THE ENERGY SPECTRUM OF PRIMARY COSMIC RAYS AND THEIR COMPOSITION IN THE ULTRA-HIGH-ENERGY RANGE

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The energy spectrum, composition, and anisotropy factor of ultra-high-energy primary cosmic rays are calculated assuming a diffusion model of the propagation in the galaxy, and taking into account the fragmentation processes during the diffusion from the source to the observation point. It is shown that the change in the spectrum exponent observed at $E \sim 5 \times 10^{15}$ eV can be explained by assuming a definite variation of the diffusion coefficient with the energy and charge of the diffusing particle. It is natural to assume that this corresponds to a definite distribution of the magnetic clouds in the galaxy with respect to lH (where l is the cloud size and H is its field strength). The calculations show also that at $E \sim 10^{17}$ eV there should be an appreciable number of protons and α particles in the primary cosmic radiation, a circumstance which does not contradict the available experimental data. The anisotropy factor calculated on the basis of the model is also consistent with the data.

DETAILED data on the extensive air showers available at present allow us to evaluate the energy spectrum and composition of cosmic rays at ultra-high energies. According to the data, the energy spectrum of primary cosmic rays in the energy range $E \sim 10^5$—10^{16} eV changes relatively fast from $F(>E) \sim E^{-\gamma_1}$ to $F(>E) \sim E^{-\gamma_2}$ where $\gamma_2 - \gamma_1 \approx 0.5$. Moreover, below 10^{15} eV and at energies from $E > 10^{16}$ eV up to $E \sim 10^{18}$ eV the values of γ_1 and γ_2, respectively, remain practically constant. Although the data concerning the composition of primary cosmic rays at ultra-high energies are so far only qualitative, intensive work undertaken in this direction will, it is hoped, soon produce quantitative results.

A possible explanation of the rapid change in the energy spectrum has already been discussed earlier. [1-3] According to [3], the change is due to a definite distribution of the magnetic clouds in the galaxy with respect to the parameter lH, where l is the cloud size, and H is its magnetic field intensity. The expected composition of cosmic rays in the range $E > 10^{16}$ eV has also been calculated in [3] where, however, the change in the composition due to nuclear interactions and fragmentation during the diffusion from the source to the point of observation has been neglected. The effects can, in principle, be essential for both the primary energy spectrum and the composition of primary cosmic rays if the particles traverse a sufficiently thick layer of interstellar matter during the diffusion process. [4] Since this is a very probable case, it is interesting to carry out a more complete calculation, taking the fragmentation into account. This, in turn, requires a more detailed model of the diffusion process of cosmic rays in the galaxy. We shall consider a model of free diffusion of cosmic rays, with the sources at the center of the spherical part of the galaxy. It is known that such a model is the most consistent one with all available data on the low-energy (several BeV/nucleon) cosmic rays.

In the following we shall assume, as in [3], that the observed change in the energy spectrum is wholly due to a change in the coefficient of diffusion of cosmic rays in the interstellar matter. We neglect possible singularities in the energy spectrum of the sources and the contribution of extragalactic cosmic rays.

The main assumptions underlying our calculation are:

1. The sources of ultra-high-energy cosmic rays are concentrated near the center of the galaxy. Their composition is similar to that of low-energy cosmic rays. The energy spectrum of cosmic rays in the sources is of the form $Q(\epsilon) \sim \epsilon^{-(\gamma+1)}$, where ϵ is the energy per nucleon, and $(\gamma+1)$ is the exponent of the differential energy spectrum, which is a constant over the whole energy range under consideration, and is the same for all nuclei.

2. Cosmic rays diffuse freely and isotropically.

Their energy does not change during the diffusion process. The diffusion coefficient D of cosmic rays is independent of the distance r from the center of the galaxy. Up to a critical energy E_{cr}, determined by the magnetic rigidity of the particle, and which is therefore different for different nuclei, the coefficient of diffusion is independent of the particle energy E and is constant. Above E_{cr} we assume that D is a function of the energy, $D = D(E)$:

$$D = D_0 \quad \text{for} \quad E < E_{cr},$$
$$D = D_0 (E/E_{cr})^{\alpha' \ln(E/E_{cr})} \quad \text{for} \quad E_{cr} < E < E'_{cr},$$
$$D = D_0 (E/E_{cr})^{\alpha} \quad \text{for} \quad E > E'_{cr}. \tag{1}$$

where

$$E_{cr,z} = zE_{cr,p}, \qquad \alpha = \alpha' \ln (E'_{cr}/E_{cr})_p, \tag{2}$$

and where z is the nuclear charge and $E_{cr,p}$ is the critical energy for protons.

