the ratio $n_\gamma(\epsilon, X \approx 0)/N_\mu(\epsilon)$ does not contain any uncertainties connected with the possible inaccuracy of the primary-particle flux. For ϵ = 470 BeV (see Fig. 3) we have

$$n_\gamma(\varepsilon, X \approx 0) = (5.6 \pm 2.0)\cdot 10^{-11}\ \text{g}^{-1}\,\text{sec}^{-1}\,\text{sr}^{-1}\,\text{BeV}^{-1},$$
$$N_\mu(\varepsilon) = 1.7\cdot 10^{-9}\ \text{cm}^{-2}\,\text{sec}^{-1}\,\text{sr}^{-1}\,\text{BeV}^{-1}$$ [15,16],

and therefore $a_0/a_1 = 0.64 \pm 0.22$ for ϵ = 470 BeV.

4. GENERATION OF PIONS IN DIFFERENT PROCESSES

As was explained above, in order to explain the observed muon and γ-quanta spectra, the process generating the pions that make the main contribution to the observed muon and high-energy quanta fluxes should be such as to ensure a_1 = 0.13 — 0.11.

Let us consider the values of a_1 obtained from the following processes: a) pionization (under the assumption that the pions observed in the laboratory system are the product of an isotropic decay of a fire-ball); b) decay of isobars and hyperons, which obtain 70% of the energy of the nucleon that interacts with the nucleus.

A. Role of pionization in the generation of high-energy $\pi^\pm$ mesons. In order to calculate a_1, it is necessary to know the spectrum of the pions generated in the elementary act of nucleon-nucleus interaction.

To this end we can take the experimental data from [19], but we encounter here the following complication. Depending on the form of the generation spectrum, a large contribution can be made to a_1 by the pions which receive a large fraction of the primary-nucleon energy. In the experiment of [19], the maximum measured particle momentum was only 12 BeV, which is less than 10% of the primary-particle energy for most interactions. Thus, there are no experimental data for the most essential region of the generation spectrum.

In order to get around this difficulty, we have decided to calculate in the laboratory frame the spectrum of the pions produced as a result of isotropic decay of a fire ball moving in the c.m.s. with a Lorentz factor $\bar{\gamma}$. We have assumed here for the momentum distribution of the pions in the fire ball system the expression

$$N(p)\,dp = Ap^2dp/[\exp(\sqrt{p^2+1}) - 1],$$

which is given in the paper of Guseva et al[20] at a scattering temperature $T = m_\pi c^2$ (the momentum p is measured in units of $m_\pi c$).

If γ_S is the Lorentz factor of the fire ball in the laboratory frame, then the energy spectrum of the pions in the laboratory frame will be in the form

$$n(E_\pi)\,dE_\pi = \frac{A\,dE_\pi}{2\beta_s\gamma_s}\int_{p_{min}}^{\infty}\frac{p\,dp}{\exp(\sqrt{p^2+1}) - 1}; \qquad (10)$$

$$p_{min} = \frac{1}{2}\left(\frac{E_\pi}{\gamma_s} - \frac{\gamma_s}{E_\pi}\right) \quad \text{for} \quad \frac{E_\pi}{\gamma_s} > 1,$$

$$p_{min} = \frac{1}{2}\left(\frac{\gamma_s}{E_\pi} - \frac{E_\pi}{\gamma_s}\right) \quad \text{for} \quad \frac{E_\pi}{\gamma_s} < 1, \tag{11}$$

where E_π—total energy of the pion in $m_\pi c^2$ units. From (10) we see that the pion spectrum is a universal function $F(E_\pi/\gamma_s)$ of E_π/γ_s, with

$$n(E_\pi)\,dE_\pi = \frac{1}{2\beta_s} F\left(\frac{E_\pi}{\gamma_s}\right)\frac{dE_\pi}{\gamma_s}. \tag{12}$$

The form of the function $F(E_\pi/\gamma_s)$ is given in Fig. 4.

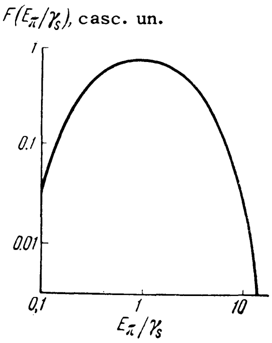

FIG. 4. Differential l.s. spectrum of pions generated as a result of the decay of a fire ball which is at rest in the c.m.s. of the colliding nucleons.

