REGGE POLES IN PHOTOPRODUCTION AMPLITUDE

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Submitted to JETP editor May 17, 1963

J. Exptl. Theoret. Phys. (U.S.S.R.) 45, 1585-1594 (November, 1963)

The spin structure of the amplitude for photoproduction of pions from nucleons is examined under the Regge hypothesis of moving poles in scattering amplitudes as functions of the orbital angular momentum j. Classification of the Regge poles by various quantum numbers is carried out. Polarization of the recoil nuclei in photoproduction of pions from polarized targets, and polarization of photons in the process of stopping of fast pions by interaction with nucleons, are calculated. It is shown that the contributions of various poles give rise to qualitatively different results for the polarization. The amplitude for the photoproduction of pions from nucleons at zero degrees is examined.

1. INTRODUCTION

A new and promising direction has been given recently to the theory of strong interactions by the hypothesis of moving poles in the scattering amplitude as a function of orbital angular momentum j. It has become clear that in relativistic theory, just like in nonrelativistic theory, the interaction amplitude is an analytic function of the angular momentum j and may have moving poles in the right half plane of the complex variable j.[1] The pole lying farthest to the right determines the asymptotic behavior of the amplitude for the transformation of two particles into two, A(s, t), for large energies $\sqrt{s}$ and fixed momentum transfer $\sqrt{-t}$. In the case of elastic processes for small $|t|$ such a pole is the vacuum pole, or the Pomeranchuk pole, which for $t = 0$ has $j = 1$, which ensures the constancy of total interaction cross sections at high energies. In the case of inelastic processes the asymptotic behavior of the interaction amplitude is determined by a pole whose quantum numbers differ from those of the vacuum. The study of the properties of such poles and of the location of their trajectories relative to that of the vacuum pole is of considerable theoretical and experimental interest.

In this work we consider the spin structure of the amplitude for pion photoproduction on nucleons, whose asymptotic behavior is governed by non-vacuum poles. As will be shown, the amplitude under consideration receives contributions from three sets of poles, which give rise to qualitatively the same results for the differential photoproduction cross section; hence these results can not be used to experimentally distinguish the various types of Regge poles. The spin structure, however, turns out to be different for each of these groups of poles giving rise to a qualitatively different dependence of the polarization of recoil nuclei on the polarization of the target nuclei. Study of the reaction inverse to photoproduction shows that the resultant photons will be completely linearly polarized in the plane of the reaction, or perpendicular to it depending on the location of poles with different quantum numbers.

Investigation of the amplitude for the photoproduction of pions on nucleons at zero degrees and at arbitrary energies leads to an expression for the amplitude that is totally different from the asymptotic one-pole expression; consequently the polarization effects are different too. This is due to the inapplicability of the asymptotic formulas in the region of momentum transfers $|t| \sim m^2 m_\pi^4/s^2$.

Consequently a study of the polarization effects by means of experiments with polarized beams and targets permits one, in principle, to clarify the order of appearance of trajectories of various poles relative to the vacuum pole and verify Chew's hypothesis on the influence of various quantum numbers on the position of Regge poles.[2]

2. PARTIAL AMPLITUDES IN THE ANNIHILATION CHANNEL $N + \bar{N} \rightarrow \pi + \gamma$

The photoproduction amplitude $A_{\pi\gamma}$ depends in general on four invariant functions $A_i(s, t)$[3] and, in accordance with the requirements of relativistic and gauge invariance, may be written in the annihilation channel in the form[1)]

$$A_{\pi\gamma} = i\bar{u}(-\mathbf{p}_2)[\gamma_5(A_1 + A_2\hat{k})(eq) + (A_3 + A_4\hat{k})(en)]\,u(\mathbf{p}_1), \quad (1)$$

[1)]We use the metric in which the scalar product of 4-vectors is $(ab) = a_0b_0 - \mathbf{a}\cdot\mathbf{b}$.

where A_i are functions of the kinematic invariants $t = (p_1 + p_2)^2$ and $s = (p_1 - k)^2$, p_1 is the 4-momentum of the nucleon, p_2 is the 4-momentum of the antinucleon, k is the 4-momentum of the photon, and e is the polarization vector of the photon in the transverse gauge,

$$q_\alpha = Q_\alpha/\sqrt{Q^2}, \quad Q_\alpha = p_{1\alpha}(p_2k) - p_{2\alpha}(p_1k),$$
$$n_\alpha = N_\alpha/\sqrt{-N^2}, \; N_\alpha = \varepsilon_{\alpha\beta\gamma\delta}p_{1\beta}p_{2\gamma}k_\delta.$$