3. A considerable change in the composition of cosmic rays occurs during the diffusion in the galaxy as a result of the fragmentation and absorption of nuclei. In the energy range $E > E_{cr,p}$ the composition changes both as a result of a different degree of fragmentation (due to a change in the effective thickness of the matter traversed), and because the critical energy is a function of z [see Eq. (2)] and, consequently, the diffusion coefficient D begins to exhibit a dependence on the energy E at different energies for different z. The latter fact causes also the storage factor of cosmic rays in the galaxy (i.e., a quantity proportional to the lifetime of cosmic rays in the galaxy before leaving its boundary) to vary strongly, with z for a given E. In the calculation, the nuclei of the primary cosmic radiation are divided into the following groups: H $(z \geq 10)$, M $(9 \geq z \geq 6)$, L $(5 \geq z \geq 3)$, α $(z = 2)$, and p $(z = 1)$ denoted, respectively, by indices $i = 1, 2, 3, 4, 5$. The fragmentation coefficients p_{ik}, i.e., the probabilities of transition of a nucleus from group i into group k for an absorption mean-free-path λ_i, are assumed to be independent of the energy.

Expressions for the concentration of nuclei of group i with energy ϵ per nucleon in cosmic rays are given in [4], assuming that $D = D_0$ and that it is independent of energy. It is easy to write similar formulae for the concentration $N_i(\epsilon)$ when $D = D(\epsilon)$.

In the case of free diffusion, neglecting fragmentation, the concentration of nuclei of group i with energy ϵ per nucleon decreases with the distance r from the source as

$$N'_i(\varepsilon) = q_i \varepsilon^{-(\gamma+1)} f_i(\varepsilon), \tag{3}$$

where

$$f_i(\varepsilon) = \frac{1}{4\pi r D_i(\varepsilon)} \exp\left\{-\sqrt{\frac{2x}{\lambda_i}}\right\}. \tag{4}$$

and where $x = \rho c r^2/2D_i(\epsilon)$ is the average thickness of matter traversed by cosmic rays before arriving at a distance r from the sources. The factor $[4\pi r D_i(\epsilon)]^{-1}$ takes account of the change in the concentration of nuclei of group i as a function of r and the diffusion coefficient D, the exponential term reflects the variation of the concentration of nuclei of group i as a result of their absorption during the diffusion process, and $q_i\epsilon^{-(\gamma+1)}$ represents the source intensity of nuclei of group i per unit energy interval.

The expression for the concentration of nuclei of group i at a distance r from the source, taking fragmentation into account, is

$$N_i(\varepsilon) = \varepsilon^{-(\gamma+1)} \sum_{k=1}^{i} f_k(\varepsilon) \sum_{l=1}^{k} a_{ikl} q_l, \tag{5}$$

where a_{ikl} is a factor depending on the fragmentation probability p_{ji} and the absorption mean-free-path λ_j. For the fifth group (protons) Eq. (5) does not apply directly, since so far there are no experimental results for p_{i5} $(i < 5)$.

The following assumptions were made to determine the proton concentration:

1) In the collisions between the nuclei and the interstellar matter the energy of the multiply-charged fragmentation products per nucleon ϵ is identical to that of the original nucleus. The energy of the free nucleons produced in the process can, on the average, be k times as small as ϵ.

2) Since the interaction mean free path of protons is relatively large and their energy spectrum is decreasing relatively rapidly, we can neglect the absorption of the secondary protons, and also the contribution of the protons which have interacted along the path from the source or the place where they were produced by fragmentation, to the total number of protons. The number of protons is thus given by the following equation:

$$N_p(\varepsilon) = N'_5(\varepsilon) + \sum_{i=1}^{4} A_i N''_i(k\varepsilon) - \sum_{i=1}^{4} A_i N_i(k\varepsilon), \tag{6}$$

where $N''_i(\epsilon) = q_i\epsilon^{-(\gamma+1)}/4\pi r D_i(\epsilon)$, i.e., differs from $N'_i(\epsilon)$ by the absence of the exponential factor.