Let us determine a_1 for the pion spectrum shown in Fig. 4. This spectrum corresponds to a fire ball at rest in the c.m.s. By definition

$$a_1 = \int_0^{E_0} \left(\frac{E}{E_0}\right)^{\gamma-1} n_1(E, E^0)\,dE.$$

In our case

$$\bar{n}_1(E, E_0)\,dE = \bar{n}_1 F(E_\pi/\gamma_s)\,\gamma_s^{-1}\,dE_\pi.$$

We shall assume that $F(E_\pi/\gamma_s)$ is normalized so that

$$\int_0^\infty F(E_\pi/\gamma_s)\,\gamma_s^{-1}\,dE_\pi = 1.$$

Putting $E/E_0 = \alpha E_\pi/\gamma_s$ we obtain

$$a_1 = \alpha^{\gamma-1} n_1 \overline{(E_\pi/\gamma_s)^{\gamma-1}}.$$

By definition, the average inelasticity coefficient is

$$\bar{K} = \int_0^{E_0} \frac{E}{E_0} n(E, E_0)\,dE,$$

i.e., $\bar{K} = \alpha\bar{n}(E_\pi/\gamma_s)$. Hence $\alpha = \bar{K}/\bar{n}(\bar{E}_\pi/\gamma_s)$ and

$$a_1 = \frac{\bar{n}_1 (\bar{K})^{\gamma-1}}{(\bar{n})^{\gamma-1}}\eta, \qquad \eta = \overline{\left(\frac{E_\pi}{\gamma_s}\right)^{\gamma-1}} \Big/ \left(\frac{\bar{E}_\pi}{\gamma_s}\right)^{\gamma-1}$$

($\bar{n}$—average multiplicity of all pions and n_1—average multiplicity of charged pions). Since $n_1 = 2n/3$, we have

$$a_1 = \frac{1}{1.5}\frac{(\bar{K})^{\gamma-1}}{(\bar{n})^{\gamma-2}}\eta. \tag{13}$$

The ratio η depends on the form of the function $F(E_\pi/\gamma_s)$ and on the exponent γ of the nuclear-active particle spectrum. Using (4), we readily calculate η for different values of γ:

γ:	2	2.7	2.9
η:	1	1.3	1.47

For primary particles of energy $\bar{E}_0 = 200$ BeV we have $n_1 = 8.5$ and $K = 0.3$[19,20]. Therefore $n = 12$ and for $\gamma = 2.7$ we obtain $a_1 = 0.152(K)^{\gamma-1} = 0.020$.

We can raise the following question: for what value of $\bar{K}$ can we obtain the necessary value $a_1 = 0.13 - 0.11$?

The average pion energy in the fire ball system is uniquely connected with the experimentally well known average value of the perpendicular component of the momentum $\bar{p}_\perp$ by the relation

$$\bar{E}_\pi = \sqrt{\bar{p}^2 + 1}, \qquad \bar{p} = \frac{4}{\pi}\bar{p}_\perp,$$

where $\bar{p}_\perp = 2.5$ and $E_\pi = 3.4$. In order to obtain $a_1 = 0.13 - 0.11$, the value of $\bar{K}$ must lie in the interval $1.95 - 1.65$. This result means there are no conditions (which do not contradict experiment) for which the decay of a fire ball at rest in the c.m.s. ensures the observed intensity of high-energy pion generation.

If we fix the average multiplicity $\bar{n}$ of the generated pions, then in order to obtain the necessary value $a_1 = 0.11 - 0.13$ it is necessary to assume for $\bar{K}$ a value $0.83 - 0.85$. However, so large a value of $\bar{K}$ corresponds to too small an absorption range for the nucleons in the atmosphere ($L_p \approx 90$ g/cm^2) and too large a momentum: $\bar{p}_\perp = 7\,m_\pi c = 1.0$ BeV/c.