We shall assume that the bispinors describing the spin states of the nucleon and antinucleon are normalized such that $\bar{u}(p)u(p) = p_0/|p_0|$, and the amplitude $A_{\pi\gamma}$ such that the differential cross section is expressed by the formula

$$d\sigma_{\pi\gamma} = \frac{k_2}{k_1}\frac{1}{W^2}|A_{\pi\gamma}|^2 d\Omega, \tag{2}$$

where k_1 and k_2 are the absolute values of the particle momenta before and after the collision, W is the total energy and Ω is the solid angle element in the center-of-mass system (c.m.s.).

To discuss the photoproduction channel (s-channel) one must replace in (1) p_2 by $-p_2$, and k by $-k$. To discuss the inverse photoproduction channel (u-channel) $\pi + N \to N + \gamma$ one must replace in (1) p_2 by $-p_2$ and s by $u = (p_2 + k)^2$.

Equation (1) presupposes a definite projection of isospin T_3 in the annihilation channel (t-channel). The experimental analysis of the asymptotic cross sections of the processes $\gamma + p \to p + \pi^0$ (mixture of states with T = 0 and T = 1 in t-channel, $T_3 = 0$) and $\gamma + p \to n + \pi^+$ (pure T = 1 state in t-channel, $T_3 = 1$) gives one information on the isotopic spin of the right-most Regge pole in the photoproduction amplitudes. As is known,[4] Regge poles characterizing the strong interactions are described by quantum numbers: parity P, signature or j-parity P_j, isotopic spin T and G-parity. In our case the electromagnetic interactions cause T and G to have no meaning as quantum numbers and only the third component of isospin T_3 is a good quantum number. In the case of the neutral system in the t-channel ($T_3 = 0$) the charge parity C is a good quantum number also. Thus in photoproduction it is convenient to classify the Regge poles by the following quantum numbers: P, P_j and C. As will be seen, in the case of interest to us specifying the C-parity is sufficient to determine the remaining quantum numbers (P and P_j) of the Regge poles that contribute to the photoproduction of pions.

In order to analyze the photoproduction amplitude by the Regge-Gribov method it is necessary to obtain the expansion of the amplitudes $A_i(s, t)$ in partial waves in the c.m.s. in the physical region of the t channel. In the following we shall obtain this expansion by making use of a technique proposed by Berestetsky.[5] To that end we go over to two-component spinors by making use of the explicit form of u(p):

$$u(\mathbf{p}_1) = \frac{1}{\sqrt{2m}}\begin{pmatrix}\sqrt{E+m}\, v \\ \sqrt{E-m}\,(\sigma\mathbf{n}_1)\, v\end{pmatrix};$$

$$u(-\mathbf{p}_2) = \frac{1}{\sqrt{2m}}\begin{pmatrix}-\sqrt{E-m}\,(\sigma\mathbf{n}_2)\, w \\ \sqrt{E+m}\, w\end{pmatrix},$$

where σ are the Pauli matrices, $E = \sqrt{t}/2$ is the nucleon energy in the t channel, $\mathbf{n}_1$ is a unit vector in the direction of motion of the nucleon in the c.m.s., and m is the nucleon mass. Using two-component spinors the amplitude (1) determining the polarization of the nucleon may be expressed in the form

$$A_{\pi\gamma} = f_1\chi\,(\mathbf{e}\mathbf{n}_1) + f_2\,(\mathbf{n}_1\boldsymbol{\chi})\,(\mathbf{n}_1\,[\mathbf{e}\mathbf{n}_2]) + f_3\,(\mathbf{e}\,[\boldsymbol{\chi}\mathbf{n}_1])$$
$$+ f_4[\mathbf{n}\,[\mathbf{n}_1\boldsymbol{\chi}]]\,[\mathbf{n}_2\mathbf{e}], \tag{3*}$$

where $\chi = w^*v$, $\boldsymbol{\chi} = w^*\boldsymbol{\sigma}v$, $\mathbf{n}_2$ is a unit vector in the direction of emission of the photon; χ describes the singlet, and $\boldsymbol{\chi}$ the triplet, state of the $N\bar{N}$ system; f_1 is the amplitude for $N\bar{N}$ annihilation from the singlet state and f_2, f_3, f_4 are amplitudes for annihilation from the triplet state.