For other values of i we have used Eq. (5). The values of the various parameters used in the calculations are given in Tables I and II. The composi-

Table I

Nuclei	H	M	L	α	p
$\bar{A}$	31	14	10	4	1
$q_i(\varepsilon)/q_1(\varepsilon)$	1	2	0	10.8	164
λ_i, g/cm²	6.1	7.8	10	34	72

Table II

Variant	γ	α	E'_{cr}/E_{cr}
I	1.5	0.5	2.7
II	1.5	0.8	2.7
III	1.7	0,5	2.7

tion of cosmic rays at the sources (q_i) given in Table I was taken from [4], where it had been obtained using the most probable set of values of p_{ik} and λ_i, and $x = \rho c r^2/2D_0 = 9$ g/cm² for the variable energy range, i.e., for $D = D_0$. The fragmentation coefficients a_{ikl} corresponding to the chosen set of p_{ik} are:

$a_{111}=1$
$a_{211}=-2.3859$
$a_{221}=2.3859$
$a_{222}=1$
$a_{311}=1.0472$
$a_{321}=-4.3618$
$a_{322}=-1.4091$
$a_{333}=2.3146$
$a_{332}=1.4091$
$a_{333}=1$
$a_{411}=-0.2661$
$a_{421}=0.0902$
$a_{422}=0.03737$
$a_{431}=-4.2636$
$a_{432}=-2.5950$
$a_{433}=-1.8417$
$a_{441}=4.4395$
$a_{442}=2.5576$
$a_{443}=1.8417$
$a_{444}=1$

The integral energy spectrum of primary cosmic rays calculated for different values of γ and α is shown in the figure. Solid lines represent the total integral energy spectrum $\sum_{i=1}^{5} N_i(>5)$ (E being the energy per particle) for different γ and α. Dotted curves represent partial integral energy spectra $N_i(>5)$ ($i = 1, 2, 5$) for $\gamma = 1.5$ and $\alpha = 0.8$. The integral spectra have been obtained by a numerical integration of the differential energy spectra $N_i(E)$ obtained using Eqs. (5) and (6).

The values of the exponent γ' of the primary integral energy spectrum for different energy ranges 1) and different γ and α are given in Table III. For comparison we have included earlier results for $\gamma = 1.5$ and $\alpha = 0.8$ from [3], in which the change in the composition of cosmic rays due to nuclear interactions was neglected, and also the experimental data on γ' obtained recently for energies up to $E \sim 10^{19}$ eV, 2) taking for E_{cr} the value $E_{cr,p} \approx 5 \times 10^{15}$ eV.

Table III. Exponent γ' of the integral primary energy spectrum for different energy ranges

Range	Variant			[3]	Experiment[1,7]	
	I	II	III		$S=1$	$S=1.15$
$\frac{1}{5}E_{cr}-E_{cr}$	1.5	1.5	1.7	1.5	1.5±0.05	1.7±0.05
$E_{cr}-10E_{cr}$	1.80	1.92	1.96	1.87	2.0±0.1	2.2±0.1
$10E_{cr}-100E_{cr}$	1.85	2.0	2.0	2.20	2.2	
$100E_{cr}-1000E_{cr}$	1.90	2.2	2.1	2.3		