If the fire ball moves in the c.m.s., the value of a_1 may change. If the fire ball moves in the c.m.s. with a Lorentz factor $\bar{\gamma}$, then $\gamma_s = \gamma_c\bar{\gamma}(1 \pm \beta_c\bar{\beta})$ (the $\pm$ signs correspond to forward or backward motion of the fire ball in the c.m.s. relative to the direction of motion of the primary particle). In this case the spectrum of the generated pions

$$n(E_\pi)\,dE_\pi \sim F_1 dE_\pi/\gamma_{s1} + F_2 dE_\pi/\gamma_{s2}$$

is no longer a universal function of E_π/γ_s, since it consists of two parts: the function $F(E_\pi/\gamma_{s1})$ where $\gamma_{s1} = \gamma_c\bar{\gamma}(1 + \beta_c\bar{\beta})$, and the function $F(E_\pi/\gamma_{s2})$, where $\gamma_{s2} = \gamma_c\bar{\gamma}(1 - \beta_c\bar{\beta})$.

We have calculated the dependence of η on $\bar{\gamma}$ for a spectrum exponent $\gamma = 2.7$. Knowing the distribution of the interactions with respect to $\bar{\gamma}$ (see [20]), we can determine the average value of $\bar{\eta}$, which was found to be $\bar{\eta} = 1.48$. Taking into account the c.m.s. motion of the fire ball we get

$$a_1 = \frac{\bar{\eta}}{1.5} \frac{(\bar{K})^{\gamma-1}}{(\bar{n})^{\gamma-2}}. \qquad (14)$$

Because the average value of $\bar{\gamma}$ is 1.12, the average inelasticity coefficient increases by 12% and amounts to $\bar{K} = 0.34$[20].

Thus, we obtain as a result of pionization

$$a_1 = 0.99 \ (\bar{K})^{\gamma-1}/(\bar{n})^{\gamma-2} = 0.027.$$

The estimates made show that the pionization (even if the multiplicity is independent of the energy) can account for only 20 — 25% of the high-energy pions generated by nucleons. Consequently, the decisive role in the generation of fast pions is due to some mechanism which differs from the decay of a fire ball that moves slowly in the c.m.s.

B. Generation of pions due to decay of isobars and hyperons. Inasmuch as the average energy losses of a nucleon interacting with a light nucleus due to pionization amount to $\sim 1/3$, the isobar or hyperon, if these carry away the unconsumed energy, receive on the average $2/3$ of the energy of the "primary" nucleon.

Let us consider for concreteness an isobar with spin and isospin $J = T = 3/2$. In the case of isotropic decay in its own coordinate system, the energy spectrum of the pions due to the decay of the isobar in the laboratory system will be

$$\frac{dn}{dE} = \begin{cases} \text{const} & \text{for } 0.027 < E/E_{\text{isob}} < 0.43, \\ 0 & \text{for } E/E_{\text{isob}} < 0.027 \quad \text{or} \quad E/E_{\text{isob}} > 0.43. \end{cases}$$

Since $E_{\text{isob}} = (1 - K) E_0 = 2E_0/3$, we get $f_1(u)\,du = G\,du$ for $0.018 < u < 0.29$, where $u = E/E_0$ (E_0—energy of the primary nucleon).

If we assume that $\bar{n}_1 \int_0^1 f_1(u)\,du = 2/3$ (this is necessary in order to obtain $a_0/a_1 = 1/2$), then $G = 2.47$ and $a_1 = 0.032$. An account of the distribution of the energy losses to pionization[20] changes a_1 by merely 3%.

Let us consider the contribution made to a_1 by pions produced in the decay of hyperons which obtain $2/3$ of the primary-nucleon energy in the interaction.

If we express the energy E of the pion, resulting from the decay of the hyperon in fractions of the primary-nucleon energy, $u = E/E_0$, then we find, as in the case of the isobar, that $f_1(u)\,du = B\,du$ in the region $0.026 \leq u \leq 0.24$. Assuming that the probability of the hyperon decay to a charged pion is $7/8$ [21], we get $0.21B = 7/8$ or $B = 4.16$, i.e., $a_1 = 0.030$.

We see that from the point of view of the production of high-energy pions, the isobars ($J = T = 3/2$) are equivalent to hyperons.

The calculations show that pionization accompanied by energy loss with $K = 1/3$, in conjunction with subsequent decay of the isobar or the hyperon, can ensure only about 40 — 45% of high-energy pions generated on the average in one interaction between the nucleon and the light nucleus. (Agreement of similar calculations with experiment was obtained by Kotov and Rozental' [21] because the intensity they assumed for the primary particles was 1.8 times larger than the value we assumed, and conservation of 80% of the energy was assumed for the isobars or hyperons.)