Comparing (1) and (3) makes it easy to find the connection between the invariant amplitudes A_i and the amplitudes f_i which, in essence, are combinations of helicity amplitudes and are convenient for expansion in partial amplitudes:

$$A_1 = \frac{-m\sqrt{1-z_t^2}}{E}\left[f_1 + \frac{im\,(f_4 - z_tf_3)}{p\,(1-z_t^2)}\right],$$

$$A_2 = \frac{im}{p\omega\sqrt{1-z_t^2}}\,[f_4 - z_tf_3],$$

$$A_3 = \frac{im\sqrt{1-z_t^2}}{p}\left[f_2 - \frac{mz_t\,(f_3 - z_tf_4)}{E\,(1-z_t^2)}\right],$$

$$A_4 = \frac{im}{E\omega\sqrt{1-z_t^2}}\,(f_3 - z_tf_4), \tag{4}$$

where ω is the photon energy and

$$z_t = (\mathbf{n}_1\mathbf{n}_2) = \frac{2\sqrt{t}\,(s - m^2 + (t - m_\pi^2)/2)}{(t - m_\pi^2)\sqrt{t - 4m^2}}.$$

Expressing $A_{\pi\gamma}$ in the form

$$A_{\pi\gamma} = \chi\,(\mathbf{e}\mathbf{F}) + \chi_ie_kF_{ik} \tag{5}$$

$^*[\mathbf{e}\mathbf{n}_2] = \mathbf{e} \times \mathbf{n}_2$.

and making use of well-known formulas (see, e.g. [5,6]) we find the expansion of the functions F and F_{ik}:

$$\mathbf{F} = 4\pi \sum_{jM\lambda} f^j_{0\lambda}(t)\, \mathbf{Y}^{*\lambda}_{jM}(\mathbf{n}_2)\, Y_{jM}(\mathbf{n}_1),$$

$$F_{ik} = 4\pi \sum_{jM\lambda\sigma} f^j_{\sigma\lambda}(t)\, [Y^{*\lambda}_{jM}(\mathbf{n}_2)]_k\, [Y^{\sigma}_{jM}(\mathbf{n}_1)]_i, \quad (6)$$

where Y_{jM} is a spherical harmonic and $\mathbf{Y}^{\lambda}_{jM}$ is a vector spherical harmonic. The index λ takes on two values corresponding to the two photon polarizations (index 2—electric photon, index 3—magnetic photon). The index σ takes on three values corresponding to the three polarization states of the $N\bar{N}$ system in the triplet state, and $\sigma = 0$ for the singlet state of the $N\bar{N}$ system. In the notation of Berestetsky[5]

$$\mathbf{Y}^1_{jM}(\mathbf{n}) = \mathbf{n}\, Y_{jM}(\mathbf{n}),$$

$$\mathbf{Y}^2_{jM}(\mathbf{n}) = \frac{1}{\sqrt{j(j+1)}} \nabla Y_{jM}(\mathbf{n}),$$

$$\mathbf{Y}^3_{jM} \frac{1}{\sqrt{j(j+1)}} [\mathbf{n}\nabla] Y_{jM}(\mathbf{n}),$$

where the operator ∇, by definition, acts as

$$\nabla Y_{jM} = \rho \frac{\partial}{\partial \rho} Y_{jM}\left(\frac{\rho}{|\rho|}\right).$$

Introducing these relations into Eq. (6) and using the addition theorem

$$\sum_M Y^*_{jM}(\mathbf{n}_2)\, Y_{jM}(\mathbf{n}_1) = \frac{(2j+1)}{4\pi} P_j(z_t)$$

