SOVIET PHYSICS JETP VOLUME 18, NUMBER 4 APRIL, 1964

ELECTRODYNAMIC INTERACTION OF A GROUP OF CHARGES DURING THE SCATTERING OF RADIATION

V. V. YANKOV

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

Submitted to JETP editor May 22, 1963

J. Exptl. Theoret. Phys. (U.S.S.R.) **45**, 1634-1637 (November, 1963)

We investigate the distribution of light pressure over the individual elements of a scattering system of free charges comprising two electrons. It is shown that dissipative radiation pressure due to electron interaction is completely concentrated on the rear particle (relative to the direction of propagation of the incident wave). If the distance between the charges is not too much greater than the wavelength, then the indicated pressure is always positive that is, it tends to make the rear electron approach the front one. Short-range quasistationary radiative forces which predominate at short distances lead to mutual repulsion of a pair of close electrons in the case of transverse polarization of the scattered wave and to mutual attraction in the case of longitudinal polarization.

THE scattering of light by a system of free electrical charges is accompanied, as is well known, by pressure on the system in the direction of radiation propagation. The purpose of the present article is to clarify some specific singularities in the distribution of the radiation pressure over the individual elements of the scattering aggregate of charged particles 1). The first to show interest in this problem was Veksler[3].

We consider as the simplest case a group of charges consisting of two electrons oscillating in the field of a monochromatic wave of frequency ω at a distance a from each other, as shown in Fig. 1. It is obvious that it is sufficient to determine for each electron separately the radiation force due to the interaction between the electrons only. The radiation pressure forces due to the identical reaction of each electron on each other, i.e., to the intrinsic radiation friction, are equally distributed between the electrons and are therefore disregarded. We likewise disregard the purely static Coulomb interaction of the electrons, the influence of which can be eliminated by adding charges of opposite sign (quasi-neutral systems).

Assuming that the instantaneous velocity of the charges is small compared with the velocity of light c, we confine ourselves in writing down the equation of motion of the electron in the high frequency field, as usual, to the electric force $m\ddot{\mathbf{r}} = e\mathbf{E}$, which includes, of course, the radiation-indúced field of the neighboring electron. Knowing thus the electric dipole moment $\mathbf{p} = e\mathbf{r}$ of the particle, we can represent the sought radiation force applied to the electron in standard form $\mathbf{F} = \overline{(\mathbf{p}\cdot\nabla)\mathbf{E} + \dot{\mathbf{p}} \times \mathbf{H}/c}$, where the bar stands for the time-averaging operation. Following the method of successive approximations, we put $\mathbf{E} = \mathbf{E}^{(0)} + \mathbf{E}^{(1)}$, $\mathbf{E}^{(1)} \ll \mathbf{E}^{(0)}$, etc.

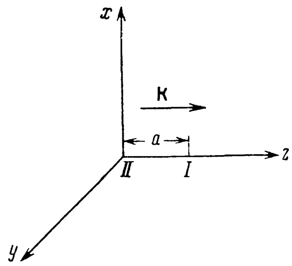

FIG. 1

The corrections to the external field (zeroth approximation), which take into account the presence of the charges, coincide in first approximation with the fields of auxiliary point oscillators, the moment of which $p_0 = p^{(0)}$ is given by the forced oscillations of the electrons in the field of the external wave. The radiation itself is calculated accurate to small quantities of order r_0^2, where $r_0 = e^2/mc^2$ is the classical radius of the electron. The criterion for the applicability of the theory are the inequalities r_0/a, $r_0/a\rho$, $r_0/a\rho^2 \ll 1$, which are practically always satisfied (ρ is the

1)The radiative interaction of electrons at relativistic velocities in accelerators was considered by Tamm.[1] Bolotovskiĭ[2] studied the distribution of radiation forces in the passage of a group of charges inside a dielectric.

dimensionless distance of the interacting electrons and $a/\lambda\!\!\!^{-} = ka$ $(k = \omega/c)$.