We must emphasize that the estimates give too high a value for a_1 for two reasons. First, during pionization the average multiplicity $\bar{n}$ increases with increasing energy and consequently a_1 should decrease with increasing energy. Second, in the calculation of the contribution of the isobars and hyperons to a_1 we have assumed that each active interaction of the nucleon with the nucleus leads to the formation of an isobar or hyperon of high energy. There are grounds for assuming[1,2] that in the energy region $10^{11} - 10^{12}$ eV the probability of production of isobars or hyperons of high energy is much smaller than assumed in our estimates. Therefore the processes considered, taken together, actually give a value $a_1 < 0.057$. Consequently, there should exist a process of generation of high-energy pions, more effective than pionization and the decay of isobars or hyperons.

5. MAIN CHARACTERISTICS OF THE PROCESS OF GENERATION OF PIONS OF HIGH ENERGY

Let us consider what main characteristics should be possessed by such a process.

As shown by the experimental data[20], when $E_0 \approx 200$ BeV the inelasticity produced in the bulk of the collisions between the nucleon and the light nucleus is small ($\bar{K} \approx 0.3$), and no high-energy pions are generated ($E/E_0 \sim 1$). Consequently, the sought process is realized with some probability W which is considerably smaller than unity. If pionization ensures a value $a_1 = 0.03$, then the hypothetic process should yield $a_1 \approx 0.1$.

Assuming that N particles carrying away a fraction K_1 of the primary-particle energy are produced in the sought process, we find, by making the substitution $K_1/N = \bar{u}$ in (13), that $0.1 = W(\bar{u})^{\gamma-2}K_1\eta/1.5$ and for $\gamma = 2.7$ we get $(\bar{u})^{0.7} = 0.15/W\eta K_1$ and $N = K_1/\bar{u}$. If all the particles have the same energy, then $\eta = 1$. If these particles have a uniform energy probability distribution, then $\eta = 1.2$.

We assume for the estimate $\eta = 1.2$. We can then determine $\bar{u}$ and $\bar{N}$ for different values of W and K_1. The results of these calculations are listed in Table II.

As can be seen from Table II, the sought process should be realized with large inelasticity $K_1 \approx 1$ and with small effective multiplicity N of the particles carrying away the greater part of the energy transferred to the pions in the interaction.

Table II

	K_1=0,9		K_1=0,7		K_1=0,5	
W	$\bar{u}$	N	$\bar{u}$	N	$\bar{u}$	N
0.1	>1	<1	>1	$\leqslant 1$	$\geqslant 1$	$\leqslant 1$
0.2	0.59	1.5	0.85	$\leqslant 1$	$\geqslant 1$	$\leqslant 1$
0.3	0.39	2.3	0.47	1.5	0.77	$\leqslant 1$
0.4	0.25	3.6	0.38	1,9	0.50	1
0.5	0,18	5.0	0,27	2.6	0.45	1

In particular, one possible process is a completely inelastic interaction between the nucleon and a light nucleus with formation (in the c.m.s.) of two fast fire balls moving in opposite directions. In this case $\bar{\gamma} \gg 1$. When $\bar{\gamma} \geq 2.5$, we get $\eta = 2.1$. Therefore such "fast" clouds will make a contribution of $2.1/1.5\,(n)^{\gamma-2}$ to a_1.

If we assume the same total multiplicity as before ($\bar{n} = 12$), then $a_1 = 0.25$, i.e., such a process would be quite effective from the point of view of generation of high-energy pions. For example, it is sufficient to assume that nucleon energy losses with inelasticity from K = 0.8 to K = 1 have a probability of 30—40% (catastrophic energy losses) with production of fast fire balls in the c.m.s. with total multiplicity of produced pions on the order of 12—15, then such collisions alone will yield $a_1 = 0.08—0.1$. Together with the contribution made to a_1 by pionization, we would obtain the necessary quantities $a_1 = 0.11—0.13$.

If the proposed almost fully inelastic interactions exist and are realized with probability 30—40%, the average inelasticity coefficient for the interaction of the nucleons at high energies will be about 0.5—0.6, which agrees with the known data on the absorption of the nucleon component in the atmosphere.