(P_j is the Legendre polynomial) we obtain for $A_{\pi\gamma}$ the expansion

$$A_{\pi\gamma} = \sum_j \frac{2j+1}{\sqrt{j(j+1)}} \Big\{ (\mathbf{e}\mathbf{n}_1)\, \chi f^j_{02}(t)\, P'_j(z_t)$$
$$+ ([\mathbf{e}\mathbf{n}_2]\, \mathbf{n}_1)(\chi \mathbf{n}_1) f^j_{13}(t)\, P'_j(z_t)$$
$$+ \frac{1}{\sqrt{j(j+1)}} (\mathbf{e}\, [\chi \mathbf{n}_1])\, [f^j_{32}(t)\, (z_t P'_j(z_t))' + f^j_{23}(t)\, P''_j(z_t)]$$
$$+ \frac{1}{\sqrt{j(j+1)}} [\mathbf{n}_2 \mathbf{e}]\, [\mathbf{n}_1 [\mathbf{n}_1 \chi]]\, [f^j_{23}(t)\, (z_t P'_j(z_t))'$$
$$+ f^j_{32}(t)\, P''_j(z_t)] \Big\} \quad (7)$$

(the dash denotes differentiation with respect to z_t), which compared with (3) yields

$$f_1 = \sum_j \frac{2j+1}{\sqrt{j(j+1)}} f^j_{02}(t)\, P'_j(z_t), \quad f_2 = \sum_j \frac{(2j+1)}{\sqrt{j(j+1)}} f^j_{13}(t)\, P'_j(z_t),$$

$$f_3 = \sum_j \frac{2j+1}{j(j+1)} [f^j_{32}(t)\, (z_t P'_j(z_t))' + f^j_{23}(t)\, P''_j(z_t)],$$

$$f_4 = \sum_j \frac{2j+1}{j(j+1)} [f^j_{23}(t)\, (z_t P'_j(z_t))' + f^j_{32}(t)\, P''_j(z_t)]. \quad (8)$$

The indices of the amplitudes $f^j_{\sigma\lambda}$ correspond to the indices in Eqs. (6). In particular, the amplitude f^j_{02} corresponds to $N\bar{N}$ annihilation from the singlet state into a photon of the electric type and a pion; the amplitude f^j_{13} corresponds to $N\bar{N}$ annihilation from the longitudinal triplet state (projection of total spin onto the direction of momentum $m_S = 0$) into a photon of the magnetic type and a pion; the amplitude f^j_{32} corresponds to $N\bar{N}$ annihilation from the transverse triplet state ($|m_S| = 1$) into an electric type photon and a pion, and the amplitude f^j_{23} to $N\bar{N}$ annihilation from the transverse triplet state ($|m_S| = 1$) into a magnetic type photon and a pion.

Taking also into account that C is the parity of the $\bar{p}p$ system, given by

$$C = (-1)^{l+S}, \quad (9)$$

where S is the total spin and $l = j$ in the singlet state ($\sigma = 0$) and the transverse triplet state with $\sigma = 3$, while $l = j \pm 1$ for the transverse triplet state with $\sigma = 2$ and the longitudinal triplet state with $\sigma = 1$, it is easy to see that for $C = \mp 1$ the amplitude f^j_{02} describes transitions between states with quantum numbers $P = \pm 1$, $P_j = \mp 1$, $C = \mp 1$ (α Regge pole), f^j_{32} describes transitions with $P = \mp 1$, $P_j = \pm 1$, $C = \mp 1$ (β pole), and f^j_{13} and f^j_{23} correspond to transitions with $P = \mp 1$, $P_j = \mp 1$, $C = \mp 1$ (γ pole). The state $\bar{p}p$ is a pure $C = -1$ state, the state $p\bar{n}$ is a mixture of states with $C = 1$ and $C = -1$.

We note, in particular, that among the poles of type γ are the poles with the quantum numbers of ω and ρ^0, ρ^+, ρ^- mesons. It is to be expected that the leading photoproduction Regge pole will describe the reaction up to $|t| \lesssim m^2$.[6]

3. CONTRIBUTION TO THE PHOTOPRODUCTION AMPLITUDE FROM POLES WITH VARIOUS QUANTUM NUMBERS

Considering separately the symmetric and antisymmetric parts of the functions $f_j(s, t)$ (depending on the signature P_j) and passing over from the summation over j to the Sommerfeld-Watson integral, we easily find the contribution from the pole with largest Re j to the amplitudes $f_j(s, t)$ and therefore, with the help of Eq. (4), to $A_j(s, t)$. At that we assume that the only moving singularities of $f^j_{\sigma\lambda}(t)$ in the complex j plane are in the form of poles lying to the right of the line Re j = 0. Passing to the limit $|z_t| \gg 1$ ($s \gg m^2$) and continuing the amplitudes $A_j(s, t)$ from the t channel to the scattering channel ($t < 0$) one can obtain asymptotic expressions for the photoproduction amplitudes. In the process of continuation one must, in particular, cross the line $(-t) = (-t_{min}) \approx m^2 m_\pi^4/s^2$ for $s \gg m^2$, on which $|z_t| \approx 1$ and asymptotic behavior is not reached. Apparently the continuation can be performed in the complex j plane, bypassing the dangerous line $(-t) = (-t)_{min}$. However this circumstance has not been rigorously proved and requires additional investigations. This same difficulty is encountered in the investigation of the asymptotic behavior of the backward scattering amplitude.[9] In this fashion our investigation of photoproduction makes sense in the region