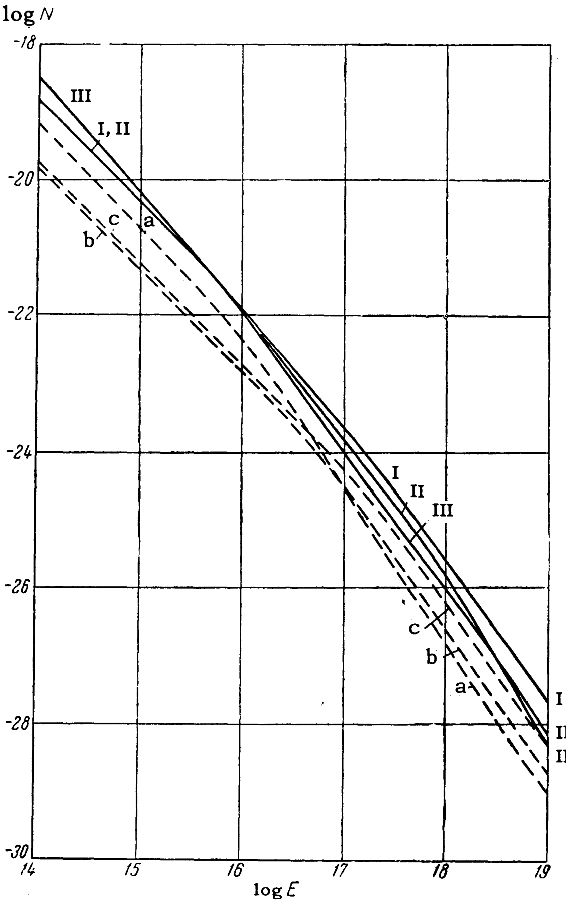
Note. 1. The experimental values of γ' were obtained from the experimental values of the EAS size spectrum exponent κ according to the relation $\gamma' = k\kappa$, where $k = 1.2 \pm 0.1$[3]. 2. The comparison with experiment was made at $E_{cr} \approx E_{cr,p} = 5 \times 10^{15}$eV.

Considering the experimental errors, we find a good agreement between the calculation for $\gamma = 1.5$ and $\alpha = 0.8$ and the experimental results up to $E \sim 10^{19}$ eV. It is clear, however, that the value $\alpha = 0.8$ is the limiting value consistent with the experimental value[5] $\gamma' = 2.3$ at $E \sim 10^{19}$ eV. Hence, it follows in particular that the value of E'_{cr}/E_{cr} cannot be much greater than the assumed one (Table II) since for a sufficiently fast change of the exponent γ' it is necessary that α increase with increasing E'_{cr}/E_{cr}.

Thus, the observed change in the form of the primary energy spectrum can be explained assuming a definite variation of the diffusion coefficient D with the energy. It is most natural to assume

1)The calculation was carried out up to $E = 10^{19}$eV. For protons of such an energy, however, the diffusion model attains the limits of its usefulness, since for $\alpha = 0.8$ the mean free path $l = l(E_{cr})(E/E_{cr})^{\alpha}$ is only several times smaller than the size of the galaxy, if we take $E_{cr,p} \approx 10^{16}$eV and $l(E_{cr}) \approx 10$ parsec. On the other hand, although according to [4] heavy nuclei can be accelerated in the sources up to $E \sim 10^{19}$eV, this is doubtful for protons. Thus, at $E \sim 10^{19}$eV our calculations can serve as a rough approximation only.

2)From the values of the fragmentation coefficient given above we can see that at $E > 10E_{cr}$, account of fragmentation decreases γ'. This can be attributed to the relatively large increase of the fraction of heavy nuclei (see Table III) when fragmentation is taken into account. The energy spectrum, which depends on the distribution with respect to A, becomes somewhat less steep at large E when fragmentation is included.



Integral energy spectra of primary cosmic rays at the top of the atmosphere, calculated under various assumptions: I—$\gamma = 1.5$, $\alpha = 0.5$; II—$\gamma = 1.5$, $\alpha = 0.8$; III—$\gamma = 1.7$, $\alpha = 0.5$. Dashed curves correspond to partial integral energy spectra: a—for group p, b—for group M, c—for group H at $\gamma = 1.5$ and $\alpha = 0.8$.

that the function $D = D(E)$ follows from the distribution of the magnetic clouds in the galaxy with respect to the parameter lH.

It should be noted that from a comparison of the results of the present article and those of [3] it follows that when account is taken of the change in the composition of cosmic rays due to fragmentation the kink in the spectrum near $E_{cr,p}$ becomes more pronounced, and the spectrum in the range $E \approx E_{cr,p} - 100\,E_{cr,p}$ less steep. Nevertheless, the variation of the diffusion coefficient with the energy should not differ much from that assumed in the model neglecting fragmentation.[3] In the scattering cloud model the variation can be explained by a specific distribution of the magnetic clouds in the galaxy with respect to lH, as it was in [3].