The foregoing analysis has shown that the most probable interactions between nucleons and light atomic nuclei, accompanied by low energy losses ($\bar{K} \approx 0.3$) and possibly by the production in many cases of an isobar or of a hyperon of high energy, cannot ensure the observed high intensity of high-energy pion generation. To explain the experimental data on the high-energy pion generation it becomes necessary to admit of the existence of interactions with large inelasticity coefficient ($K \approx 1$) and high energy concentration in a small number of pions.

Thus, the decisive role in the generation of high-energy pions is apparently played by the large fluctuations of the inelasticity of interaction.

[1] Babayan, Babecki, Boyadzhyan, Buja, Grigorov, Loskiewicz, Mamidzhanyan, Massalski, Oles, Tret'yakova, and Shestoperov, Izv. AN SSSR ser. fiz. **26**, 558 (1962), Columbia Tech. Transl. p. 558.

[2] Babecki, Buja, Grigorov, Loskiewicz, Massalski, Oles, and Shestoperov, JETP **40**, 1551 (1961), Soviet Phys. JETP **13**, 1089 (1961).

[3] Babayan, Grigorov, Dubrovin, Mishchenko, Murzin, Sarycheva, Sobinyakov, and Rapoport, Trans. Intl. Conf. on Cosmic Rays, IUPAP, July 1959, v. 1, AN SSSR, 1960, p. 176.

[4] N. L. Grigorov and V. Ya. Shestoperov, JETP **34**, 1539 (1958), Soviet Phys. JETP **7**, 1061 (1958).

[5] N. L. Grigorov, UFN **48**, 499 (1956).

[6] Baradzei, Rubtsov, Smorodin, Solov'ev, and Tolkachev, Izv. AN SSSR ser. fiz. **26**, 575 (1962), Columbia Tech. Transl. p. 573.

[7] S. I. Nikol'skii, UFN **78**, 365 (1962), Soviet Phys. Uspekhi.

[8] B. Rossi, op. cit. [3], v. 2, p. 17.

[9] Kocharyan, Saakyan, and Kirakosyan, JETP **35**, 1335 (1958), Soviet Phys. JETP **8**, 933 (1959).

[10] W. B. Fretter, Phys. Rev. **76**, 511 (1949). J. Tinlot, Phys. Rev. **73**, 1476 (1948) and **74**, 1197 (1948). T. G. Walsh and O. Piccioni, Phys. Rev. **80**, 619 (1950).

[11] Christy, Biehl, Inonu, and Neher, Phys. Rev. **81**, 647 (1951). M. F. Kaplon and D. M. Ritson, Phys. Rev. **88**, 386 (1952).

[12] Babayan, Boyadzhyan, Grigorov, Tret'yakova, and Shestoperov, JETP **44**, 22 (1963), Soviet Phys. JETP **17**, 15 (1963).

[13] A. N. Charakhch'yan and T. N. Charakhch'yan, op. cit. [3], v. 3, p. 144.

[14] Bowler, Duthie, Fowler, Kaddoura, and Perkins, Proc. Intern. Conf. Kyoto, (1961) **3**, p. 423.

[15] Ashton, Brooke, and Gardener et al, Nature **185**, 364 (1960).

[16] P. Babu and Y. Pal, Proc. Intern. Conf. Kyoto, (1961) **3**, p. 322.

[17] Bowler, Duthie, Fowler, Kaddoura, Perkins, Pinkau, and Wolter, Proc. Intern. Conf. Kyoto, (1961) **3**, p. 424.

[18] Akashi, Shimizu, Watanabe, Fujimoto, Hasegawa, Nishimura, Niu, Yokoi, Ogata, and Ogita et al. Proc. Intern. Conf. Kyoto, (1961) **3**, p. 427.

[19] Grigorov, Kuseva, Dobrotin, Lebedev, Kotel'nikov, Murzin, Rapoport, Ryabikov, and Slavatinskii, op. cit. [3], v. 1, p. 140.

[20] Guseva, Dobrotin, Zelevinskaya, Kotel'nikov, Lebedev, and Slavatinskiĭ, Izv. AN SSSR ser. fiz. **26**, 549 (1962), Columbia Tech. Transl. p. 550.

[21] Yu. D. Kotov and I. L. Rozental', JETP **43**, 1411 (1962), Soviet Phys. JETP **16**, 1001 (1963).

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