$$(-t)_{min} \ll (-t) \lesssim m^2.$$

A. The pole α is contained in the amplitude f^j_{02}. According to Eqs. (8) and (4) it contributes only to the amplitudes f_1 and A_1. In this case the matrix element for photoproduction in the c.m.s. for $|z_t| \gg 1$ may be written as

$$A_{\pi\gamma} = \bar{u}(p_2)\gamma_5 u(p_1) \times 2ms^{3/2}(\mathbf{en}_2) f_1(s, t)/\sqrt{t(t-4m^2)}\,(m_\pi^2 - t). \tag{10}$$

Here $\mathbf{n}_2 = \mathbf{p}_2/|\mathbf{p}_2|$ and m_π is the mass of the pion. Thus the pole α contributes only to the pseudoscalar covariant.

The form of $f_1(s, t)$ for $|z_t| \gg 1$ follows from the Sommerfeld-Watson integral:

$$f_1(s, t) = -\pi \frac{\alpha C^\pm(\alpha)}{\sin \pi\alpha} \rho_{02}^\pm(t)\, z_t^{\alpha-1}, \tag{11}$$

where $\alpha(t)$ is the position of the α pole in the complex j plane, with the residue being $r^\pm_{02}(t) = \sqrt{\alpha(\alpha+1)}\,\rho^\pm_{02}(t)$, and

$$C^\pm(\alpha) = (2\alpha+1)\,\Gamma(2\alpha+1)\,(1 \pm e^{i\pi\alpha})/2^{\alpha+1}\,[\Gamma(\alpha+1)]^2.$$

The ± signs correspond to positive or negative signature.

Based on Eqs. (2) and (10) it is easy to obtain the expression for the differential photoproduction cross section in the c.m.s.:

$$\frac{d\sigma}{d\Omega} = \frac{2(-t)(1+\xi_3)s|f_1|^2}{(4m^2-t)(m_\pi^2-t)^2} = (1+\xi_3)\, s^{2\,\mathrm{Re}\,\alpha-1}\varphi_\alpha(t). \tag{12}$$

Here ξ_3 is the Stokes parameter that determines the linear polarization of the incident photon.

The polarization 4-vector of the recoil proton is given by the formula

$$a_\mu^{(2)} = -a_\mu^{(1)} + 2t^{-1}q_\mu\,(a^{(1)}p_2), \tag{13}$$

where $q_\mu = (p_2 - p_1)_\mu$ and $a_\mu^{(1)}$ is the polarization 4-vector of the target proton.

From Eq. (13) there follow for the polarization vector ζ_2 of the recoil nucleon the asymptotic ($|z_t| \gg 1$) relations:

$$\zeta_z^{(2)} = -\zeta_z^{(1)}, \quad \zeta_x^{(2)} = \zeta_x^{(1)}, \quad \zeta_y^{(2)} = -\zeta_y^{(1)}. \tag{14}$$

Here the z-axis was taken along the direction of the vector $\mathbf{n}_1$, the x-axis lies in the reaction plane, and the y-axis is perpendicular to the reaction plane. It follows from Eq. (14) that the longitudinal polarization of the recoil nucleon, as well as one of the components of transverse polarization, has a sign opposite to the sign of the corresponding component of the initial nucleon.

In the u channel, i.e. in the channel of the reaction $\pi + N \to N + \gamma$, the expressions for the analogous quantities are obtained by the substitution $s \to u$ (accurate up to unimportant numerical factors related to averaging over photon polarizations).

In this case the relations (13) and (14) for the polarization of the recoil nucleon are preserved. It is easy to show that the photon in such a reaction is fully linearly polarized in the reaction plane, i.e., that the Stokes parameter is

$$\xi_3 = 1. \tag{15}$$

This result is independent of the initial polarization of the nucleons.