From the partial energy spectra $N_i(E)$ we can deduce the composition of primary cosmic rays at different E (see Table IV). A comparison of the results of our calculations with the data of [3] shows that the fragmentation affects considerably the composition of primary cosmic rays at $E > E_{cr}$. At $E < E_{cr}$ cosmic rays near the earth are richer in protons and have relatively few heavy nuclei as compared with the source. At sufficiently large energies ($E_{cr} < E < zE_{cr}$) the diffusion mean free path for heavy nuclei and, therefore, the role of fragmentation and diffusion, do not change; their concentration, therefore, does not change either, while the proton concentration decreases due to the decrease in the storage factor for protons. Thus the fraction of heavy nuclei

Table IV. Composition of primary cosmic rays (in %) on the top of the atmosphere in various energy ranges

Nuclei	Total energy range						Composition at the source
	$E < E_{cr}$	$E_{cr}/2$ $2\,E_{cr}$	$5\,E_{cr}$	$20\,E_{cr}$	$200\,E_{cr}$	$2\cdot10^3\,E_{cr}$	
			Variant I: $\gamma = 1{,}5$, $\alpha = 0{,}5$				
p	49.5	40.5	31	24.6	20.5	16.3	30
α	24	28,6	28.4	21.4	16,3	12,2	16,5
L	2	2.7	3,6	2.7	1,4	1,3	0
M	11.5	13.2	17	21	22,8	26,2	20.3
H	13	15	20	28.5	39	44	33,2
			Variant II: $\gamma = 1{,}5$, $\alpha = 0{,}8$				
p	49.5	37,2	28.8	16.5	10,5	8,4	30
α	24	30,2	27	16	9,8	8	16,5
L	2	2,8	3.6	2,5	0,9	0.4	0
M	11.5	14	19	25	23.8	21	20,3
H	13	15,8	21.6	40	55	62,2	33,2
			Variant III: $\gamma = 1{,}7$, $\alpha = 0{,}5$				
p	38	31,5	26,4	18,5	14,1	11	21
α	24.5	27.4	24,6	18,4	13,8	10,8	14
L	2.8	3,2	3.1	2,8	1.7	0,9	0
M	15	16.4	18.5	23,5	24,5	23,2	22
H	19.7	21,5	27,4	37,1	45,9	54,1	43

Table V. Anisotropy coefficient δ calculated for variant II ($\gamma = 1.5$, $\alpha = 0.8$) for different nuclei [1)]

E, eV	δ_H	δ_M	δ_L	δ_α	δ_p	$\delta_{exp.}$ [8-10]
10^{15}	2.5	2	1.2	1.3	1	1
10^{17}	2.5	3.8	2.2	6.2	12	10
10^{18}	11	17	4.8	36	48	100

δ is given in arbitrary units. For comparison with the experiment, $\delta = 10^{-3}$ was taken as the unit, which corresponds to $D_0 = 2 \times 10^{29}$ cm^2/sec.

somewhat increases. At still higher energies, $E > zE_{cr,p}$, the role of fragmentation is less pronounced because of the decrease in the diffusion mean free path x, as a result of which the composition is more similar to that in the sources, i.e., richer in heavy nuclei. Moreover the storage factor for heavy nuclei remains considerably larger than for protons and light nuclei.[3)] As a result, at these energies cosmic rays near the earth should be richer in heavy nuclei than at the source.

Experiments are being presently carried out to study the composition of primary cosmic rays in the range $E \sim 10^{15} - 10^{16}$ eV.[5,6] According to the authors,[5,6] at $E \sim 10^{17}$ eV the fraction of protons and α particles is considerable, which is in complete agreement with our calculations (Table IV).

For the sake of completeness, let us mention the results concerning the anisotropy coefficient

$$\delta = \frac{3D}{c} \frac{1}{N_i} \frac{dN_i}{dr} = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}, \qquad (7)$$

calculated using our model for nuclei of different groups and for different energies. The results are given in Table V, together with the experimental data for all primary rays in the range 10^{15}—10^{18} eV. It can be seen that the experimental and theoretical values of δ are consistent.