We shall henceforth designate individual physical quantities belonging to the front electron (relative to the incident wave) by a subscript I, and the rear electron by II. The time factor $e^{-i\omega t}$ is omitted throughout.

We stop to discuss two specific examples.

1. Plane linearly polarized (transverse wave), $E_x = H_y = E_0e^{ikz}$. In accordance with the foregoing, we have

$$E^{(0)}_{\mathrm{II}x} = H^{(0)}_{\mathrm{II}y} = E_0, \quad E^{(0)}_{\mathrm{I}x} = H^{(0)}_{\mathrm{I}y} = E_0e^{ika}, \quad p^{(0)}_{\mathrm{II}x} = -(r_0/k^2)E_0,$$

$$p^{(0)}_{\mathrm{I}x} = -(r_0/k^2)E_0e^{ika}, \qquad E^{(1)}_{\mathrm{I}x} = E_0\frac{r_0}{k^2}\left(\frac{1}{a^3} - \frac{ik}{a^2} - \frac{k^2}{a}\right)e^{ika},$$

$$H^{(1)}_{\mathrm{I}y} = -E_0\frac{-ir_0}{k}\left(\frac{1}{a^2} - \frac{ik}{a}\right)e^{ika}, \qquad E^{(1)}_{\mathrm{II}x} = E^{(1)}_{\mathrm{I}x}e^{ika},$$

$$H^{(1)}_{\mathrm{II}y} = -H^{(1)}_{\mathrm{I}y}e^{ika}, \quad p^{(1)}_{\mathrm{I}x} = -(r_0/k^2)E^{(1)}_{\mathrm{I}x}, \quad p^{(1)}_{\mathrm{II}x} = -(r_0/k^2)E^{(1)}_{\mathrm{II}x}.$$

The summary formulas for the electrodynamic radiation interaction forces for the electrons in the field of a plane wave assume the form

$$F^{\perp}_{\mathrm{I}} = \tfrac{3}{2}F_0\,(3/\rho^4 - 1/\rho^2),$$

$$F^{\perp}_{\mathrm{II}} = -\tfrac{3}{2}F_0\,(3\rho^{-4}\cos 2\rho + 4\rho^{-3}\sin 2\rho - 3\rho^{-2}\cos 2\rho - 2\rho^{-1}\sin 2\rho),$$

where $F_0 = r_0^2E_0^2/3$ denotes the radiation pressure on a single (isolated) electron. Plots of the functions $F^{\perp}_{\mathrm{I}}$ and $F^{\perp}_{\mathrm{II}}$ and their sum $F^{\perp}_{\mathrm{I}} + F^{\perp}_{\mathrm{II}}$ are shown in Fig. 2 (curves I, II, and III, respectively). In the limiting coherent case $\rho \ll 1$ we obtain, after expanding $\cos 2\rho$ and $\sin 2\rho$ in power series, the asymptotic formula

$$F^{\perp}_{\mathrm{I}} + F^{\perp}_{\mathrm{II}} = 2F_0\,[1 - 11\rho^2/10 + \ldots].$$

2. Longitudinal wave of the electric type, $E_z = E_0e^{ikz}$. After calculating the radiation force in analogy with Sec. 1, we arrive at the final formulas

$$F^{\parallel}_{\mathrm{I}} = -3F_0\cdot 3/\rho^4,$$

$$F^{\parallel}_{\mathrm{II}} = 3F_0\,(3\rho^{-4}\cos 2\rho + 4\rho^{-3}\sin 2\rho - 2\rho^{-2}\cos 2\rho).$$

The behavior of the functions $F^{\parallel}_{\mathrm{I}}$, $F^{\parallel}_{\mathrm{II}}$, and $F^{\parallel}_{\mathrm{I}} + F^{\parallel}_{\mathrm{II}}$ is illustrated by Fig. 3 (curves I, II, and III). For $\rho \ll 1$ the sum can be replaced by the series $F^{\parallel}_{\mathrm{I}} + F^{\parallel}_{\mathrm{II}} = 2F_0\,[1 - 4\rho^2/5 + \ldots]$.