B. The pole β is contained in the amplitude f^j_{32}. According to Eqs. (8) and (4) it contributes to the amplitudes f_3 and f_4, and, consequently, to all the A_i. However, taking it into account that f^j_{32} and f_4 appear in front of the functions $P''_j(z_t)$, and that the scalar and pseudoscalar covariants contribute to the cross section by a power s less than the vector and pseudovector, we obtain using (4) for $|z_t| \gg 1$

$$A_{\pi\gamma} = \bar{u}(p_2)\gamma_5 \hat{k}\, u(p_1) \frac{4m\sqrt{t}\,(\mathbf{en}_2) f_3(s, t)}{\sqrt{t-4m^2}\,(m_\pi^2-t)\sin\theta_s},$$

$$f_3 = -\pi\beta C^\pm(\beta)\, r_{32}^\pm(t)\, z_t^{\beta-1}/(1+\beta)\sin\pi\beta, \tag{16}$$

where θ_s is the angle of emission of the photon in the s channel, $r^\pm_{32}(t)$ is the residue of the β pole, $\beta(t)$ is the position of the β pole, and the ± signs correspond to positive or negative signature. Thus the β pole contributes predominantly to the pseudovector covariant.

In this case the expression for the differential cross section does not differ qualitatively from the previous one and is obtained from (12) by the substitution $f_1 \to f_3$.

The polarization 4-vector of the recoil nucleon depends on the polarization 4-vector of the target nucleon according to the following formula:

$$a_\mu^{(2)} = -a_\mu^{(1)} + \frac{(a^{(1)}k)(p_1+p_2)_\mu}{(kp_1)} + \frac{(a^{(1)}p_2)}{(kp_2)} k_\mu - \frac{(a^{(1)}k)(p_1p_2+m^2)}{(kp_1)(kp_2)} k_\mu. \quad (17)$$

From Eq. (17) result the following relations between various components of the polarization vectors of the initial and final nucleons for $|z_t| \gg 1$:

$$\zeta_z^{(2)} = \zeta_z^{(1)}, \qquad \zeta_x^{(2)} = -\zeta_x^{(1)}, \qquad \zeta_y^{(2)} = -\zeta_y^{(1)}. \quad (18)$$

The coordinate axes are chosen here in the same way as in the derivation of Eq. (14).

It is seen from Eq. (18) that the transverse polarization of the recoil nucleon is opposite in sign to the polarization of the initial nucleon, whereas the longitudinal polarization is preserved.

For the u channel the situation is as in the case A, the recoil nucleon polarization being determined as before by Eqs. (17) and (18). The photon in such a reaction is polarized in the reaction plane, i.e. $\xi_3 = 1$ independently of the initial polarization of the nucleons.

C. The pole γ is contained in the amplitudes f_{13}^j and f_{23}^j. Arguing as under B we arrive at the conclusion that the γ pole for $|z_t| \gg 1$ contributes to the amplitudes A_3 and A_4, which in this case have the form, according to Eq. (4),

$$A_3 = \frac{2mz_t}{\sqrt{4m^2-t}}\left[if_2 - \frac{2m}{\sqrt{-t}} f_4\right], \qquad A_4 = \frac{4m}{m_\pi^2 - t} f_4. \quad (19)$$

Then

$$A_{\pi\gamma} = \bar{u}(\mathbf{p}_2)(A_3 - A_4\hat{k})\, u(\mathbf{p}_1)(\mathbf{en})/\sin\theta_s, \quad (20)$$

where $\mathbf{n} = \mathbf{n}_1 \times \mathbf{n}_2$.

Thus the γ pole contributes predominantly to the scalar and vector covariants.

On the basis of Eqs. (19) and (20), and taking into account that the functions A_3 and A_4 have the same complex parts [8] for $t < 0$ and $s \gg m^2$, we obtain the formula for the differential cross section with only the γ pole included, which differs from (12) by the substitutions $|f_1|^2 \to |f_2|^2 + |f_4|^2$ and $\xi_3 \to -\xi_3$, where

$$f_2 = -\pi\gamma C^{\pm}(\gamma)\, \rho_{13}^{\pm}(t)\, z_t^{\gamma-1}/\sin\pi\gamma, \quad (21)$$

$$f_4 = -\pi\gamma C^{\pm}(\gamma)\, r_{23}^{\pm}(t)\, z_t^{\gamma-1}/(1+\gamma)\sin\pi\gamma. \quad (22)$$

Here $\gamma(t)$ is the position of the γ pole, the residue at the γ pole of the amplitude f_{13}^j is $r_{13}^{\pm}(t) = \sqrt{\gamma(\gamma+1)}\rho_{13}^{\pm}(t)$, $r_{23}^{\pm}(t)$ is the residue at the γ pole of the amplitude f_{23}^j, and the $\pm$ signs refer to positive and negative signatures.