Thus, available experimental data are fully consistent with the hypothesis of a galactic origin of even the ultra-high-energy cosmic rays ($E \sim 10^{18} - 10^{19}$ eV) if we assume a definite variation of the diffusion coefficient D with energy E and charge z of the cosmic-ray particles. It can be seen from the figure that the assumption that protons constitute about 30% of the cosmic-ray nuclei in the sources up to the extreme energy $E \sim 10^{19}$ does not affect greatly the high-energy end of the spectrum, since in that range protons contribute only little to all particles of a given E. In agreement with the observed form of the spectrum and degree of anisotropy, we could therefore assume that there are no protons at all at energies above 10^{18} eV. The final answer will come only from a measurement of the composition in the ultra-high energy range, $E \geq 10^{18}$ eV.

[3)]In the range $E > zE'_{cr,p}$ the diffusion coefficient $D \sim (E/z)^\alpha$, and for a given energy E is therefore z^α times smaller for nuclei with charge z than for protons. The storage factor is proportional to the lifetime in the galaxy and in the diffusion model is therefore proportional to $[D(E)]^{-1}$.

[1] Vernov, Khristiansen, Atrashkevich, Belyaeva, Dmitriev, Kulikov, Nechin, Solov'eva, Fomin, and Khrenov, J. Phys. Soc. Japan 17, Suppl. A3, 118 (1962).

[2] Goryunov, Dedenko, and Zatsepin, Izv. AN SSSR ser. fiz. 26, 685 (1962), Columbia Tech. Transl. p. 684.

[3] Yu. A. Fomin and G. B. Khristiansen, JETP 44, 666 (1963), Soviet Phys. JETP 17, 451 (1963).

[4] V. L. Ginzburg and S. I. Syrovat-skiĭ, Proiskhozhdenie kosmicheskikh lucheĭ (Origin of Cosmic Rays), AN SSSR, 1963.

[5] L. Linsley and L. Scarsi, Phys. Rev. Letters 9, 123 (1962).

[6] Vernov, Solov'eva, Khrenov, and Khristiansen, Proceedings of the First All Union Conference on the Cosmophysical Aspects of Cosmic Rays, AN SSSR (in press).

[7] Linsley, Scarsi, and Rossi, J. Phys. Soc. Japan 17, Suppl. A3, 91 (1962).

[8] G. Clark, Phys. Rev. 108, 450 (1957).

[9] Clark, Earl, Kraushaar, Linsley, Rossi, Scherb, and Scott, Phys. Rev. 122, 637 (1961).

[10] Linsley, Scarsi, Eccles, and Rossi, Phys. Rev. Letters 8, 286 (1962).

Translated by H. Kasha

ERRATA

Volume 16 (Russ. v. 43)

No. 1, p. 81 (Russ. p. 112), article by B. M. Smirnov.

The article contains an error. In the calculation of the matrix element $(\partial H/\partial t)_{km}$ contained in the formula of the adiabatic perturbation theory, an error was made in the sign of one of the terms, leading to a non-zero result, and the order of the expansion in the small parameter is lower than actual. A corrected paper will be published in "Optika i spekroskopiya."

Volume 17 (Russ. v. 44)

No. 2, p. 518 (Russ. p. 766), article by E. P. Shabalin

Right hand side of Eq. (3) should read

$$\frac{f_1 f_2 G^2 \sin(\varphi_0 - \varphi_1)}{2^8 \pi^4 7!! M}(Q^2 - 4m^2)(M - Q)^5\left(1 + \frac{5Q}{M} + \frac{Q^2}{M^2}\right)$$

No. 5 p. 999 (Russ. p. 1485), article by D. K. Kopylova et al.

Caption to Fig. 7 should read:

Distribution of two-prong stars by "target mass": Continuous histogram - cases with $M_X^2 > 0$, dashed - with $M_X^2 < 0$.

Volume 18 (Russ. v. 45)

No. 4, p. 1100 (Russ. p. 1598), article by S. I. Syrovat-skiĭ et al.

Values of the fragmentation coefficient: in place of $a_{321} = -4.3618$ read $a_{321} = -3.3618$.