Summarizing the results obtained, we state, first, that charges I and II are in non-equivalent states. Whereas quasi-static forces $\sim\rho^{-4}$, which obey the "action equal to reaction" principle, prevail at close distances $\rho \ll 1$, the singular position of the rear electron (II) with respect to the distribution of the light pressure on the system as a whole between the particles becomes noticeable already when $\rho \sim 1$. After subtracting from F_{I} and F_{II} the symmetrically distributed internal quasi-stationary forces, we are left with the external radiation (wave) pressure proper $F_{\mathrm{I}} + F_{\mathrm{II}}$, due exclusively to the interference process and concentrated completely on the rear electron. In the case of a pair of close electrons $(\rho \ll 1)$ the indicated pressure is equal to $2F_0$.

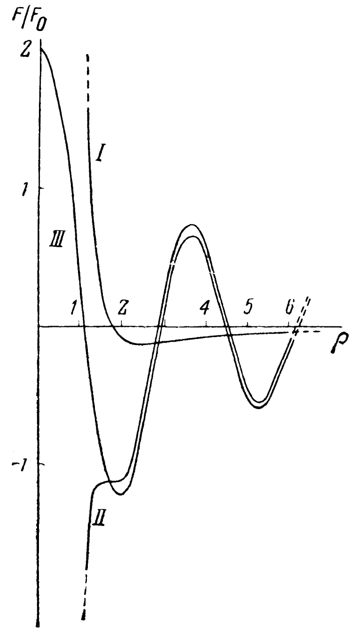

FIG. 2

Generalization to the case of a one dimensional system of N particles entails no difficulty: the

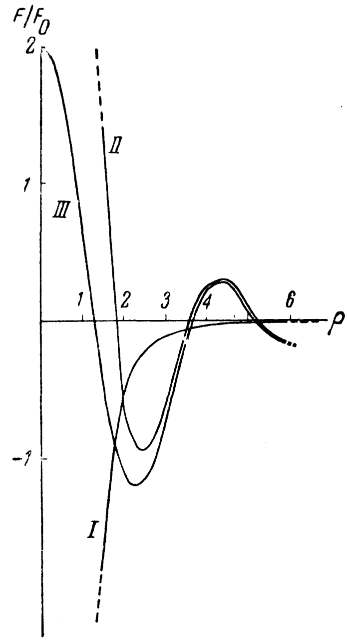

FIG. 3

pressure on the $\acute{N}$-th particle amounts to $2(N-1)F_0$, i.e., it increases uniformly towards the end of the chain of charges. Whereas the dissipative radiation forces always tend to reduce the distance between the closely lying charges ($\rho \lesssim 1$), the character of interaction of the quasi-stationary forces depends essentially on the polarization of the scattered wave. A transverse (plane) wave leads to a mutual repulsion of the charges at relatively short distance, whereas a longitudinal wave leads always to mutual attraction. Thus, in the latter case there are on hand additional favorable factors contributing to longitudinal focusing of a group of charged particles by the radiation forces.

It is easy to show that perfectly analogous effects occur in the case of dipole-magnetic scattering of a magnetic longitudinal wave by a group of charges.

In conclusion, the author is indebted to Academician V. I. Veksler and to Professor M. S. Rabinovich for a useful discussion.

[1] I. E. Tamm, Trudy, Phys. Inst. Acad. Sci. 18, 3 (1962).

[2] B. M. Bolotovskiĭ, Dissertation, Phys. Inst. Acad. Sci. 1954.

[3] V. I. Veksler, Atomnaya énergiya 2, 427 (1957).

Translated by J. G. Adashko
262