From (20) we obtain an expression for the polarization of the recoil nucleon assuming that $(-t)_{min} \ll (-t) \lesssim m^2$:

$$\zeta_z^{(2)} = \frac{(|f_2|^2 - |f_4|^2)\zeta_z^{(1)} - 2\,\mathrm{Im}\, f_2 f_4^* \zeta_x^{(1)}}{|f_2|^2 + |f_4|^2},$$

$$\zeta_x^{(2)} = \frac{(|f_2|^2 - |f_4|^2)\zeta_x^{(1)} + 2\,\mathrm{Im}\, f_2 f_4^* \zeta_z^{(1)}}{|f_2|^2 + |f_4|^2},$$

$$\zeta_y^{(2)} = \zeta_y^{(1)}. \quad (23)$$

Thus if the γ pole dominates for $|z_t| \gg 1$ then the polarization of the nucleon in the plane perpendicular to the reaction plane is preserved.

Passage to the u channel is accomplished, as usual, by the substitution $s \to u$. The polarization of the recoil nucleon for $(-t)_{min} \ll (-t) \lesssim m^2$ is determined by the relation (27). The photon, in contrast to cases A and B, turns out to be polarized in the plane perpendicular to the reaction plane, i.e. the Stokes parameter of the photon is

$$\xi_3 = -1. \quad (24)$$

In conclusion of this section we note that the angles of emission of the pions in the laboratory system in the photoproduction reaction for $(-t) \lesssim m^2$ are of the order of $\theta_L \lesssim \sqrt{-t/\omega_L^2} \approx m/\omega_L$ (ω_L—photon energy in the laboratory system); $\theta_L \approx 10°$ for $\omega_L = 5$ BeV. The situation is the same in the inverse photoproduction process. For not too large photon energies the effects here discussed are fully accessible for experimental studies.

4. PION PHOTOPRODUCTION ON NUCLEONS AT 0 DEGREES

The amplitude for the photoproduction of pions on nucleons in the forward direction ($\cos\theta_s = 1$) is determined, because of the selection of the unique direction, by a single invariant amplitude instead of four. This is easy to understand since of the four independent helicity photoproduction amplitudes [9]

$$\langle \tfrac{1}{2}0|A|\tfrac{1}{2},1\rangle, \langle \tfrac{1}{2}0|A|\tfrac{1}{2},-1\rangle,$$
$$\langle -\tfrac{1}{2}0|A|\tfrac{1}{2},1\rangle, \langle -\tfrac{1}{2}0|A|\tfrac{1}{2},-1\rangle,$$

all but the last one vanish for $\cos\theta_s = 1$ as a consequence of conservation of the component of the total angular momentum along the direction of relative motion.[10]

Expressing the helicity amplitudes in terms of the invariant ones and making use of the behavior of the d-functions for $\cos\theta_s \to 1$ it is not hard to see that all the $A_i(s, t)$ have a kinematic singularity of the form $1/\sin\theta_s$, and that there exist between them the following relations:

$$A_1/A_2 = 2p_1(E_1 + p_1)(E_2 + p_2)/m(E_2 + p_2 - E_1 - p_1),$$

$$A_1/A_3 = -1,$$

$$A_1/A_4 = 2p_1(E_1 + p_1)(E_2 + p_2)/m(E_1 + p_1 + E_2 + p_2). \quad (25)$$

It follows from (25) that for $\cos\theta_s = 1$ the amplitude $A_{\pi\lambda}$ is of the form $A_{\pi\lambda} \sim f(s)\boldsymbol{\sigma}\cdot\mathbf{e}$. Since only one of the invariant functions is independent the polarization states of the final particles are given in terms of only the polarizations of the initial particles.

Thus, for the photoproduction reaction and in the notation of Sec. 3 there follows:

$$\zeta_z^{(2)} = -\frac{\zeta_z^{(1)} + \xi_3}{1 + \zeta_z^{(1)}\xi_3}, \qquad \zeta_x^{(2)} = \frac{\xi_1\zeta_x^{(1)} - \xi_2\zeta_y^{(1)}}{1 + \zeta_z^{(1)}\xi_3},$$

$$\zeta_y^{(2)} = -\frac{\xi_1\zeta_y^{(1)} + \xi_2\zeta_x^{(1)}}{1 + \zeta_z^{(1)}\xi_3}, \quad (26)$$

and for the inverse reaction

$$\xi_3 = \zeta_z^{(1)}, \qquad \zeta_z^{(2)} = -\zeta_z^{(1)}. \quad (27)$$

All remaining components of the vectors $\boldsymbol{\xi}$ and $\boldsymbol{\zeta}^{(2)}$ vanish. It is seen from (27) that the exact formulas for the polarization vectors for $\cos\theta_s = 1$ are substantially different from the asymptotic expressions obtained on the assumption that the process is dominated by some one leading Regge pole. As was indicated above, all the asymptotic formulas are applicable only for $|z_t| \gg 1$ or for $|t| \gg m^2 m_\pi^4/s^2$ and, consequently, are not valid for the description of photoproduction at 0 degrees.

We note that the physical regions in s and t channels of the photoproduction process are determined by the same equation of the form (see [11])

$$s^2 t + t^2 s - st(2m^2 + m_\pi^2) + tm^2(m^2 - m_\pi^2) + m_\pi^4 m^2 = 0. \quad (28)$$

Taking into account Eq. (28) and making use of the law of conservation of the projection of the total angular momentum onto the direction of relative motion in the t channel one can show that the reaction amplitude in the s channel at zero degrees is the analytic continuation in s and t of the reaction amplitude in the t channel at 180°. At that only the partial amplitudes $f_{32}^j(t)$ and $f_{23}^j(t)$ contribute to the invariant amplitudes, and $f_{02}^j(t)$ and $f_{13}^j(t)$ do not contribute and are in this sense special.

On the basis of the results of Gribov and Volkov,[4] who clarified the location of the Regge pole trajectories for $t = 0$, one can satisfy the exact relations (25) by imposing limitations on the behavior of the residues at the poles of the partial photoproduction amplitudes for $t \approx cm^2 m_\pi^4/s^2 \to 0$, $|c| \gg 1$. However experimental confirmation of relations (26) and (27) cannot serve as verification of the picture of location of the poles obtained in [4].

5. DISCUSSION OF RESULTS

The analysis of the amplitudes for photoproduction of pions on nucleons and photon emission in the stopping of fast pions in collisions with nucleons based on the Regge-Gribov method shows that Regge poles with different quantum numbers give rise to qualitatively different polarization effects in the processes here considered. This circumstance allows one to determine experimentally the order in which the Regge poles are distributed depending on their quantum numbers, to show which of the poles give the dominant contribution to the asymptotic photoproduction cross section, and to test the Chew[2] hypothesis on the relative importance of different quantum numbers in the determination of the asymptotic behavior of cross sections for various interaction processes between high-energy particles.

In particular, if the α pole dominates photoproduction at high energies then the longitudinal component of the polarization of the recoil nucleon has opposite sign to that of the target nucleon; if it is the β pole then the longitudinal polarization of the recoil nucleon is the same as the longitudinal polarization of the target nucleon, and the transverse polarization changes sign; if it is the γ pole then the recoil nucleon conserves the magnitude of the polarization of the incident nucleon perpendicular to the reaction plane. The polarization effects for the recoil nucleons in the reaction inverse to photoproduction turn out to be the same, except that in this case the produced photon is fully linearly polarized in the plane of the reaction (α or β poles) or perpendicular to it (γ pole). Photoproduction at zero degrees at any energy is determined by partial amplitudes of type β and γ, which gives rise to substantially different polarization effects than the ones obtained on the basis of a single pole.

In this manner experiments with polarized beams and targets at high energies, namely experiments on determination of the polarization of the secondary particles, are of exceptional importance for the verification of the Regge pole hypothesis and for the study of the pole properties.

The authors are grateful to V. N. Gribov, B. L. Ioffe, L. B. Okun', I. Ya. Pomeranchuk, and K. A. Ter-Martirosyan for useful discussions of questions touched upon in this work.

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Translated by A. M. Bincer
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